SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

2013

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics

General Instructions

- Reading time – 5 minutes.
- Working time – 180 minutes.
- Write using black or blue pen.
- Board approved calculators may be used.
- Show all necessary working in Questions 11–16

Total Marks - 100 Marks

Section I **10 Marks**

- Attempt Questions 1–10
- Allow about 15 minutes for this section.

Section II **90 Marks**

- Attempt Questions 11–16
- Allow about 2 hour 45 minutes for this section.

Examiner: *External Examiner*

This is an assessment task only and does not necessarily reflect the content or format of the Higher School Certificate.

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

Section I **Objective-response Questions**
Total marks – 10
Attempt Questions 1 – 10

Answer each question on the multiple choice answer sheet provided.

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1) $\frac{\tan^2\theta}{1+\tan^2\theta}+\cos^2\theta$ equals

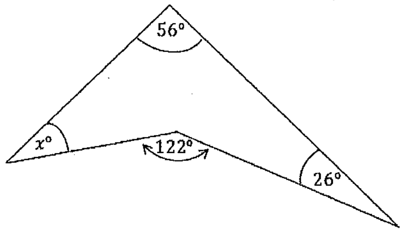
(A) 1

(B) $\frac{1}{2}+\cos^2\theta$

(C) $1+\tan^2\theta$

(D) $1+\cos^2\theta$

2)



In the figure above, x equals

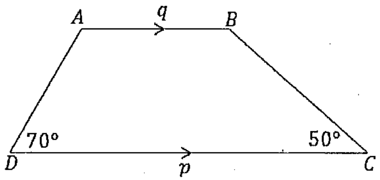
(A) 31°

(B) 34°

(C) 40°

(D) 48°

3)



In the figure above $AB||DC$, $AB=q$ and $DC=p$. BC equals

(A) $\frac{(p+q)\sin 50°}{2\sin 70°}$

(B) $\frac{(p+q)\sin 70°}{2\sin 50°}$

(C) $\frac{(p-q)\sin 70°}{\sin 60°}$

(D) $\frac{(p-q)\sin 50°}{\sin 70°}$

4) The period of the function $f(x)=\sin\left(3x-\frac{\pi}{3}\right)$, $x\epsilon R$ is

(A) $\frac{\pi}{9}$

(B) $\frac{2\pi}{3}$

(C) 2π

(D) $\frac{\pi}{3}$

5) The solution(s) of the equation $e^x+e^{-x}=-\frac{3}{2}$, where $x\in R$, is (are)

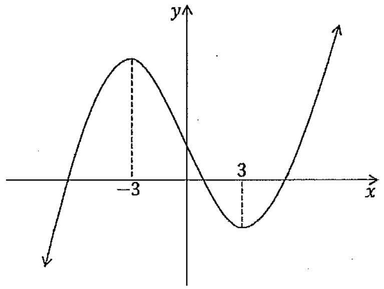
(A) ln 2 only

(B) $\pm\ln 2$

(C) $-\ln 2$ only

(D) None of these

6) From the graph of $y = f(x)$, when is $f'(x)$ negative?



(A) $x < -3$ or $x > 3$

(B) $-3 < x < 3$

(C) $x \leq -3$ or $x \geq 3$

(D) $-3 \leq x \leq 3$

7) If M is decreasing at an increasing rate, what does this suggest about $\frac{dM}{dt}$ and $\frac{d^2M}{dt^2}$?

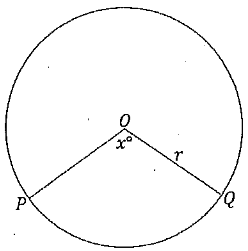
(A) $\frac{dM}{dt} < 0$ and $\frac{d^2M}{dt^2} < 0$

(B) $\frac{dM}{dt} > 0$ and $\frac{d^2M}{dt^2} < 0$

(C) $\frac{dM}{dt} < 0$ and $\frac{d^2M}{dt^2} > 0$

(D) $\frac{dM}{dt} > 0$ and $\frac{d^2M}{dt^2} > 0$

8)



In the figure, the radius of the sector is r and $\angle POQ = x°$. If the area of the sector is A then x equals

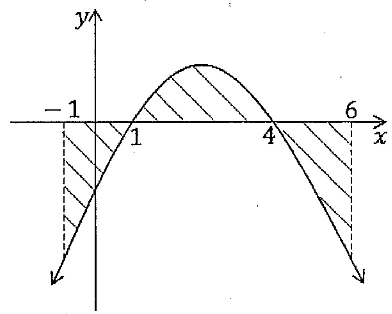
(A) $\frac{2A}{r^2}$

(B) $\frac{360A}{\pi r^2}$

(C) $\frac{180A}{\pi r^2}$

(D) $\frac{180A}{r^2}$

9) Which of the following expressions gives the total area of the shaded region in the diagram?



