ASCHAM SCHOOL

YEAR 12
MATHEMATICS EXTENSION 1
TRIAL EXAMINATION 2011

TIME ALLOWED: 2 HOURS PLUS 5 MINUTES' READING TIME

INSTRUCTIONS

ALL QUESTIONS MAY BE ATTEMPTED.
ALL QUESTIONS ARE OF EQUAL VALUE (12 MARKS).
ALL NECESSARY WORKING MUST BE SHOWN.
MARKS MAY NOT BE AWARDED FOR CARELESS WORK.
APPROVED CALCULATORS AND TEMPLATES MAY BE USED.

COLLECTION

START EACH QUESTION IN A NEW BOOKLET.
IF YOU USE A SECOND BOOKLET FOR A QUESTION, PLACE IT INSIDE THE FIRST.
WRITE YOUR NAME/NUMBER, TEACHER'S NAME AND QUESTION NUMBER ON EACH BOOKLET.

Standard Integrals

$$\int x^n dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}dx = \ln x, \quad x > 0$$

$$\int e^{ax}dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}dx = \ln\left(x+\sqrt{x^2-a^2}\right) \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$

$$\int \frac{1}{\sqrt{1-x^2}} = \sin^{-1} x + c$$

Question 1

a) Find the value of $\tan^{-1}\sqrt{3}$ in radians (1)

b) If α, β and γ are the roots of the equations $x^3 - 2x + 5 = 0$, find the value of $\alpha\beta\gamma$ (1)

c) Find $\frac{d}{dx}(\sin^{-1} 2x)$ (2)

d) Find the remainder when $P(x) = x^3 - 2x^2 - 2x + 1$ is divided by $x - 2$ (2)

e) Find $\int \cos^2 4x \, dx$ (2)

f) Find the co-ordinates of the point P which divides the interval joining A(-3,4) and B(2,-8) externally in the ratio 2:5. (2)

g) Find the acute angle between the lines $y = -x$ and $\sqrt{3}y = x$ (2)

Question 2 Begin a new booklet

a) Find $\int \frac{dx}{4x^2 + 1}$ (2)

b) Differentiate $4\sec^2 x$ (2)

c) Find the general solution to $\cos x = -\frac{\sqrt{3}}{2}$ (2)

d) Use the substitution $x = \cos\theta$ to evaluate (4)

$$\int_{\frac{1}{2}}^{1} \frac{\sqrt{1-x^2}}{x^2} dx$$

e) Find the Cartesian equation of the curve with parametric equations $x = \sin t$ and $y = 2 + \cos t$ (2)

$\sin 45 = x$
$\sin^{-1} x = 45$

Question 3 Begin a new booklet

a) Find the exact value of $\sin\left(2\sin^{-1}\frac{3}{4}\right)$ (2)

b) Solve $\ln(2x+3) + \ln(x-2) = 2\ln(x+4)$ (3)

c) Solve the inequation $\frac{2x+1}{x-2} \geq 1$ (3)

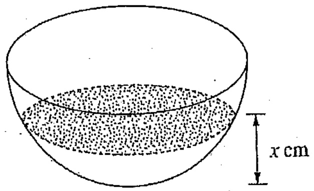
d) A particle is moving in a straight line so that its displacement x from the origin at time t, in seconds, is given by $x = \sqrt{3}\cos 2t - \sin 2t$, $t \geq 0$

i) Show that the particle moves in simple harmonic motion (2)

ii) Find the velocity the first time the particle is at the origin. (2)

Question 4 **Begin a new booklet**

a)



