Ascham School

Mathematics Form 6 - 2 Unit Trial Examination

2004

July 2004

Time allowed: 3 Hours
Plus 5 minutes reading time.

Instructions

1. Attempt ALL questions
2. All questions are of equal value.
3. All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
4. Standard integrals are printed on page 11.
5. Board approved calculators may be used.
6. Answer each question in a *separate* writing booklet.

Question 1 (12 marks) — Marks

(a) Find the value of $\log_e 3$ correct to 3 significant figures. — 2

(b) Simplify $\frac{5}{x+1}-\frac{3}{x^2-1}$ — 2

(c) Solve $15x^3 = 10x$ — 2

(d) Find the primitive function of $\sec^2 5x$ — 2

(e) Solve the pair of simultaneous equations — 2

$x + y = 2$

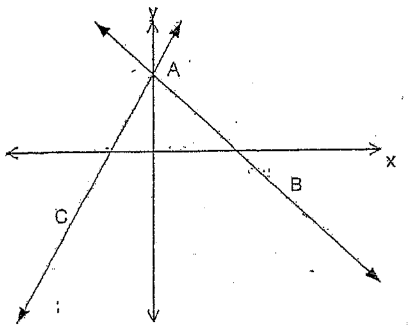
$2x - y = 7$

(f) Graph the solution of $|2x-1| < 7$ on a number line. — 2

Question 2 (12 marks)

(a) Find the equation of the normal to the curve $y = e^{2x}$ at the point (0, 1). **3**

(b)



not to scale

The diagram shows the points A(0,2), B (4, -2) and C(-3,-4) on the number plane.

(i) Calculate the gradient of AB. **1**

(ii) Show that the equation of AB is $x + y - 2 = 0$ **1**

(iii) Find the angle of inclination of the line AB . **2**

(iv) Calculate the exact perpendicular distance of C from the line AB **2**

(v) Find the area of ΔABC. **2**

(vi) If ABCD is a parallelogram, find the area of ABCD. **1**

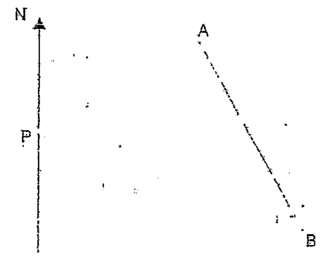
Question 3 (12 marks)

(a) Differentiate with respect to x:

(i) $2\cos(3x)$ **2**

(ii) xe^{2x} **2**

(b)



Ship A is 20 nautical miles from a port P and is on a bearing of 055°T.
Ship B is 27 nautical miles from P and is on a bearing of 115°T.

(i) Copy the diagram and explain why $\angle APB = 60^{0}$. **2**

(ii) Calculate the distance between the two ships, giving your answer correct to one decimal place. **2**

(c) (i) Find $\int \sin\frac{x}{3}\,dx$ **2**

(ii) Find the exact value of $\int_0^1 \frac{dx}{1+2x}$ **2**

Question 4 (12 marks)

(a) Consider the function $f(x) = x^4 - 2x^3$.

(i) Show that $f'(x) = 2x^2(2x-3)$. **1**

(ii) Find the coordinates of the stationary points of the curve $y = f(x)$, and determine their nature. **3**

(iii) Find any points of inflexion. **2**

(iv) Sketch the graph of the curve $y = f(x)$, showing the significant points. Use a reasonably big scale. **1**

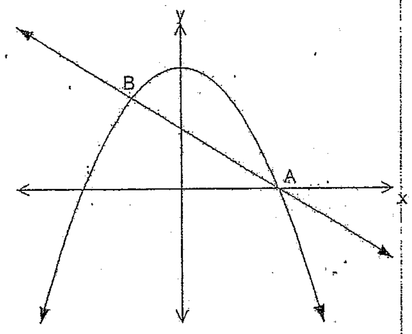
(v) Find the values of x for which the graph of $y = f(x)$ is concave down. **2**

(b) The area under the curve $y = x + \frac{1}{x}$, for $1 \le x \le 3$ is rotated about the x axis. Find the exact volume of the solid of revolution **3**

Question 5 (12 marks)

(a) Find the values of k for which the quadratic equation $4x^2 - kx + 9 = 0$ has real roots. **3**

(b)



The graphs of $y = 4 - x^2$ and $y = 2 - x$ intersect at the points A (2, 0) and B, as shown in the diagram.