(A) $\int_{-1}^{6} f(x)\,dx$

(B) $-\int_{-1}^{0} f(x)\,dx + \int_{0}^{6} f(x)\,dx$

(C) $-\int_{-1}^{1} f(x)\,dx + \int_{1}^{4} f(x)\,dx - \int_{4}^{6} f(x)\,dx$

(D) $\int_{1}^{4} f(x)\,dx + 2\int_{4}^{6} f(x)\,dx$

10) Which of the following is the derivative of $y = \ln[f(x)]$

(A) $\frac{f(x)}{f'(x)}$

(B) $\frac{f'(x)}{f(x)}$

(C) $\frac{1}{f'(x)}$

(D) $\frac{f''(x)}{f'(x)}$

End of Section I

Section II **Free response questions**
Total marks – 90
Attempt Questions 11 – 16

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

Question 11 (15 marks) Use a SEPARATE writing booklet.

(a) Factorise $x^2 - 2x + 1 - 4y^2$ **1**

(b) Simplify **1**

$$\sqrt{\frac{3^{5k+2}}{27^k}}$$

(c) Simplify **1**

$$\frac{\log(a^3b^2) - \log(ab^2)}{\log\sqrt{a}}$$

(d) Solve $x^2 + 2x - 8 > 0$ **1**

(e) By considering the cases $x \leq 1$ and $x > 1$, or otherwise, solve **2**

$$|1 - x| = x - 1$$

(f) For the parabola $(x - 3)^2 = -4y$.

(i) Find the coordinates of the vertex. **1**

(ii) State the equation of the directrix of the parabola. **1**

(g) Prove **2**

$$\frac{\sin\theta}{1 - \cos\theta} = \frac{1 + \cos\theta}{\sin\theta}$$

(h) Evaluate **1**

$$\lim_{x \to 2} \frac{x - 2}{x^2 + x - 6}$$

(i) Find the equation of a straight line passing through the point of intersection of the lines l_1: $2x - y - 4 = 0$ and l_2: $2x + 3y - 12 = 0$ and perpendicular to the line $2x - 3y + 1 = 0$. **2**

(j) Find the equation of the tangent to the curve $y = 2\sin 2x$ at the point $\left(\frac{\pi}{8}, \sqrt{2}\right)$. **2**

End of Question 11

Question 12 (15 marks) Use a SEPARATE writing booklet.

(a) Draw a number plane and mark the points $P(-2, 2)$ and $Q(-4, -2)$.

(i) Show that the equation of the line through P perpendicular to PQ is given by **2**

$$x + 2y - 2 = 0$$

(ii) The line perpendicular to PQ through P intersects the x-axis at R. Find the coordinates of R. **2**

(iii) Show that the mid-point of QR is $(-1, -1)$. Mark this point T on your diagram. **1**

(iv) Find the perpendicular distance from T to the interval PR. **2**

(b) x and y are positive numbers. $x, -2, y$ are consecutive terms of a geometric series, and $-2, y, x$ are consecutive terms of an arithmetic series.

(i) Find the value of xy. **1**

(ii) Find the values of x and y. **3**

(iii) Find the sum to infinity of the geometric series

$$x - 2 + y \ldots$$ **2**

(c) Given that α and $m\alpha$ are the roots of the equation $x^2 + px + q = 0$, show that

$$mp^2 = (m + 1)^2 q$$ **2**

End of Question 12

Question 13 (15 marks) Use a SEPARATE writing booklet.

(a) Differentiate with respect to x:

(i) $x \sin x$ **1**

(ii) $\ln(x^2 + 4)$ **1**

(iii) $e^{5x} + x$ **1**

(b) The graph of $y = f(x)$ passes through the point $(3, 5)$, and $f'(x) = 2x - 3$. Find $f(x)$. **2**

(c) Find:

(i)

$$\int \sqrt{x + 10}\,.\,dx$$ **1**

(ii)

$$\int_0^{\frac{\pi}{8}} \sec^2 2x\,.\,dx$$ **1**

(d) Consider the curve $y = \cos 2x$:

(i) Sketch $y = \cos 2x$ for $0 \le x \le 2\pi$. **1**

(ii) Find the area between the curve $y = \cos 2x$ and the x-axis from $x = 0$ to $x = \pi$. **2**

(e) The population P of Newcastle after t years is given by the exponential equation

$$P = 50000e^{-0.08t}$$

(i) Find the time to the nearest year for the initial population to halve. **1**

(ii) Find the number of people who leave Newcastle during the tenth year. **2**

(f) A continuous curve $y = f(x)$ has the following properties for the closed interval $-3 \le x \le 5$: $f(x) > 0, f'(x) > 0, f''(x) < 0$. Sketch a curve satisfying these conditions. **2**

End of Question 13

Question 14 (15 marks) Use a SEPARATE writing booklet.

