Assignment 11 - Integrals, Areas + Volumes

1 Find the general primitive for:

(a) $\sqrt{x^3} - \dfrac{2}{\sqrt{x}}$

(b) $(3x-1)^2$.

2 Find the following integrals:

(a) $\int 7\,dx$

(b) $\int_{-2}^{-1} \dfrac{y^3+2}{y^2}\,dy$

(c) $\int_{-1}^{1} 2\pi r^2(1-r^2)\,dr$.

3 (a) Find an approximate value for the definite integral

$$\int_0^1 \frac{3}{(x+1)^2}\,dx$$

using the trapezoidal rule with three function values.

If the exact value of the integral is 1.5, then calculate:

(b) the relative error of the answer in part (a)

(c) the percentage error of the answer in part (a).

4 The curve $y = f(x)$ passes through (2, 1) and the gradient function is given by:

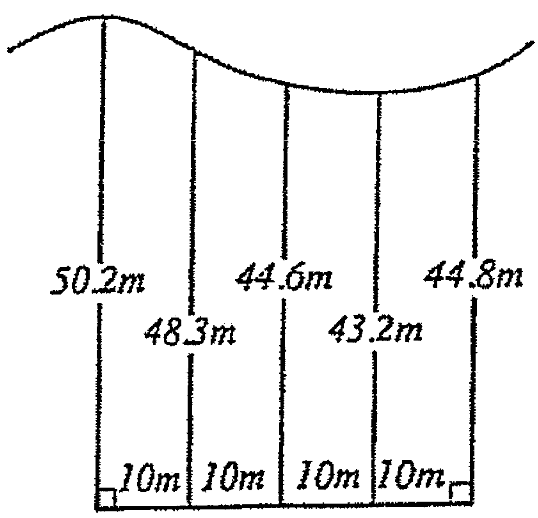
$$f'(x) = 3x^2 - 2x + 1$$

(a) Find the value of $f(x)$.

(b) Find the exact value of $\int_{-2}^{2} (3x^2 - 2x + 1)\,dx$.

(c) Evaluate $f(0)$ and $f'(2)$.

5 A surveyor calculates the area of a block of land which is bounded on one side by a winding river. Use Simpson's rule in conjunction with all of the measurements made to calculate the area as accurately as possible.



6 (a) Evaluate $\int_{-1}^{2} x^3\,dx$.

(b) Find the area bounded by the graph of $y = x^3$, the x-axis and the lines $x = -1$ and $x = 2$.

(c) Explain why the answers for (a) and (b) are different.

7 (a) Find the area enclosed by the y-axis, the line $y = 3$ and the curve $x = y^2$.

(b) This area is then rotated 360 degrees about the y-axis. Find the volume of the solid of revolution that is formed.

8 Find the area enclosed by the curves $y = x(x-1)$ and $y = x(2-x)$.

ASSIGNMENT 11

(1) (a) $\int x^{\frac{3}{2}} - 2x^{-1/2}\,dx$

$= \dfrac{x^{\frac{5}{2}}}{\frac{5}{2}} - \dfrac{2x^{\frac{1}{2}}}{\frac{1}{2}} + c$

$= \underline{\frac{2}{5}x^{\frac{5}{2}} - 4x^{\frac{1}{2}} + c}$

(b) $\int (3x-1)^2\,dx$

$= \dfrac{(3x-1)^3}{3\times 3} + c$

$= \underline{\frac{1}{9}(3x-1)^3 + c}$

(2) (a) $\int 7\,dx = \underline{7x + c}$

(b) $\int_{-2}^{-1} \dfrac{y^3}{y^2} + \dfrac{2}{y^2}\,dy$

$= \int_{-2}^{-1} y + 2y^{-2}\,dy$

$= \left[\frac{y^2}{2} + \frac{2y^{-1}}{-1}\right]_{-2}^{-1}$

$= \left[\frac{1}{2}(-1)^2 - 2(-1)\right] - \left[\frac{1}{2}(-2)^2 - 2(-2)\right]$

$= \left[\frac{1}{2} + 2\right] - \left[2 + 4\right]$

$= \underline{3\frac{1}{2}}$

(c) $\int_{-1}^{1} 2\pi\,(r^2 - r^4)\,dr$

$= 2\pi\left[\frac{r^3}{3} - \frac{r^5}{5}\right]_{-1}^{1}$

$= 2\pi\left[\left(\frac{1}{3} - \frac{1}{5}\right) - \left(-\frac{1}{3} + \frac{1}{5}\right)\right]$

$= \underline{\frac{8\pi}{15}}$

(3) (a) $\int_0^1 \dfrac{3}{(x+1)^2}\,dx$

x	0	$\frac{1}{2}$	1
y	3	$\frac{4}{3}$	$\frac{3}{4}$

$A_{Trapezoidal} = \dfrac{\frac{1}{2}}{2}\left[\left(3 + \frac{3}{4}\right) + 2\left(\frac{4}{3}\right)\right]$

$= 1.604\ldots$

(b) Relative error $= 1.604 - 1.5 = 0.104$ (to 3 dp)

(c) Percentage error $= \dfrac{0.104}{1.5} \times 100\%$

$= 6.9\%$ (to 1 dp).