A hemispherical bowl of radius r cm is initially empty. Water is poured into it at a constant rate of k cm^3 per minute. When the depth of the water in the bowl is x cm, the volume, V cm^3, of the water in the bowl is given by

$$V=\frac{\pi}{3}x^2(3r-x) \qquad \textit{(do not prove this)}$$

i) Show that $\dfrac{dx}{dt}=\dfrac{k}{\pi x(2r-x)}$ (2)

ii) Find an expression for t as a function of x (2)

b) A function is defined as $f(x)=1-\cos\frac{x}{2}$ where $0\le x\le a$

i) Find the largest value of a for which the inverse function $f^{-1}(x)$ exists. (2)

ii) Show that $f^{-1}(x)=2\cos^{-1}(1-x)$ (2)

iii) Sketch the graph of $y = f^{-1}(x)$ (2)

iv) Find the area enclosed between the curve $y = f^{-1}(x)$, the x axis and $x = 2$. (2)

Question 5

a) A ball is projected from a point 6m above the ground at an angle of 30^0 from the horizontal with velocity of V metres per second. The equations for acceleration in the horizontal and vertical direction are given by $\ddot{x}=0$ *and* $\ddot{y}=-g$ respectively.

i) Using Calculus show that $x=\dfrac{Vt\sqrt{3}}{2}$ and $y=\dfrac{-gt^2}{2}+\dfrac{vt}{2}+6$ (3)

ii) Hence find the Cartesian equation of the path of the ball (2)

iii) Assuming that $g = 9.8$ m/s^2, will the ball clear a 50 m tall building which is 355m away if the ball is projected with a velocity of 65 m/s ? Justify your answer. (3)

b) Use mathematical induction to prove that for all integers $n\ge 3$ (4)

$$\left(1-\frac{2}{3}\right)\left(1-\frac{2}{4}\right)\left(1-\frac{2}{5}\right)\cdots\cdots\left(1-\frac{2}{n}\right)=\frac{2}{n(n-1)}$$

Question 6

a) P($8p,4p^2$) and Q($8q,4q^2$) are variable points on the parabola $x^2 = 16y$ The tangent at P and Q meet at R.

i) Show that the equation of the chord PQ has equation $y=\frac{1}{2}(p+q)x-4pq$ (2)

ii) The chord PQ produced passes through the fixed point (4,0). Show that $p + q = 2pq$ (2)

iii) Find the coordinates of R (2)

iv) Find the Cartesian equation for the locus of R (2)

b) The function $f(x) = e^{2x} - x - 3$ has a zero near $x = 0.8$. Use one application of Newton's method to find a second approximation to the zero. Write your answer correct to 3 significant figures. (2)

c) By finding any asymptotes and intercepts draw the graph of $y=\dfrac{x}{(x-1)^2}$, showing significant features. *(do not use calculus)* (2)

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MATHEMATICS WRITING BOOKLET

Name.	Question Number
Form Class:	1
Teacher's Initials	

- Enter the information requested in each of the boxes above
- Take a new writing booklet for each question
- You may ask for an extra writing booklet if you need one. Label it as above and put it inside the first booklet for the question.
- If you do not attempt a question still hand in the booklet with "NOT ATTEMPTED" written on the front.
- Write the question part in the margin
- Write on the ruled lines in black or blue ink

Do not write in this box

$10\frac{1}{2}$

a) $\tan^{-1}\sqrt{3} = \frac{\pi}{3}$ 1

b) $x^3 - 2x + 5 = 0$ 1

$\alpha\beta\gamma = -5 = \frac{-d}{a}$

c) $\frac{d}{dx}(\sin^{-1}2x) = \frac{2}{\sqrt{1-4x^2}}$ 2

d) $P(2) = 2^3 - 2(2)^2 - 2(2) + 1$ 1

$= -3$

e) $\int \cos^2 4x\, dx = \frac{1}{2}\int 1 + \cos 8x\, dx$ 2

$= \frac{1}{2}\left(x + \frac{1}{8}\sin 8x\right) + C$

f) $A(-3,4)$ $B(2,-8)$

$2:5 = k:l$

$-2:5 = k:l$ since external.