(a) Given that $f(x) = x(x-2)^2$

(i) Show that $f'(x) = 3x^2 - 8x + 4$. **1**

(ii) Find 2 values of x for which $f'(x) = 0$, and give the corresponding values of $f(x)$. **1**

(iii) Determine the nature of the turning points of the curve $y = f(x)$. **2**

(iv) Sketch the curve $y = f(x)$ showing all essential features. **2**

(v) Use your sketch to solve the inequation $x(x-2)^2 \geq 0$. **1**

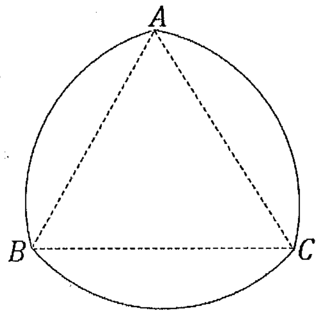
(b) An economist predicts that over the next few months, the price of crude oil, p dollars a barrel, in t weeks time will be given by the formula

$$P = 0.005t^3 - 0.3t^2 + 4.5t + 98$$

(i) What is the price at present, and how rapidly is it going up? **2**

(ii) How high does she expect the price to rise? **2**

(c) A coin is made by starting with an equilateral triangle ABC of side 2 cm. With centre A an arc of a circle is drawn joining B to C. Similar arcs join C to A and A to B.



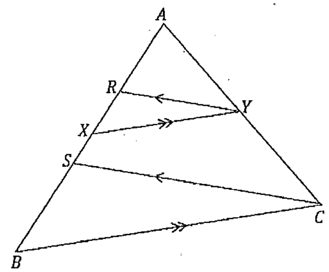
(i) Find, exactly, the perimeter of the coin. **2**

(ii) Find area of one of its faces. **2**

End of Question 14

Question 15 (15 marks) Use a SEPARATE writing booklet.

(a) Given that in $\Delta ABC, XY||BC$ and $RY||SC$,



Prove $AX:XB = AR:RS$. **2**

(b) A couple plan to buy a home and they wish to save a deposit of $40 000 over five years. They agree to invest a fixed amount of money at the beginning of each month during this time. Interest is at 12% per annum compounded monthly.

(i) Let $P be the monthly investment. Show that the total investment $A after five years is given by **2**

$$A = P(1.01 + 1.01^2 + \cdots + 1.01^{60})$$

(ii) Find the amount $P needed to be deposited each month to reach their goal. Answer correct to the nearest dollar. **2**

(c) A train is travelling on a straight track at 48 ms^{-1}. When the driver sees an amber light ahead, he applies the brakes for a period of 30 seconds, producing a deceleration of $\frac{1}{125}t(30-t)$ ms^{-2}, where t is the time in seconds after the brakes are applied.

(i) Find how fast the train is moving after 30 seconds. **2**

(ii) How far it has travelled in that time. **2**

(d) Two ordinary dice are thrown. Find the probability that the sum of the numbers on the uppermost faces is at least 10. **2**

(e) Consider the function $f(x) = x^2 lnx - \frac{x^2}{2}$:

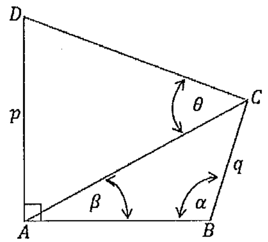
(i) Show that $f'(x) = 2xlnx$. **1**

(ii) Hence find $\int_1^2 xlnx.dx$. **2**

End of Question 15

Question 16 (15 marks) Use a SEPARATE writing booklet.

(a)



$ABCD$ is a quadrilateral with AD perpendicular to AB. Given that $\angle CAB = \beta$, $\angle ABC = \alpha$, $\angle ACD = \theta, AD = p$ and $BC = q$.

(i) Show that $\angle ADC = 90 - (\theta - \beta)$ **1**

(ii) Using the sine rule, prove that **3**

$$q = \frac{p \sin\beta \cos(\theta - \beta)}{\sin\theta \sin\alpha}$$

(b) Consider the function $y = f(x) = 1 + e^{2x}$.

(i) Find $f(0), f(1), f(2)$. **1**

(ii) Show that $x = \frac{1}{2}\ln(y - 1)$. **2**

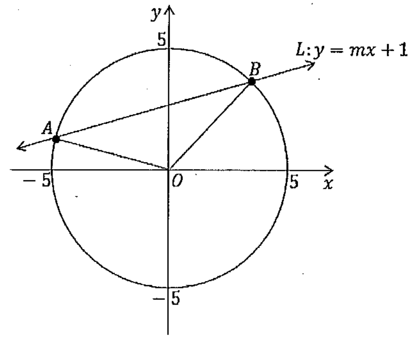
(iii) The volume V formed when the area between $y = 1 + e^{2x}$, the y-axis, and the lines $y = 2$ and $y = 4$ is rotated about the y-axis is given by:

$$V = \frac{\pi}{4}\int_2^4 [\ln(y-1)]^2 . dy$$

Use Simpson's rule with 3 function values to estimate this volume. Leave your answer rounded to 3 significant figures. **3**

Question 16 continues on the next page

(c)



In the above figure, the line $L: y = mx + 1$ cuts the circle $x^2 + y^2 = 25$ at two points $A(x_1, y_1)$ and $B(x_2, y_2)$.