(i) Find the coordinates of B. **2**

(ii) Find the shaded region bounded by $y = 4 - x^2$ and $y = 2 - x$ **3**

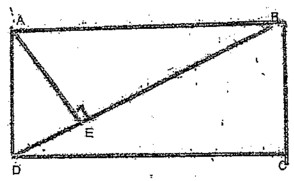
(c) Sally joined a superannuation scheme at the beginning of 1964. She invested \$1000 at the beginning of each year earning 8% per annum which compounded annually.

Find the value of Sally's superannuation when she retires on 31 December 2004. **4**

Question 6 (12 marks)

(a) Can there be an infinite series with a limiting sum of 5/8 and a first term of 2? All working and reasoning must be shown. 2

(b) ABCD is a rectangle and E is a point on the diagonal BD so that AE is perpendicular to BD.



(i) Prove that ΔADE is similar to ΔDBC. 2

(ii) If AD=5cm and DE=2cm, find the length of the diagonal BD. 2

(c) A bottle of solvent is open and the solvent evaporates in such a way that the amount remaining, V ml, in the bottle is given by $V = 2000e^{-0.005t}$, where t is time in hours.

(i) How much solvent has evaporated out of the bottle after 24 hours? 2

(ii) How long is it before half the initial amount of solvent has evaporated from the bottle? (answer in hours and minutes, to the nearest minute) 2

(iii) If the solvent continues to evaporate will the bottle ever empty? Explain. 2

Question 7 (12 marks)

(a) Solve $2\ln x = \ln(5+4x)$ 2

(b) A tap and n water troughs are in a straight line. The tap is first in line, 6 metres from the first trough. A person fills the troughs by carrying a bucket of water from the tap to each trough and then returning to the tap. They repeat this process with each successive trough 4 metres further from the tap than the previous trough.

(i) How many troughs would there be if the trough furthest from the tap is 346m away from the tap? 2

(ii) How many troughs would there be if the person walks 1760 metres to fill all the troughs? 3

(c) Draw a parabola with the following properties:

$f(3) = 4$ $f'(x) < 0$ for $x < 3$ $b^2 - 4ac < 0$ 2

(d) Find the co-ordinates of the focus and directrix of the parabola $y = x^2 - 2x + 5$ 3

Question 8 (12 marks)

(a) (i) Use the trapezoidal rule with three function values to estimate $\int_0^2 \sqrt{4-x^2}\,dx$ (to 2 decimal places) 3

(ii) Is this approximation an over or under estimate of the exact value? Explain. 1

(iii) What is the exact value of the definite integral? 2

(b) Given that $x = 3$ is one root of the quadratic equation $mx^2 - 20x + m = 0$, find the exact value of the other root. 2

(c) If $\frac{d^2x}{dt^2} = 6 + e^{-t}$ and $\frac{dx}{dt} = -1$ at $t = 0$, and $x = 0$ at $t = 0$, find an expression for x in terms of t. 4

Question 9 (12 marks)

(a) (i) On the same set of axes, draw the graphs of $y = 2\sin x$ and $y = \tan x$, $0 \le x \le 2\pi$. **2**

(ii) Use your graphs to determine the number of solutions to $2\sin x = \tan x$ within this domain. **1**

(iii) From the equation $2\sin x = \tan x$, show that $\sin x(2\cos x - 1) = 0$. **1**

(iv) Find all the values of x, where $0 \le x \le 2\pi$, that are solutions to the equation $2\sin x = \tan x$. **2**

(b) A particle P moves along a straight line so that at time, its displacement from a fixed point O on the line is given by $x = 3 + 4t - 5\sqrt{t^2 + 4}$ where t is time in seconds.

(i) Show that the velocity of the particle at time t is $4 - \dfrac{5t}{\sqrt{t^2 + 4}}$ **2**

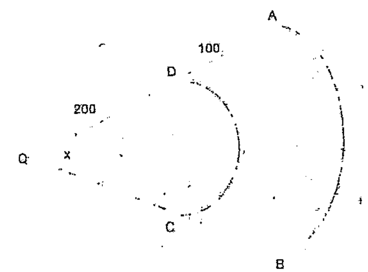
(ii) Find the time when the particle is momentarily at rest. **2**

(iii) How far has the particle travelled in the first 4 seconds? **2**

Question 10 (12 marks)

(a) A vertical line x = p is drawn to cross the two graphs $y = \log_e x$ and $y = \log_e 2x$ at the points A and B. Show that the distance AB is constant. **2**

(b)



In the figure, AB and CD are circular arcs which subtend an angle x radians at the center Q where $0 < x < \pi$. The length AD is 100 metres and DQ is 200 metres.