$x = \frac{lx_1 + kx_2}{l+k}$ $y = \frac{ly_1 + ky_2}{l+k}$

$= \frac{5(-3)+(-2)(2)}{3}$ $= \frac{5(4)+(-2)(-8)}{3}$

$= \frac{-15-4}{3}$ $= \frac{20+16}{3}$

$= -\frac{19}{3}$ $= 12$ 2

$\therefore$ ~~a~~ Point is $(-6\frac{1}{3}, 12)$

g) $y = -x$ $\sqrt{3}y = x \longrightarrow y = \frac{x}{\sqrt{3}}$

~~$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1m_2}\right|$~~

~~$= \left|\frac{-1 - \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}}\right|$~~

~~$= \left|\frac{-\sqrt{3}-1}{\sqrt{3}-1}\right|$~~

~~$=$~~

$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1m_2}\right|$

$= \left|\frac{-1 - \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}}\right|$

$\theta = 75°$ ½

+ A poly ... 2 correct gradients

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MATHEMATICS WRITING BOOKLET

Name:	Question Number 2
Form Class:	Teacher's Initials

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- You may ask for an extra writing booklet if you need one. Label it as above and put it inside the first booklet for the question.
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- Write the question part in the margin
- Write on the ruled lines in black or blue ink

Do not write in this box

$11\frac{1}{2}$

a) $\int \frac{dx}{4x^2+1} = \int \frac{dx}{4(x^2+\frac{1}{4})}$

$= 2x \frac{1}{4} \tan^{-1} \frac{x}{\frac{1}{2}} + C$

$= \frac{1}{2} \tan^{-1} 2x + C$

b) $\frac{d}{dx} 4\sec^2 x = 4 \times 2 (\sec x) \cancel{\sec x \tan x}$ $\frac{d}{dx} 4(1+\tan^2 x)$

$= \cancel{8\sec^2 x \tan x}$ $\frac{d}{dx} 4 + 4\tan^2 x$

$= 8\tan x \cdot \sec^2 x$

c) $\cos x = \frac{-\sqrt{3}}{2}$

$x = \frac{\pm 5\pi}{6} + 2k\pi$

d) $\int_{1/2}^{1} \frac{\sqrt{1-x^2}}{x^2}\, dx$

$x = \cos\theta$

$\frac{dx}{d\theta} = -\sin\theta$

$dx = -\sin\theta\, d\theta$

when $x = 1$ $\quad 1 = \cos\theta$

$\theta = 0$

$= \frac{1}{2}$ $\quad \frac{1}{2} = \cos\theta$

$\theta = \frac{\pi}{3}$

$\int_{\pi/3}^{0} \frac{\sqrt{1-\cos^2\theta}}{\cos^2\theta} \times -\sin\theta\, d\theta = \int_0^{\pi/3} \frac{\sqrt{\sin^2\theta}}{\cos^2\theta} \sin\theta\, d\theta$

$= \int_0^{\pi/3} \frac{\sin^2\theta}{\cos^2\theta}\, d\theta$

$= \int_0^{\pi/3} \tan^2\theta\, d\theta$

$= \int_0^{\pi/3} \sec^2\theta - 1\, d\theta$

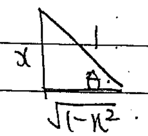
$= [\tan\theta - \theta]_0^{\pi/3}$

$= \sqrt{3} - \frac{\pi}{3}$

e) $x = \sin t$ ① $\quad y = 2 + \cos t$ ②

From ① $t = \sin^{-1} x$. sub → ②

$y = 2 + \cos \sin^{-1} x$



$\therefore y = 2 \pm \sqrt{1-x^2}$ $\quad -\frac{1}{2}$

Do not write in this box

11

Q3

a) $\sin(2\sin^{-1}\frac{3}{4}) = 2\sin(\sin^{-1}\frac{3}{4})\cdot\cos(\sin^{-1}\frac{3}{4})$

