Applications of Series and Sequences – Financial Maths

1) Calculate the interest earned on $5000 if it is invested for 4 years at 12% p.a. and interest compounds monthly?
2) A car valued at $35 000 depreciates at a rate of 5% p.a. What will be its value after 7 years?
3) An investor has $40 000 to invest. Her goal is to make it grow to an amount of $90 000.
   a) If she invests for 10 years, what rate of interest p.a. must she secure? (Assume interest compounds monthly)
   b) If she can only secure 9% p.a. (int. comp. Monthly), how long must she invest for?
   c) Because the Banks could not offer an attractive lump sum investment service, she decides on a different course of action: She invests an amount on the 1st January and the 1st July every year over an 8 year period. She secures an interest rate of 6% p.a. with interest compounding every 6 months. How much should she invest each time so that she reaches her goal of $90 000?
4) A couple borrows $50 000 at 6% p.a. (comp. Monthly) They wish to repay the loan over 12 years.
   a) Show that the monthly repayment is given by:

$$M = \frac{50\,000\,(1\cdot005)^{144}\cdot 0\cdot005}{1\cdot005^{144} - 1}$$

   b) Calculate the equivalent simple interest rate p.a.
5) A builder borrows $250 000 which is to be repaid over a period of 10 years. He is charged 9% p.a. interest (interest comp. Monthly).
   a) In the first 2 years, he wishes to pay off an amount of $80 000. If he makes equal monthly instalments, what should each instalment be?
   b) After the first two years, he decides to repay the remainder over the eight year period in equal bi-annual instalments. What should each instalment be?

6) An investor contributes to a particular fund which requires you to initially invest $1000 then 4/5 of this amount in the second year. The third year requires you to invest 4/5 of the previous investment and so on until one's death. Show that the total will never exceed $5000.

Solutions

1) $A = P(1+r)^n$

$= 5000(1.01)^{48}$

$= \$8061.13$

$\therefore \text{Int} = \$3061.13$

2) $A = P(1-r)^n$

$= 35000(0.95)^7$

$= \$24\,441.81$

3) a) $90\,000 = 40\,000(1+r)^{120}$

$2.25 = (1+r)^{120}$

$\therefore 1+r = \sqrt[120]{2.25}$

$= 1.006780637$

$\therefore r = 0.006780637$ p.m

$= 0.081367643$ p.a

$= 8.14$ (2dp) % p.a.

b) $90\,000 = 40\,000(1.0075)^n$

$2.25 = 1.0075^n$

$\ln 2.25 = \ln 1.0075^n$

$\therefore n = \dfrac{\ln 2.25}{\ln 1.0075}$

$= 108.53$ months

$\doteqdot 9$ yrs

c) $A_1 = P(1.03)^{16}$ $\quad \therefore \text{Total} = P(1.03 + 1.03^2 + \ldots + 1.03^{16})$

$A_2 = P(1.03)^{15}$ $\quad = P\left[\frac{1.03(1.03^{16}-1)}{0.03}\right]$

$\vdots$

$A_{16} = P(1.03)^1$ $\quad = P(20.76158774)$

let Tot = \$90 000

$\therefore P = \$4334.93$ every 6 months.

4)a) $A_1 = 50000(1.005) - M$

$A_2 = [50000(1.005) - M] \times 1.005 - M$

$= 50\,000(1.005)^2 - 1.005m - m$

$= 50\,000(1.005)^2 - m(1.005 + 1)$

$A_3 = [50\,000(1.005)^2 - m(1.005+1)] \times 1.005 - m$

$= 50000(1.005)^3 - m(1.005^2 + 1.005) - m$

$= 50\,000(1.005)^3 - m(1.005^2 + 1.005 + 1)$

$\therefore A_{144} = 50\,000(1.005)^{144} - m(1.005^{143} + \ldots + 1)$

$= 50\,000(1.005)^{144} - m\left[\frac{1((1.005)^{144} - 1)}{1.005 - 1}\right]$ using $S_n = \frac{a(r^n - 1)}{r-1}$

let $A_{144} = 0$

$\therefore 50\,000(1.005)^{144} = \frac{m[(1.005)^{144} - 1]}{0.005}$

$\therefore m = \frac{50000 \ (1.005)^{144} . 0.005}{1.005^{144} - 1}$

b) $M = \$487.93$ per month

$\therefore$ 12 yrs, Total repaid = \$70 261.22

$\therefore$ Total interest = \$20 261.22

$\therefore$ Int. p.a = \$1688.43

$\therefore$ Int % p.a = 3.38 % p.a. (2dp)

5) $A_1 = 250\,000(1.0075) - M$

A_2, A_3, etc. (Use same working as for Q4)

$$\therefore A_{24} = 250\,000(1.0075)^{24} - M(1 + 1.0075 + \ldots + 1.0075^{23})$$

Let $A_{24} = 80000$

$$\therefore 170\,000 = 250\,000(1.0075)^{24} - M\left(\frac{1.0075^{24} - 1}{0.0075}\right)$$

$$\therefore M = \frac{\left[250\,000(1.0075)^{24} - 170\,000\right]0.0075}{1.0075^{24} - 1}$$

$= \$4929.78$

b) P.T.O.

b) Amount owing = \$170 000

Time Period = 8 yrs

$A_1 = 170\,000(1.0075)$

$A_2 = [170\,000(1.0075)] \times 1.0075$

$= 170\,000(1.0075)^2$

$A_6 = 170\,000(1.0075)^6 - m$

$A_7 = [170\,000(1.0075)^6 - m] \times 1.0075$

$= 170\,000(1.0075)^7 - 1.0075m$

$A_{12} = [170\,000(1.0075)^{11} - 1.0075^5 m] \times 1.0075 - m$

$= 170000(1.0075)^{12} - 1.0075^6 m - m$

$= 170\,000(1.0075)^{12} - m(1.0075^6 + 1)$

$\therefore A_{96} = 170\,000(1.0075)^{96} - m(1 + 1.0075^6 + \ldots + 1.0075^{90})$

let $A_{96} = 0$

$$170\,000(1.0075)^{96} = m\left[\frac{1.0075^{6\times16} - 1}{1.0075^6 - 1}\right]$$

$$\therefore m = \frac{170\,000(1.0075)^{96}(1.0075^6 - 1)}{1.0075^{96} - 1}$$

= \$15 226.21 every 6 months.

6) $S_\infty = \dfrac{a}{1-r}$

$= \dfrac{1000}{1 - 4/5}$

= \$5000