(i) Show that x_1 and x_2 are the roots of $(1 + m^2)x^2 + 2mx - 24 = 0$. **2**

(ii) Show that area of $\Delta OAB = \frac{1}{2}(x_2 - x_1)$. **3**

End of paper.

2013 MATHEMATICS TRIAL - SOLUTIONS (2 - UNIT)

1. $\dfrac{\tan^2\theta}{1+\tan^2\theta} + \cos^2\theta$

$= \dfrac{\tan^2\theta}{\sec^2\theta} + \cos^2\theta$

$= \sin^2\theta + \cos^2\theta$

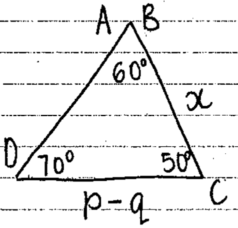
$= 1$ $\boxed{A}$

2. $122^\circ = x^\circ + 56^\circ + 26^\circ$

$122^\circ = x^\circ + 82^\circ$

$x^\circ = 40^\circ$ $\boxed{C}$

3. Eliminate AB:



$\dfrac{x}{\sin 70^\circ} = \dfrac{p-q}{\sin 60^\circ}$

$x = \dfrac{(p-q)\sin 70^\circ}{\sin 60^\circ}$ $\boxed{C}$

4. $f(x) = \sin\left(3x - \dfrac{\pi}{3}\right)$

Period $= \dfrac{2\pi}{3}$ $\boxed{B}$

5. $e^x + e^{-x} = -\dfrac{3}{2}$

None of these $\boxed{D}$

6. $f'(x) < 0$

$-3 < x < 3$ $\boxed{B}$

7. $\dfrac{dM}{dt} < 0$ and $\dfrac{d^2M}{dt^2} > 0$

OR $\dfrac{dM}{dt} < 0$ and $\dfrac{d^2M}{dt^2} < 0$

$\boxed{C}$ & $\boxed{A}$

8. $A = \dfrac{x}{360} \times \pi \times r^2$

$\dfrac{360A}{\pi r^2} = x$ $\boxed{B}$

9. $A = \int_{-1}^{1} f(x)\,dx + \int_{1}^{4} f(x)\,dx - \int_{4}^{6} f(x)\,dx$ $\boxed{C}$

10. $y = \ln[f(x)]$

$y' = \dfrac{f'(x)}{f(x)}$ $\boxed{D}$

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2 unit Trial 2013 — /15

11 (a) $x^2 - 2x + 1 - 4y^2$

$(x-1)^2 - 4y^2 \Rightarrow A^2 - B^2$

$(x-1-2y)(x-1+2y)$ ①

(b) $\sqrt{\dfrac{3^{5k+2}}{3^{3k}}} = \left(3^{5k+2-3k}\right)^{\frac12}$

$= \left(3^{2k+2}\right)^{\frac12} = 3^{k+1}$ ①

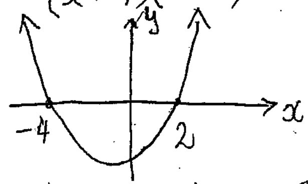
(c) $\dfrac{\log a^3 + \log b^2 - (\log a + \log b^2)}{\log a^{\frac12}}$

$\dfrac{3\log a + 2\log b - \log a - 2\log b}{\frac12 \log a}$

$\dfrac{2\log a}{\frac12 \log a} = 4$ ①

(d) $x^2 + 2x - 8 > 0$

$(x+4)(x-2) > 0$



$x < -4$ and $x > 2$ ①

(e) $|1-x| = x-1$

$1-x = x-1$ or $-(1-x) = x-1$

$2 = 2x$ $\qquad -1+x = x-1$

$x = 1$ $\qquad 0 = 0$

Test $x=1$, LHS $|0| = 0$, RHS $1-1=0$ So $x=1$ ②

But for $x>1$, $|1-x| = x-1$ is also true so $x \geq 1$ is the solution.