$4^2 = 3^2 + x^2$

3, 4, θ, $\sqrt{7}$

$= 2\cdot\frac{3}{4}\cdot\frac{\sqrt{7}}{4}$

$= \frac{6\sqrt{7}}{4}$

$= \frac{3\sqrt{7}}{2}$

2

b) $\ln(2x+3)+\ln(x-2)=2\ln(x+4)$

$\cancel{\ln\frac{2x+3}{x+}}$ $\ln(2x+3)(x-2) = \ln(x+4)^2$

$(2x+3)(x-2) = (x+4)^2$

$2x^2-4x+3x-6 = x^2+8x+16$

$x^2-9x-22=0$

$(x+2)(x-11)=0$

$x=11$ since $x\neq 2$ ~~log(x-2)~~

c) $\frac{2x+1}{x-2} \geq 1$

$(2x+1)(x-2) \geq (x-2)^2$

$2x^2 \cancel{-4x} + x - 2 \geq x^2 \cancel{-4x} + 4$

$x^2+x-6 \geq 0$

$(x+3)(x-2) \geq 0$ -3 2

$x \leq -3$ or $x > 2$. since $x \neq 2$

d) i) $\dot{x} = -\frac{\sqrt{3}}{2}\sin 2t - \frac{1}{2}\cos 2t$

$\ddot{x} = -\frac{\sqrt{3}}{4}\cos 2t + \frac{1}{4}\sin 2t$ incorrect coefficients!

$= -\frac{1}{4}(\sqrt{3}\cos 2t - \sin 2t)$

$= -\frac{1}{4}x$ ouch! $\ddot{x} = -4x$

$\ddot{x} = -n^2 x$

$\therefore$ in SHM

ii) $\sqrt{3}\cos 2t - \sin 2t = 0$ when at origin.

Auxiliary $\angle$ method.

$R = \sqrt{3+1}$

$= 2$.

~~$\cos\sin(2t-\alpha)$ sin sin 2t cos~~

$\cos(2t+\alpha) = \cos 2t\cos\alpha - \sin 2t\sin\alpha$.

comparing coefficients

$\cos\alpha = \sqrt{3}$ $\sin\alpha = 1$.

$\frac{\sin\alpha}{\cos\alpha} = \frac{1}{\sqrt{3}}$

$\tan\alpha = \frac{1}{\sqrt{3}}$

$\alpha = \frac{\pi}{6}$

$\therefore 2\cos(2t+\frac{\pi}{6}) = 0$

$2t+\frac{\pi}{6} = \frac{\pi}{2}$

$2t = \frac{\pi}{3}$

$t = \frac{\pi}{6}$ incorrect from above!

$\therefore v$ when first at origin $\dot{x} = -\frac{\sqrt{3}}{2}\sin\frac{\pi}{3} - \frac{1}{2}\cos\frac{\pi}{3}$

$= -\frac{\sqrt{3}}{2}\times\frac{\sqrt{3}}{2} - \frac{1}{4}$

$= -\frac{3}{4} - \frac{1}{4}$

$= -1 ms^{-1}$ X c

1

1

1

5

4

d) i) $x = \sqrt{3}\cos 2t - \sin 2t$

~~$\dot{x} = \frac{-\sqrt{3}}{2}\sin 2t + -\frac{1}{2}\cos 2t$~~

~~$\ddot{x} = -\frac{\sqrt{3}}{4}$~~

$x = \sqrt{3}\cos 2t - \sin 2t$

$\dot{x} = -2\sqrt{3}\sin 2t - 2\cos 2t$

$\ddot{x} = -4\sqrt{3}\cos 2t + 4\sin 2t$

$= -4(\sqrt{3}\cos 2t - \sin 2t)$

$= -4x$

ii) v is first at the origin when $t = \frac{\pi}{6}$

$\dot{x} = -2\sqrt{3}\sin\frac{\pi}{3} - 2\cos\frac{\pi}{3}$

$= ~~-2\sqrt{3} \times \frac{1}{2} - 2 \times \frac{\sqrt{3}}{2}~~ \quad -2\sqrt{3} \times \frac{\sqrt{3}}{2} - 2 \times \frac{1}{2}$