Alice lives at A and there is a bus stop at B, with paths AB, BC, CD and DA in the figure forming a road system. For what values of x is it shorter for Alice to walk along the route ADCB rather than along the arc AB? **2**

(c) A straight line passing through the point (2,3) cuts the x and y axes at $(p,0)$ and $(0,q)$ respectively where p>0 and q>0.

(i) Show that $3p + 2q = pq$. **2**

(ii) Show that $A = \dfrac{q^2}{q-3}$ where A is the area of the triangle formed by the line and the coordinate axes. **2**

(iii) Find the minimum value of A. **4**

END OF EXAM

FORM 6 MATHEMATICS (2U) TRIAL 2004

QUESTION 1

a) $\log_e 3 \doteqdot 1.0986\ldots$

$= 1.10$ (3 sf)

b) $\frac{5}{x+1} - \frac{3}{x^2-1} = \frac{5x-5-3}{(x+1)(x-1)}$

$= \frac{5x-8}{(x+1)(x-1)}$

c) $15x^2 = 10x$

$15x^2 - 10x = 0$

$5x(3x-2) = 0$

$x = 0$ or $x = \frac{2}{3}$

d) $\int \sec^2 5x\,dx = \frac{1}{5}\tan 5x + c$

e) $x + y = 2$ — ①

$2x - y = 7$ — ②

①+②: $3x = 9$

$x = 3$

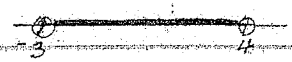
Sub in ① $y = -1$

f) $|2x-1| < 7$

$-7 < 2x-1 < 7$

$-6 < 2x < 8$

$-3 < x < 4$



QUESTION 2

a) $y = e^{2x}$

$y' = 2e^{2x}$ ✓

when $x=0$ grad of tangent $= 2e^0$

$\therefore$ grad of normal $= -\frac{1}{2}$ ✓

eqn of normal:

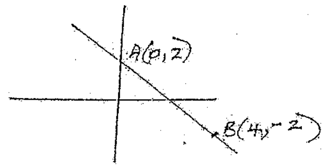
$y - 1 = -\frac{1}{2}(x-0)$

$y - 1 = -\frac{1}{2}x$

$2y - 2 = -x$ ✓ ③

$x + 2y - 2 = 0$

b)



i) grad AB $= \frac{y_2-y_1}{x_2-x_1}$

$= \frac{2+2}{0-4}$ ①

$= -1$

ii) Eqn AB: $y - y_1 = m(x-x_1)$

$y - 2 = -1(x-0)$ ✓ ①

$x + y - 2 = 0$

iii) Let ∠ of inclination be θ

$\tan\theta = -1$ ✓

$\theta = 135°$ ✓ ②

∴ ∠ of inclination is 135°

iv) Perp dist $= \frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$

$= \frac{|1\times -3 + 1\times -4 - 2|}{\sqrt{1^2+1^2}}$ ✓

$= \frac{|-9|}{\sqrt{2}}$

$= \frac{9}{\sqrt{2}}$ units ✓ ②

v) $AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{4^2+(-4)^2}$

$= \sqrt{32}$ ✓

Area of ΔABC $= \frac{1}{2}\sqrt{32} \times \frac{9}{\sqrt{2}}$

$= 18u^2$ ✓ ②

vi) Area of llgram = 2 Area ΔABC

$= 36u^2$ ①

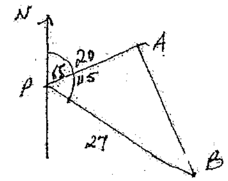
QUESTION 3

a) i) $\frac{d}{dx} 2\cos 3x = -6\sin 3x$

ii) $\frac{d}{dx} xe^{2x} = x.2e^{2x} + e^{2x}$

$= 2xe^{2x} + e^{2x}$

b)



i) $\angle APB = 115° - 55°$ (adj ∠s)