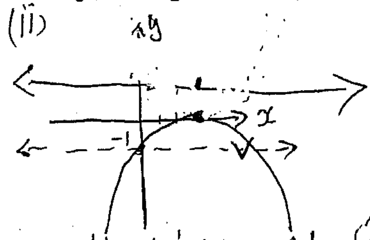
(f) $(x-3)^2 = -4y$

$(x-h)^2 = -4a(y-k)$

(i) $h=3$, $k=0$, $a=1$

$V(3,0)$ ①

(ii)



directrix $y = +1$ ①

(g) $\dfrac{\sin\theta}{1-\cos\theta} = \dfrac{1+\cos\theta}{\sin\theta}$

LHS $\dfrac{\sin\theta}{(1-\cos\theta)} \times \dfrac{(1+\cos\theta)}{(1+\cos\theta)} = \dfrac{\sin\theta(1+\cos\theta)}{1-\cos^2\theta}$

$= \dfrac{\sin\theta(1+\cos\theta)}{\sin^2\theta}$

$= \dfrac{1+\cos\theta}{\sin\theta}$

$=$ RHS. ②

(h) $\lim_{x\to 2} \dfrac{\cancel{(x-2)}}{(x+3)\cancel{(x-2)}}$

$\to \dfrac{1}{5}$ ①

11 (i) $2x - y - 4 = 0$

$2x + 3y - 12 = 0$

$-4y + 8 = 0$

$4y = 8 \Rightarrow y = 2$

So $2x - 2 - 4 = 0$

$2x - 6 = 0$

$x = 3$

Pt of intersection $(3, 2)$.

now $2x - 3y + 1 = 0$

$2x + 1 = 3y$

$y = \frac{2}{3}x + \frac{1}{3}$

Line $\perp$ to this has $m = -\frac{3}{2}$.

now $(y - y_1) = m(x - x_1)$

$(y - 2) = -\frac{3}{2}(x - 3)$

$2y - 4 = -3x + 9$

$3x + 2y - 13 = 0$

Or $y = -\frac{3}{2}x + \frac{9}{2} + 2$ 2

$y = -\frac{3}{2}x + 6\frac{1}{2}$.

11 (j) $y = 2\sin 2x$

$y' = 2 \times \cos 2x \times 2$

$= 4\cos 2x$.

At $x = \frac{\pi}{8}$, $m = 4 \times \cos\frac{\pi}{4}$

$= 4 \times \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{4\sqrt{2}}{2} = 2\sqrt{2}$.

using $(y - y_1) = m(x - x_1)$

$(y - \sqrt{2}) = 2\sqrt{2}\left(x - \frac{\pi}{8}\right)$

$y = 2\sqrt{2}x - \frac{\sqrt{2}\pi}{4} + \sqrt{2}$ 2

$(y - \sqrt{2}) = \frac{4}{\sqrt{2}}\left(x - \frac{\pi}{8}\right)$

$\sqrt{2}y - 2 = 4x - \frac{\pi}{2}$

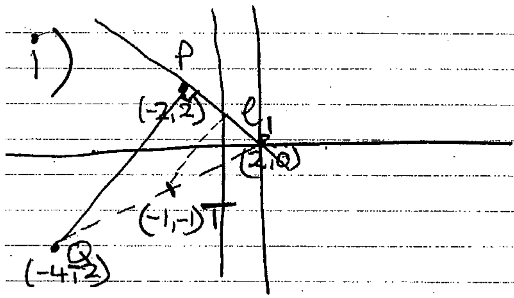
$0 = 4x - \sqrt{2}y - \frac{\pi}{2} + 2$.

or $0 = 8x - 2\sqrt{2}y - \pi + 4$

3 versions accepted!

QUESTION 12. 2U 2013 TRIAL.

i)



$m_{PQ} = \frac{2 - -2}{-2 - -4} = \frac{4}{2} = 2$.

$m_{l_1} = -\frac{1}{2}$. $P(-2,2)$.

l_1: $y - 2 = -\frac{1}{2}(x - -2)$.

$\boxed{2y - 4 = -x - 2}$

$\boxed{PR}$ $x + 2y - 2 = 0$ ✓

ii) when $\boxed{y=0}$ $x = 2$ $\therefore$ crosses at $(2,0)$.

iii) $MP_{QR} = \left(\frac{-4+2}{2}, \frac{-2+0}{2}\right)$

$= (-1, -1)$.

iv) $(x + 2y - 2) = 0$ $(-1, -1)$

$d = \left|\frac{ax_1 + by_1 + c}{\sqrt{a^2+b^2}}\right| = \frac{1(-1) + 2(-1) - 2}{\sqrt{1^2+2^2}} = \frac{-5}{\sqrt{5}} = -\sqrt{5}$