$= ~~\sqrt{3} - \sqrt{3}~~ \quad -3-1$

$= ~~-2\sqrt{3}~~ \quad -4\ \text{ms}^{-1}$

Q4

a) i) $V = \frac{\pi}{3}x^2(3r - x) = \frac{\pi}{3}(3r)x^2 - \frac{\pi}{3}x^3 = \pi r x^2 - \frac{\pi}{3}x^3$

$\frac{dx}{dt} = \frac{dV}{dt} \times \frac{dx}{dV}$

$\frac{dV}{dx} = 2\pi r x - \pi x^2$

$\frac{dx}{dt} = k \times \frac{1}{2\pi r x - \pi x^2}$ ✓

$= \frac{k}{\pi x(2r - x)}$ qed

ii) $\frac{dt}{dx} = \frac{\pi x(2r - x)}{k}$

$t = \int \frac{\pi x(2r - x)}{k}\, dx$

$= \frac{\pi}{k}\int x(2r - x)\, dx$

$= \frac{\pi}{k}\int 2rx - x^2\, dx$

$= \frac{\pi}{k}\left(rx^2 - \frac{x^3}{3}\right) + C$

when $t = 0$ $x = 0$

$0 = \frac{\pi}{k}(0 - 0) + C$

$C = 0$

$\therefore t = \frac{\pi}{k}\left(rx^2 - \frac{x^3}{3}\right)$ ✓

b) $f(x) = 1-\cos\frac{x}{2}$ let $y = 1-\cos\frac{x}{2}$.

i) Swap x and y.

$x = 1-\cos\frac{y}{2}$.

$\cos\frac{y}{2} = 1-x$

$\frac{y}{2} = \cos^{-1}(1-x)$

$\therefore y = 2\cos^{-1}(1-x)$ qed. ✓

$\cancel{a \le 1-x \le 2\pi \quad 1-x \le \pi \quad -1 \le 1-x \le 1}$

$\cancel{-x \le -2}$

x

~~Largest value of a is 2. since -1≤1-x≤1~~

~~i) $f^{-1}(x) = 2\pi$~~

~~ii)~~

i) For inverse to exist.

$0 \le x \le a$.

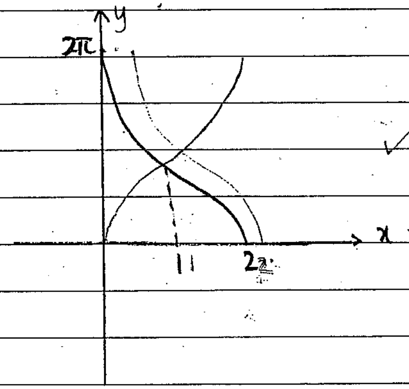
$0 \le \frac{x}{2} \le \pi$.

$0 \le x \le 2\pi$

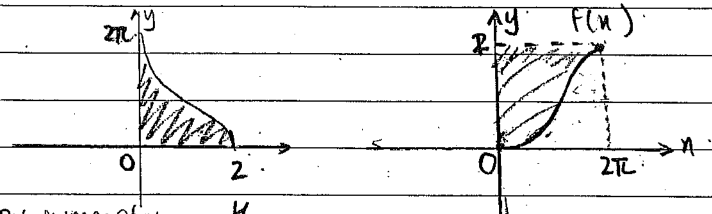
$\therefore$ max a is 2π.

I don't follow your logic but correct answer.

iii).



✓



By symmetry - they are same.

Area $= 4\pi - \int_0^{2\pi} 1-\cos\frac{x}{2}.\,dx$.

$= 4\pi - \left[x - \cancel{4}\, 2\sin\frac{x}{2}\right]_0^{2\pi}$

$= 4\pi - 2\pi - 2\sin\pi - 0 + 0$

$= 4\pi - 2\pi \ u^2$.

$= 2\pi u^2$.

Q5 left out a bit!

a) i) $\ddot{x}=0$ ← missed $\dot{x}=C_1$

$\dot{x}=V\cos 30°$

$=\frac{V\sqrt{3}}{2}$

$x=\frac{V\sqrt{3}}{2}t+C_2$

when $t=0$ $x=0$.