Assignment 11

(4) $f'(x) = 3x^2 - 2x + 1$

(a) $f(x) = \dfrac{3x^3}{3} - \dfrac{2x^2}{2} + x + c$

$\therefore f(2) = 2^3 - 2^2 + 2 + c = 1$

$c = 1 - 6 = -5$

$\therefore \underline{f(x) = x^3 - x^2 + x - 5}$

(b) $\displaystyle\int_{-2}^{2} (3x^2 - 2x + 1)\, dx$

$= \Big[x^3 - x^2 + x \Big]_{-2}^{2}$

$= (8 - 4 + 2) - (-8 - 4 - 2)$

$= \underline{20}$

(c) $f(0) = \underline{-5}$ and $f'(2) = 3(2)^2 - 2(2) + 1$

$= \underline{9}$

(5) $A_{\text{simpson's}} = \dfrac{h}{3}\left[y_0 + y_4 + 4(y_1 + y_3) + 2(y_2) \right]$

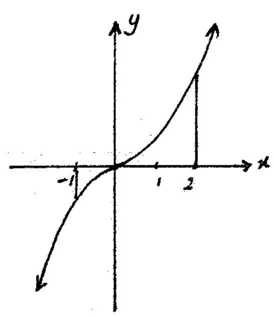
$= \dfrac{10}{3}\left[50.2 + 44.8 + 4(48.3 + 43.2) + 2(44.6) \right]$

$= \underline{1834 \text{ m}^2}$

(6) (a) $\left[\dfrac{x^4}{4} \right]_{-1}^{2} = \dfrac{1}{4}\left[(2^4) - (-1)^4 \right]$

$= \dfrac{1}{4}\left[16 - 1 \right] = \underline{\dfrac{15}{4}} = \underline{3\tfrac{3}{4}}$

(b) Area $= \left| \displaystyle\int_{-1}^{0} x^3\, dx \right| + \displaystyle\int_{0}^{2} x^3\, dx$



$= \left| \left[\dfrac{x^4}{4} \right]_{-1}^{0} \right| + \left[\dfrac{x^4}{4} \right]_{0}^{2}$

$= \left| (0) - \left(\dfrac{(-1)^4}{4} \right) \right| + \left(\dfrac{2^4}{4} - 0 \right)$

$= \dfrac{1}{4} + 4$

$= \underline{4\tfrac{1}{4}}$

(6) (c) Answer in part (a) is just finding the integral with no consideration for negative + positive area.
Part (b) is the approach to finding "areas under a curve".

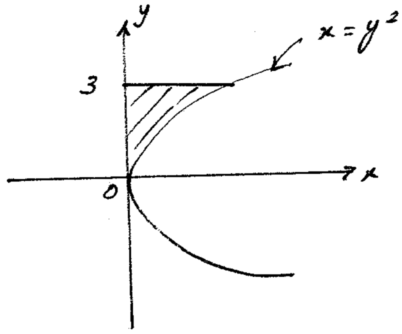
(7) (a) Shaded area

$$= \int_0^3 x \, dy$$

$$= \int_0^3 y^2 \, dy$$

$$= \left[\frac{y^3}{3} \right]_0^3 = \frac{3^3}{3} - 0$$

$$= \underline{9 \text{ sq. units}}$$



(b) Volume req^d. $= \pi \int x^2 \, dy$

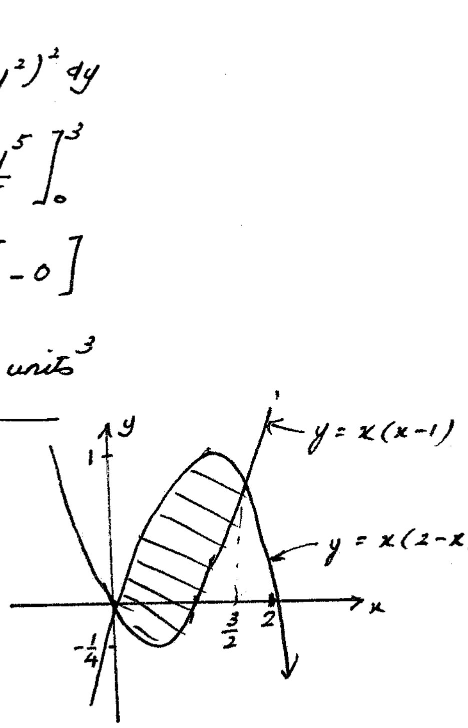
$$= \pi \int_0^3 (y^2)^2 \, dy$$

$$= \pi \left[\frac{y^5}{5} \right]_0^3$$

$$= \pi \left[\frac{3^5}{5} - 0 \right]$$

$$= \underline{\frac{243\pi}{5} \text{ units}^3}$$

(8) Solve for points of intersection first



$$x(x-1) = x(2-x)$$

$$x^2 - x = 2x - x^2$$

$$\therefore 2x^2 - 3x = 0$$

$$\therefore x(2x-3) = 0$$

$$\therefore x = 0 \text{ or } \frac{3}{2}$$

Area shaded $= \int_0^{\frac{3}{2}} x(2-x) - x(x-1) \, dx$

$$= \int_0^{\frac{3}{2}} 2x - x^2 - x^2 + x \, dx$$

$$= \int_0^{\frac{3}{2}} -2x^2 + 3x \, dx = \left[-\frac{2x^3}{3} + \frac{3x^2}{2} \right]_0^{3/2}$$

$$= \left[-\frac{2}{3}\left(\frac{3}{2}\right)^3 + \frac{3}{2}\left(\frac{3}{2}\right)^2 - 0 \right]$$

$$= \frac{9}{8} = \underline{1\frac{1}{8} \text{ units}^2}$$