Name: Date:

Topic: **APPLICATIONS OF INTEGRATION**

Question 1 [3 + 1 + 3 = 7 marks]

The acceleration of a particle undergoing rectilinear motion is given by

$$a = \frac{2}{\sqrt{t+4}} \ ms^{-2}.$$

The particle has a velocity of 12 ms^{-1} when t = 5

Find:

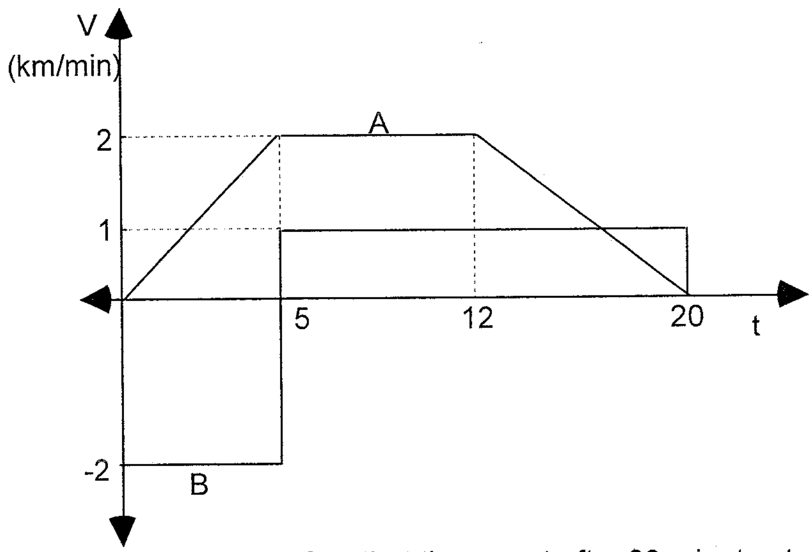
(a) the velocity when t = 12.

(b) if and when the particle is at rest.

(c) the distance covered by the particle in the first 5 seconds.

Question 2 [1 + 1 + 1 + 2 = 5 marks]

The velocity - time graph below shows the journey taken by two different cyclists, A and B, along the same straight stretch of road.



Use the v - t graph and the fact that they meet after 20 minutes to find:

(a) the acceleration of A between t = 0 and t = 5.

(b) the displacement of B from his starting position after 20 mins.

(c) the total distance travelled by A.

(d) the distance apart the cyclists were initially.

Question 3 [4 marks]

A region is bounded by the curve $y = \ln(x+1)$, the line $x = 1$ and the x – axis. Find the volume of the solid of revolution formed when this region is rotated about the y – axis.

Question 4 [3 + 3 + 1 + 2 + 1 = 10 marks]

A train slows down with an acceleration which is proportional to its velocity.

(a) Show that the velocity at any time t is given by $v(t) = v_0 e^{kt}$, where v_0 is the initial velocity.

Given that initially the particle has a velocity of 60 km h^{-1}, and that the velocity after 5 seconds is 40 km h^{-1}, then:

(b) show that the value of k is $\frac{1}{5}\ln\frac{2}{3}$.

Hence, find:

(c) the velocity after 10 seconds.

(d) the time taken to reduce the velocity to 20 km h^{-1}

(e) the acceleration after 5 seconds.

Question 5 [2 + 2 = 4 marks]

A particle is moving in a straight line, its velocity at any time t, is given by

$$v = 6\cos 3t$$

The particle is initially at the origin.

(a) Find the displacement at any time t.

(b) Show that this particle is undergoing Simple Harmonic Motion.

(7 + 5 + 4 + 10 + 4 = 30 marks)

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Question 1

(a) $v(t) = \int \frac{2dt}{\sqrt{t+4}} = 4\sqrt{t+4} + c$ [1]

when $t = 5, v = 12 \Rightarrow c = 0$ [1]

$v(12) = 16 \text{ ms}^{-1}$ [1]

(b) particle is never at rest since $4\sqrt{t+4} \neq 0$ [1]

(c) distance travelled $= \int_0^5 4\sqrt{t+4}\,dt$ [1]

$= \frac{8}{3}(t+4)^{\frac{3}{2}}\Big]_0^5$ [1]

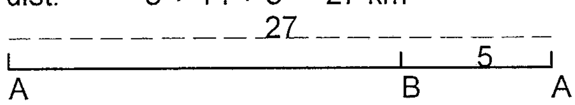
$= 50\frac{2}{3}$ m [1]

Question 2

(a) $a = \frac{\text{rise}}{\text{run}} = 0.4 \text{ km min}^{-2}$ [1]

(b) $x = -10 + 15 = 5$ km to the right of his starting point. [1]

(c) dist. $= 5 + 14 + 8 = 27$ km [1]



(d) A and B finish [1]

initially 22 km apart [1]

Question 3

$$V_y = \pi\int_0^{\ln 2} 1\,dy - \pi\int_0^{\ln 2}\left(e^y - 1\right)^2 dy \quad [1]$$

$$= \pi\int_0^{\ln 2} 1\,dy - \pi\int_0^{\ln 2}(e^{2y} - 2e^y + 1)\,dy$$

$$= \pi\int_0^{\ln 2}(2e^y - e^{2y})\,dy \quad [1]$$

$$= \pi\left[2e^y - \frac{1}{2}e^{2y}\right]_0^{\ln 2} \quad [1]$$

$$= \pi\left((4-2) - (2 - \frac{1}{2})\right) = \frac{\pi}{2} \quad [1]$$

Question 4

(a) $\frac{dv}{dt} = kv$ [1]

$\ln|v| = kt + c$ [1]

$v = e^{kt+c}$ [1]

$v = V_o e^{kt}$

(b) $40 = 60e^{5k}$ [1]

$\ln e^{5k} = \ln\frac{2}{3}$ [1]

$5k = \ln\frac{2}{3}$ [1]

$k = \frac{1}{5}\ln\frac{2}{3}$

(c) $v = 60\, e^{\frac{1}{5}\ln\frac{2}{3}(10)} = 26.7$ (1dec.pl) [1]

(d) $20 = 60\, e^{\frac{1}{5}\ln\frac{2}{3}t}$ [1]

$t = 13.6$ (1dec.pl) [1]

(e) $a = kv(5) = \frac{1}{5}\ln\frac{2}{3}\,40 = -3.2$(1dec.pl) [1]

Question 5

(a) $x = 2\sin 3t + c$ [1]

when $t = 0$, $x = 0 \Rightarrow c = 0$ [1]

$\therefore\ x = 2\sin 3t$

(b) $a = -18\sin 3t$ [1]

$= -9\{2\sin 3t\}$

$= -n^2x$ [1]

$\therefore$ S.H.M.

(7 + 5 + 4 + 10 + 4 = 30 marks)