$= 60°$

ii) $AB^2 = 20^2 + 27^2 - 2\times20\times27\cos 60°$

$= 589$

$AB \doteqdot 24.269\ldots$

∴ Distance between 2 ships is 24.3 n.m. (1 dp)

c) i) $\int \sin\frac{x}{3}\,dx = -3\cos\frac{x}{3} + c$

ii) $\int_0^1 \frac{dx}{1+2x} = \frac{1}{2}\left[\ln(1+2x)\right]_0^1$

$= \frac{1}{2}\ln 3 - \frac{1}{2}\ln 1$

$= \frac{1}{2}\ln 3$

QUESTION 4

a) $f(x) = x^4 - 2x^3$

i) $f'(x) = 4x^3 - 6x^2$

$= 2x^2(2x-3)$

ii) For stat pts $f'(x) = 0$

$\therefore x = 0$ or $x = 1\frac{1}{2}$

$y = 0$ or $y = -1.6875$

∴ stat pts are $(0,0)$ $(1\frac{1}{2}, -1.6875)$

Nature of stat pts

$f''(x) = 12x^2 - 12x$

If $x = 0$ $f''(x) = 0$

∴ (0,0) is a poss horiz pt of infl

Test for change in concavity

x	-1	0	$\frac{1}{2}$
$f''(x)$	24	0	-3

∴ change in concavity

∴ (0,0) a horiz pt of inflexion

If $x = 1.5$

$f''(x) = 9$

∴ (1.5, -1.6875) is a min turning pt

OR

x	-1	0	1	1.5	2
$f'(x)$	-10	0	-2	0	12

with conclusions.

iii) For poss pts of inflexion $f''(x) = 0$

$12x(x-1) = 0$

$x = 0$ or $x = 1$

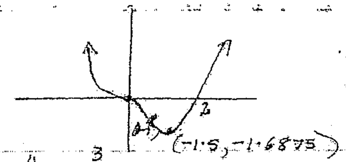
If $x = 1$ check for change in conc.

x	$\frac{1}{2}$	1	2
$f''(x)$	-3	0	24

∴ Change in concavity

∴ (1, -1) is pt. of inflexion

iii)



$y = x^4 - 2x^3$

on x axis: $x^3(x-2) = 0$

$x = 0$ or 2.

iv) Curve is concave down for $0 < x < 1$

b) $\text{Volume} = \pi\int_1^3 \left(x + \frac{1}{x}\right)^2 dx$

$= \pi\int_1^3 x^2 + 2 + \frac{1}{x^2}\, dx$

$= \pi\int_1^3 x^2 + 2 + x^{-2}\, dx$

$= \pi\left[\frac{x^3}{3} + 2x + \frac{x^{-1}}{-1}\right]$

$= \pi\left[9 + 6 - \frac{1}{3} - \left(\frac{1}{3} + 2 - 1\right)\right]$

$= \pi\left[13\frac{1}{3}\right]$

$= \frac{40\pi}{3}$ units^3

QUESTION 5

a) $4x^2 - kx + 9 = 0$

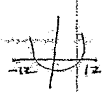
For real roots $\Delta \geq 0$

$b^2 - 4ac \geq 0$

$k^2 - 4 \times 4 \times 9 \geq 0$

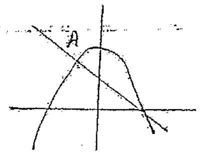
$k^2 - 144 \geq 0$

$(k-12)(k+12) \geq 0$



$k \leq -12$ or $k \geq 12$

b)



i) $y = 4 - x^2$ — ①

$y = 2 - x$ — ②

at pts of int: $4 - x^2 = 2 - x$

$x^2 - x - 2 = 0$

$(x+1)(x-2) = 0$

$x = -1$ or $x = 2$

$y = 3$ or $y = 0$

$\therefore$ A is point $(-1, 3)$

ii) Shaded region $= \int_{-1}^{2} 4 - x^2 - (2 - x)\, dx$

$= \int_{-1}^{2} 4 - x^2 - 2 + x\, dx$

$= \int_{-1}^{2} 2 - x^2 + x\, dx$

$= \left[2x - \frac{x^3}{3} + \frac{x^2}{2}\right]_{-1}^{2}$

$= 4 - \frac{8}{3} + 2 - \left(-2 + \frac{1}{3} + \frac{1}{2}\right)$

$= 3\frac{1}{3} - \left(-1\frac{1}{6}\right)$

$= 4\frac{1}{2}\ u^2$

c) Let A_n be value of n^{th} investment

$A_1 = 1000(1.08)^{41}$

$A_2 = 1000\ (1.08)^{40}$

$A_{41} = 1000(1.08)^1$

$\text{Total} = 1000(1.08 + 1.08^2 + \ldots\ 1.08^{41})$

$= 1000 \times 1.08 \times \frac{1.08^{41} - 1}{0.08}$

$= \$303\ 243.52$

QUESTION 6

a) $S_\infty = \frac{a}{1-r}$

$\frac{5}{8} = \frac{2}{1-r}$

$5 - 5r = 16$

$-5r = 11$

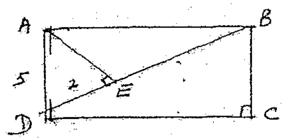
$r = -\frac{11}{5}$

For lim sum to exist $-1 < r < 1$

But $r < -1$

$\therefore$ not possible to have a limiting sum with these conditions

b)



i) In Δs ADE, DBC

$\angle AED = \angle BCD$ (rt $\angle$s)