$x, -2, y$ geo. $\frac{-2}{x} = \frac{y}{-2} = r$

$-2 \quad y \quad x$ arith. $y - {}^-2 = x - y = d$

i) $xy = +4$

$2y - x + 2 = 0$

ii) ~~a~~ $y = \frac{4}{x}$

$2\left(\frac{4}{x}\right) - x + 2 = 0$

$\frac{8}{x} - x + 2 = 0$

$8 - x^2 + 2x = 0$

$x^2 - 2x - 8 = 0$

$(x^2 - 4x + 2x - 8$

$x(x-4) + 2(x-4)$

$(x+2)(x-4) = 0$

$x = (4)$ or (-2)
$y = (1)$ or (-2) $\quad x>0 \;\; y>0$ discard $(-2, -2)$

$(x - 2 + y)$ $\quad (4 - 2 + 1)$ $\quad r = \frac{-2}{4} = \frac{-1}{2}$

$a = x \qquad r = -\frac{2}{x}$

$\boxed{S_\infty = \frac{4}{1 - {}^-\frac{1}{2}} = 2\frac{2}{3}}$

$S_\infty = \frac{a}{1-r} = \frac{x}{1 - {}^-2/x}$

$\frac{x}{\frac{x+2}{x}} \div = \frac{x^2}{x+2} = S_\infty$

$\frac{8}{\cancel{3}} = \frac{16}{6}$
$= 2\frac{2}{3}$

$(x - \alpha)(x - m\alpha) \qquad x^2 + px + q$

$mp^2 = (m+1)^2 q$

$\alpha + m\alpha = -p$

$m\alpha^2 = q$

$\boxed{\text{rearrange } \alpha(1 + m) = -p \quad \therefore \alpha = \frac{-p}{1+m}}$

$m\left(\frac{(-p)^2}{(1+m)^2}\right) = q$

$mp^2 = q(1+m)^2$

2 Unit – Solutions

13. (a)

(i) $y = x\sin x$

$\frac{dy}{dx} = x.\cos x + \sin x$ ✓

(ii) $y = \ln(x^2+4)$

$\frac{dy}{dx} = \frac{2x}{x^2+4}$ ✓

(iii) $y = e^{5x} + x$

$\frac{dy}{dx} = 5e^{5x} + 1$ ✓

(b) $f'(x) = 2x - 3$

$f(x) = x^2 - 3x + C$

$f(3) = 5 \Rightarrow 5 = 9 - 9 + C \Rightarrow C = 5$ ✓ 2

$\therefore \underline{f(x) = x^2 - 3x + 5}$ ✓

(c) (i) $\int \sqrt{x+10}\,dx$

$= \frac{2(x+10)^{\frac{3}{2}}}{3} + C$

$= \frac{2\sqrt{(x+10)^3}}{3} + C$ ✓ 2

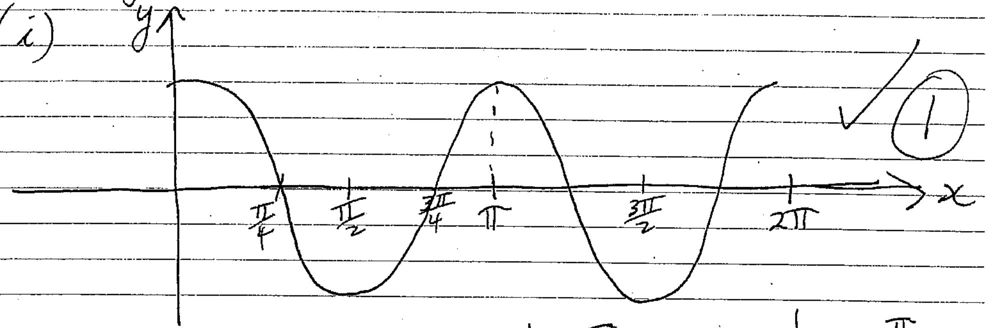
(ii) $\int_0^{\frac{\pi}{8}} \sec^2 2x\,dx = \left[\frac{1}{2}\tan 2x\right]_0^{\frac{\pi}{8}}$

$= \frac{1}{2}\left[\tan\frac{\pi}{4} - \tan 0\right] = \frac{1}{2}(1-0)$

$= \frac{1}{2}$ ✓

13(d) $y = \cos 2x$

(i)



✓ 1

(ii) $\text{Area} = \int_0^{\pi/4} \cos 2x\,dx + \left|\int_{\pi/4}^{3\pi/4} \cos 2x\,dx\right| + \int_{3\pi/4}^{\pi} \cos 2x\,dx$

$= \left[\frac{\sin 2x}{2}\right]_0^{\pi/4} + \left|\left[\frac{\sin 2x}{2}\right]_{\pi/4}^{3\pi/4}\right| + \left[\frac{\sin 2x}{2}\right]_{3\pi/4}^{\pi}$ ✓

$= \frac{1}{2}(1-0) + \frac{1}{2}\left|(-1-1)\right| + \frac{1}{2}\left[(0-{-1})\right]$

$= \frac{1}{2} + 1 + \frac{1}{2}$ ✓ 2

$= 2$ square units.

(e) $P = 50000e^{-0.08t}$

(i) $t = ?$ $P = 25000 \Rightarrow 25000 = 50000e^{-0.08t}$

$0.5 = e^{-0.08t}$

$\ln 0.5 = -0.08t$

$t = 8.66433$ years.