$0=\frac{V\sqrt{3}}{2}(0)+C_2$

$C_2=0$

$\therefore x=\frac{V\sqrt{3}t}{2}$ qed ①

$\ddot{y}=-g$ $\cancel{y=-gt+C_2}$

$\dot{y}=-gt+C_1$

when $t=0$ $\dot{y}=V\sin 30°$

$V\sin 30°=0+C_1$

$C_1=V\sin 30°$

$\dot{y}=-gt+V\sin 30°$

$=-gt+\frac{V}{2}$

$y=\frac{-gt^2}{2}+\frac{V}{2}t+C_3$

when $y=6$ $t=0$

$6=0+0+C_3$

$C_3=6$

$\therefore y=\frac{-gt^2}{2}+\frac{V}{2}t+6$ qed ②

$2\frac{1}{2}$

ii) cartesian

From ① $t=\frac{2x}{V\sqrt{3}}$ ③

sub ③ → ②:

$y=-\frac{g}{2}\left(\frac{4x^2}{V^2 3}\right)+\frac{V}{2}\left(\frac{2x}{V\sqrt{3}}\right)+6$

$=\frac{-2gx^2}{3V^2}+\frac{x}{\sqrt{3}}+6$

2

iii) LHS = 50

RHS $=\cancel{-2g(50}\ \frac{-2g(355)^2}{3(65)^2}+\frac{355}{\sqrt{3}}+6$

$=16.08\ldots<50$

$\therefore$ the ball will not clear.

3

b) $\left(1-\frac{2}{3}\right)\left(1-\frac{2}{4}\right)\left(1-\frac{2}{5}\right)\cdots\left(1-\frac{2}{n}\right)=\frac{2}{n(n-1)}$ $n\geq 3$

Step 1: Prove true for $n=3$.

LHS $=1-\frac{2}{3}$ $=\frac{1}{3}$

RHS $=\frac{2}{3(3-1)}$ $=\frac{1}{3}$ $=$ LHS

$\therefore$ True for $n=3$

Step 2: Assume true for $n=k$ i.e. $\left(1-\frac{2}{3}\right)\left(1-\frac{2}{4}\right)\left(1-\frac{2}{5}\right)\cdots\left(1-\frac{2}{k}\right)=\frac{2}{k(k-1)}$

RTP true for $n=k+1$ i.e. $\left(1-\frac{2}{3}\right)\left(1-\frac{2}{4}\right)\left(1-\frac{2}{5}\right)\cdots\left(1-\frac{2}{k}\right)\left(1-\frac{2}{k+1}\right)=\frac{2}{(k+1)k}$.

LHS $=\left(1-\frac{2}{3}\right)\left(1-\frac{2}{4}\right)\cdots\left(1-\frac{2}{k}\right)\left(1-\frac{2}{k+1}\right)$

$=\frac{2}{k(k-1)}\left(1-\frac{2}{k+1}\right)$

$=\frac{2}{k(k-1)}\left(\frac{k+1-2}{k+1}\right)$

$=\frac{2}{k(k-1)}\left(\frac{k-1}{k+1}\right)$

$=\frac{\cancel{2k+2}\ 2k-2}{k(k+1)(k-1)}$

$=\frac{2\cancel{(k-1)}}{k(k+1)\cancel{(k-1)}}$

$=\frac{2}{k(k+1)}=$ RHS

$\therefore$ True for $n=k+1$ if

Step 3: $\therefore$ by the principle of mathematical induction the result holds true for all $n\geq 3$. ?

true for $n=k+1$

true for $n=3$

true for all $n\geq 3$

Summary statement?

$\cancel{3\frac{1}{2}}$ 4

Q6

a) $(8p, 4p^2)$ $(8q, 4q^2)$.

$x^2 = 16y$.

i) $m_{PQ} = \frac{4q^2-4p^2}{8q-8p}$

$= \frac{\cancel{8}4\cancel{(q-p)}(p+q)}{2\cancel{8}\cancel{(q-p)}}$

$= \frac{p+q}{2}$

$y - 4p^2 = \frac{p+q}{2}(x-8p)$

$y = \frac{1}{2}(p+q)x - 4p(p+q) + 4p^2$

$= \frac{1}{2}(p+q)x - 4pq$ qed ①

ii). Sub $(4,0) \rightarrow$ ①

$0 = \frac{1}{2}(p+q)4 - 4pq$.

$4pq = 2(p+q)$

$p+q = 2pq$ qed.

iii) Tangent P

$m = \frac{8p}{8}$

$= p$.

$y - 4p^2 = p(x-8p)$

$y = px - 8p^2 + 4p^2$

$y = px - 4p^2$

Similarly Tangent Q : $y = qx - 4q^2$

Find R solve simultaneously.

$px - 4p^2 = qx - 4q^2$.

$x(p-q) = 4p^2 - 4q^2$

$x\cancel{(p-q)} = 4(p+q)\cancel{(p-q)}$

$x = 4(p+q)$

$y = 4p(p+q) - 4p^2$

$= 4p^2 + 4pq - 4p^2$.

$= 4pq$.

$\therefore R(4(p+q), 4pq)$

iv) $x = 4(p+q)$ ① $y = 4pq$ ②

$= 4(2pq)$

$= 8pq$.

$pq = \frac{x}{8}$ ③ $\rightarrow$ ②

$\therefore y = \frac{4x}{8}$ is the locus of R.

$= \frac{x}{2}$ Simplify

b) $x_0 = 0.8$

$f(x) = e^{2x} - x - 3$

$f'(x) = 2e^{2x} - 1$

$x_1 = 0.8 - \frac{f(x_0)}{f'(x_0)}$

$= 0.8 - \frac{e^{0.8\times2} - 0.8 - 3}{2e^{2\times0.8} - 1}$

$\doteq 0.6705\ldots$

$= 0.671$ (to 3 sf)

c) $y = \frac{x}{(x-1)^2}$

~~At~~ vert. Asymptote $x = 1$.

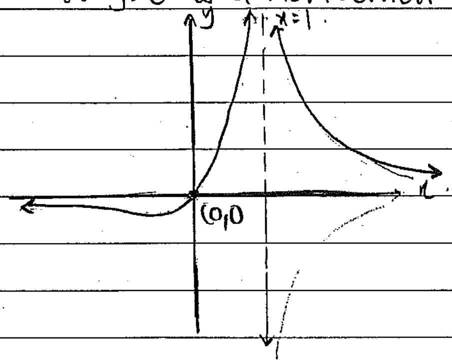
$y = \frac{x}{x^2 - 2x + 1}$

$= \frac{\frac{1}{x}}{1 - \frac{2}{x} + \frac{1}{x^2}}$

For horizontal asymp. as $x \to \infty$ $y \to \frac{\frac{1}{\infty}}{1 - \frac{2}{\infty} + \frac{1}{\infty}}$

$= 0$

$\therefore y = 0$ is a horizontal asymptote.



Q7

a) i) $\frac{dP}{dt} = 0 + kAe^{kt}$

$= k(P - 4000)$ since $Ae^{kt} = P - 4000$

$\therefore$ satisfies eqn.