$\therefore \underline{t = 9 \text{ years}}$ (to nearest year) ✓ 1

(ii) $t = 10$, $P = ?$ $\Rightarrow P = 50000e^{-0.08\times 10}$ ✓ 2

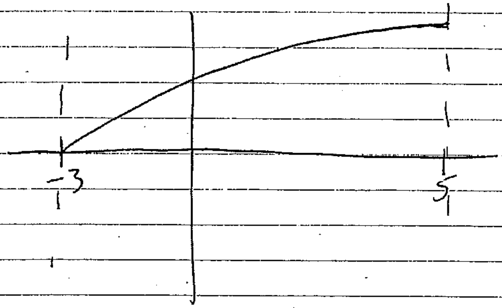
$= 22466$ people

$t = 9$, $P = 50000e^{-0.08\times 9} = 24338$ people

$\therefore$ Popn that left $= 24338 - 22466 = \underline{1872 \text{ people}}$ ✓

13.(f) $-3 \le x \le 5$, $f(x) > 0$, $f'(x) > 0 \Rightarrow f(x)$ increasing

$f''(x) < 0 \} \Rightarrow f(x)$ is concave down.



positive and increasing ✓

concave down ✓✓

2

QUESTION FOURTEEN. MATHEMATICS 2013.

a) i $f(x) = x(x-2)^2$

$= x^3 - 4x^2 + 4x$

$f'(x) = 3x^2 - 8x + 4$

ii) $f'(x) = 0$

when $3x^2 - 8x + 4 = 0$

$(3x-2)(x-2) = 0$

$x = 2,\ y = 0$

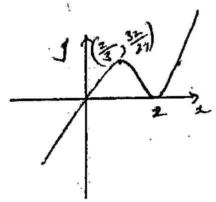
$x = 2/3,\ y = \frac{32}{27}$

iii) $f''(x) = 6x - 8$

$= 4$ when $x = 2$. MIN

$= -4$ when $x = 2/3$ MAX

iv)



v) From sketch

$x > 0$

b) $P = .005t^3 - .3t^2 + 4.5t + 98$

i) When $t = 0$ $P = 98$, \$98

$\frac{dP}{dt} = .015t^2 - .6t + 4.5$

When $t = 0$, $\frac{dP}{dt} = 4.5$, \$4.50/w

ii) $\frac{dP}{dt} = 0$ when

$\frac{3}{200}t^2 - \frac{6}{10}t + \frac{9}{2} = 0$

$3t^2 - 120t + 900 = 0$

$t^2 - 40t + 300 = 0$

$(t-10)(t-30) = 0$

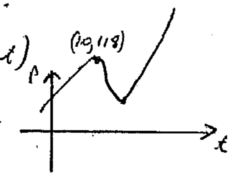
$t = 10, 30$

$\frac{d^2P}{dt^2} = .03t - 0.6$

$= -0.3$ when $x = 10$

Hence Relative MAX.

14b (cont)



When $t = 10$, $P = 118$

NOTE

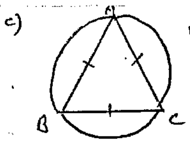
A relative maximum occurs when $t = 10$ but there is no specific given domain so $P \to \infty$ as t increases

HOWEVER

Question does state "over the next few months" so the inference is $t = 10$ weeks.

When $t = 10$, $P = 118$, \$118

c)



NOT A CIRCLE !!

$r = 2$

$\theta = \frac{\pi}{3}$

i) Perimeter consists of 3 arcs

$P = 3r\theta$

$= 3 \times 2 \times \frac{\pi}{3}$

$= 2\pi$ cm.

ii) Area consists of 1 triangle + 3 segments

$A = \frac{1}{2}r^2\sin\theta + 3 \times \frac{1}{2}r^2(\theta - \sin\theta)$

$= \frac{3}{2}r^2\theta - r^2\sin\theta$

$= \frac{3}{2} \times 4 \times \frac{\pi}{3} - 4\frac{\sqrt{3}}{2}$

$= 2\pi - 2\sqrt{3}$ cm^2

$= 2(\pi - \sqrt{3})$ cm^2

Q15

(a) $AX:XB = AY:YC$ (proportional division theorem)

$AR:RS = AY:YC$ (" " ")