ii) $t = 0$ $P = 5000$

$5000 = 4000 + Ae^0$

$A = 1000$

$t = 10$ $P = 15000$

$15000 = 4000 + 1000e^{10k}$

$11000 = 1000e^{10k}$

$11 = e^{10k}$

$\ln 11 = 10k$

$k = \frac{\ln 11}{10}$

iii) $4000 + 1000e^{\frac{\ln 11}{10}t} > 125000$

$\cancel{1000}e^{\frac{\ln 11}{10}t} > 121\cancel{000}$

$e^{\frac{\ln 11}{10}t} > 121$

$\frac{\ln 11}{10}(t) > \ln 121$

21st $t > 20$

$\therefore$ in the 20th year it will exceed \$125 000

$-\frac{1}{2}$

b) i) $PD = DQ = CD$ (Tangents from a common point) → radii of a circle.
∴ PQ is the ~~radius~~ diameter of a circle.
$\angle PCQ = 90°$ ($\angle$ in a semicircle).
ii) $\angle RCQ = 90°$ ($\angle$ in a semicircle)
$\angle RCQ + \angle PCQ = 90 + 90$ (adjacent $\angle$s)
$= 180°$
∴ ~~CQ~~ RCP are collinear.
iii) BD ∥ RCP ~~&~~ BD bisects $\angle CDQ$ (common tangents have the same $\angle$ of elevation).
~~BD ⊥ CQ (∠ from radii~~
~~BD bisects CQ.~~
Prove $\triangle CDA \equiv \triangle DAQ$
1. $\angle CDA = \angle ADQ$ (proven above).
~~iv) ∠BQD = 90°~~ 2. $CD = CQ$ (~~common~~ tangents from a common point).
3. AD is common
∴ $\triangle CDA \equiv \triangle DAQ$ (SAS)
$CA = AQ$ (matching sides of congruent $\triangle$s).
$BD \perp CQ$ (~~ra~~ line from centre bisecting chord CQ is ⊥ to the chord)
∴ PCB ∥ BD (~~cont~~ cointerior angles are supplementary)
$\angle PCA + DAC = 90 + 90 = 180°$.
~~iv) ∠CPD = 90 − ∠CQP (∠ sum of △CPQ).~~
~~∠SRB = ∠CQP (∠ between tangent and chord is equal to ∠ in alternate segment).~~
~~∠SRC~~
~~iv) ∠SRB = 90° (radius ⊥ tangent).~~
~~∠SRC = 90 − ∠CRB (adjacent ∠s).~~
Need to prove that $\angle PSR$ is supplementary to $\angle BQD$
(~~∠s~~ interior $\angle$s of cyclic quadrilaterals are supplementary)

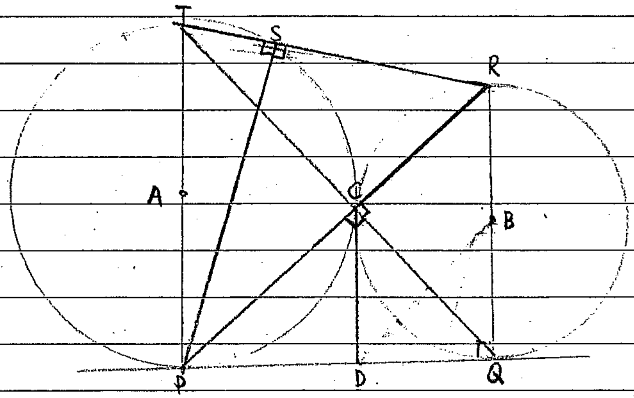
-2

$\angle RQD = 90°$ ($\angle$ between radius and tangent).
~~∠PRQ = Let ∠PRQ = x~~
Let $\angle CQP = x$.
$\angle PRQ = x$ (~~alt~~ $\angle$ in alternate segment = $\angle$ ~~in~~ between chord and tangent).
$\angle RPQ = 90 - x$ ($\angle$ sum $\triangle PRQ$).

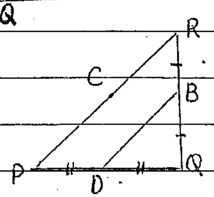
$-1\frac{1}{2}$

Q7

~~(ii) Join PC to R.~~



(iii) Join PC to R and join D to B.



PCR is a straight line.

$\frac{RB}{BQ} = 1$ (equal radii)

$\frac{PD}{DQ} = 1$ (equal tangents)

$\therefore$ RCP ∥ BD (BD divides RQ and PQ in the same ratio and is thus parallel to the 3rd side).

(iv) $\angle TSP = 90°$

$\angle RSP = 90°$ (straight line)

$\angle RQP = 90°$ (tangent ⊥ Radius)

$\angle RSP = \angle RQP = 90°$

$\therefore$ SRQP is concyclic (opposite ∠s are supplementary).