$\therefore AX:XB = AR:RS$ 2

(b) (i) Amt after 1 month

$= P\times 1.01$

Amt after 2 months

$= P\times 1.01^2 + P\times 1.01$

Amt after 3 months

$= (P\times 1.01^2 + P\times 1.01)\times 1.01 + P\times 1.01$

$= P\,(1.01^3 + 1.01^2 + 1.01)$

$\therefore A =$ Amt after 60 months

$= P(1.01^{60} + 1.01^{59} + \cdots + 1.01)$

as required 2

(ii) $A = \dfrac{P\times 1.01\times(1.01^{60}-1)}{1.01-1}$

$\therefore 40000 = P\times 101\times(1.01^{60}-1)$

$\therefore P = \dfrac{40000}{101\,(1.01^{60}-1)}$

$= 484.9286\ldots$ 2

$\approx \$485$

(c) $\ddot{x} = -\dfrac{t}{125}(30-t)$

$= \dfrac{t^2}{125} - \dfrac{6t}{25}$

$\dot{x} = \dfrac{t^3}{375} - \dfrac{3t^2}{25} + c$

when $t=0$ $48 = c$

$\therefore \dot{x} = \dfrac{t^3}{375} - \dfrac{3t^2}{25} + 48$

$\therefore x = \dfrac{t^4}{1500} - \dfrac{t^3}{25} + 48t + c_1$

when $t=0$: $0 = c_1$

$\therefore x = \dfrac{t^4}{1500} - \dfrac{t^3}{25} + 48t$

(i) when $t=30$, $\dot{x} = \dfrac{30^3}{375} - \dfrac{3\times 30^2}{25} + 48$ 2

$= 12\ \text{ms}^{-1}$

(ii) when $t=30$, $x = \dfrac{30^4}{1500} - \dfrac{30^3}{25} + 48\times 30$ 2

$= 900\ \text{m}$

(d) $P(\text{at least } 10) = P(10) + P(11) + P(12)$

$= P(46, 55, 64) + P(56, 65) + P(66)$

$= \dfrac{6}{36}$ 2

$= \dfrac{1}{6}$

(e) $f(x) = x^2\ln x - \dfrac{x^2}{2}$

(i) $f'(x) = \ln x . 2x + x^2 . \dfrac{1}{x} - \dfrac{2x}{2}$

$= 2x\ln x + x - x$ 1

$= 2x\ln x$

(ii) $\displaystyle\int_1^2 x\ln x\,dx = \frac{1}{2}\int_1^2 2x\ln x\,dx$

$= \dfrac{1}{2}\left[x^2\ln x - \dfrac{x^2}{2}\right]_1^2$

$= \dfrac{1}{2}\left\{\left[4\ln 2 - \dfrac{4}{2}\right] - \left[1.0 - \dfrac{1}{2}\right]\right\}$

$= \dfrac{1}{2}\left\{4\ln 2 - 2 + \dfrac{1}{2}\right\}$ 2

$= 2\ln 2 - \dfrac{3}{4}$

Question 16 THSC 2unit.

(a)(i) $\angle ACB = 180 - (\alpha+\beta)$ ($\angle$ sum $\triangle$)

$\angle ADC = 360 - 90 - \alpha - \theta - (180 - (\alpha+\beta))$ ($\angle$ sum quad)

$= 90 - \alpha - \theta + \alpha + \beta$

$= 90 - (\theta - \beta)$ 1

(ii) ① $\dfrac{a}{\sin\beta} = \dfrac{AC}{\sin\alpha}$ ② $\dfrac{P}{\sin\theta} = \dfrac{AC}{\sin D}$

④ $a = AC\,\dfrac{\sin\beta}{\sin\alpha}$ $AC = \dfrac{P\sin D}{\sin\theta}$

$= \dfrac{P\sin D\sin\beta}{\sin\theta\sin\alpha}$ ③ $\sin D = \sin(90 - (\theta-\beta))$

$= \cos(\theta-\beta)$

$= \dfrac{P\sin\beta\cos(\theta-\beta)}{\sin\theta\sin\alpha}$ 3 (b)(i) $f(0) = 1+1 = 2$

$f(1) = 1+e^2$

$f(2) = 1+e^4$

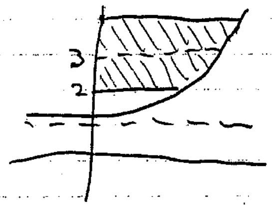
(ii) $y = 1+e^{2x}$

$e^{2x} = y-1$

$2x = \ln(y-1)$

$x = \frac{1}{2}\ln(y-1)$

(iii)



$V = \frac{\pi}{4}\int_2^4 [\ln(y-1)]^2\,dy$

$= \frac{\pi}{4} \times \frac{1}{3} \times 1 \times \left((\ln(1))^2 + 4(\ln(2))^2 + (\ln(3))^2\right)$

≈ 0.819 units^3

(c)(i) $y_1 = mx_1 + 1$ ①
$x_1^2 + y_1^2 = 25$ ② are true since (x_1, y_1) is on both curves.

$(1+m^2)x_1^2 + 2mx_1 - 24 = x_1^2 + m^2x_1^2 + 2mx_1 + 1 - 25$

$= x_1^2 + (mx_1+1)^2 - 25$

$= x_1^2 + y_1^2 - 25$ from ①

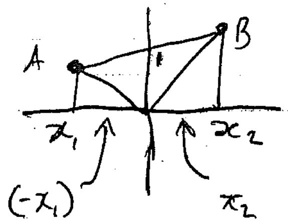
$= 25 - 25$ from ②

$= 0.$

2

Similarly for x_2.

(ii)



Area $= \frac{1}{2} \times 1 \times (-x_1) + \frac{1}{2} \times 1 \times x_2$

$= \frac{1}{2}(x_2 - x_1).$

3