$\angle ADE = \angle DBC$ (alt $\angle$s, AD ∥ BC)

$\therefore \Delta ADE\ |||\ \Delta DBC$ (equiangular)

ii) $\therefore \frac{AD}{DB} = \frac{DE}{BC} = \frac{AE}{DC}$ (corresp sides of sim Δs)

$\frac{5}{DB} = \frac{2}{5}$

$DB = \frac{25}{2}$

$= 12.5$ cm.

c) $V = 2000e^{-0.005t}$

i) $t = 24$ $\quad V \doteqdot 1773.84\ldots$

$\therefore$ Amt evaporated $= 2000 - 1773.84\ldots$

$= 226.16$ ml (2dp)

ii) $V = 1000$: $\quad 1000 = 2000 \cdot e^{-0.005t}$

$\frac{1}{2} = e^{-0.005t}$

$\ln\frac{1}{2} = -0.005t$

$t = \frac{\ln\frac{1}{2}}{-0.005}$

$\doteqdot 138.63$ hrs.

$\therefore$ it takes 138 hrs + 38 min (to nearest min)

iii) as $t \to \infty$: $V \to \frac{2000}{e^{0.005t}}$

$V \to 0$

$\therefore$ bottle will eventually be almost empty.

QUESTION 7

a) $2\ln x = \ln(5 + 4x)$

$\ln x^2 = \ln(5 + 4x)$

$x^2 = 5 + 4x$

$x^2 - 4x - 5 = 0$

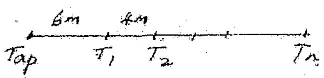
$(x-5)(x+1) = 0$

$x = 5$ or -1

but $x \neq -1$ (can't have log of neg no)

$\therefore x = 5$

b)



i) From T_1 to T_n $= 346 - 6$

$= 340$ m.

$\therefore$ no of taps $= 340 \div 4 + 1$

$= 85 + 1$

$= 86$

ii) $2 \times S_n = 1760$

$S_n = 880$

$\frac{n}{2}[2a + (n-1)d] = 880$

$n[12 + (n-1) \times 4] = 1760$

$n[12 + 4n - 4] = 1760$

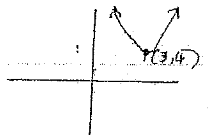
$4n^2 + 8n - 1760 = 0$

$n^2 + 2n - 440 = 0$

$(n+22)(n-20) = 0$

$n = 20$ only

c)



d) $y = x^2 - 2x + 5$

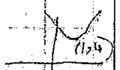
$y - 5 = x^2 - 2x$

$y - 5 + 1 = x^2 - 2x + 1$

$(x-1)^2 = y - 4$

$(x-1)^2 = 4 \times \frac{1}{4}(y-4)$

$a = \frac{1}{4}$ Vertex $= (1,4)$



Focus $= (1, 4\frac{1}{4})$

Directrix: $y = 3\frac{3}{4}$

QUESTION 8

a) i) $\int_0^2 \sqrt{4-x^2}\,dx$

$\doteqdot \frac{h}{2}\{y_0 + y_2 + 2y_1\}$

x	0	1	2
$\sqrt{4-x^2}$	2	$\sqrt{3}$	0
	y_0	y_1	y_2

$\doteqdot \frac{1}{2}(2 + 0 + 2\sqrt{3})$

$\doteqdot 1 + \sqrt{3}$

$\doteqdot 2.73$

ii) Under exact value because curve concave down so trapezia less in area.

iii) Exact value $= \frac{1}{4}\pi r^2$

$= \frac{1}{4}\pi \times 2^2$

$= \pi$

b) $mx^2 - 20x + m = 0$

$x = 3$ is root:

$9m - 60 + m = 0$

$10m = 60$

$m = 6$

$\therefore 6x^2 - 20x + 6 = 0$

$3x^2 - 10x + 3 = 0$

$(3x-1)(x-3) = 0$

$x = \frac{1}{3}$ or $x = 3$

$\therefore$ other root is $\frac{1}{3}$.

OR

Let other root be α.

$3\alpha = 1$

$\alpha = \frac{1}{3}$.

$\therefore$ other root $= \frac{1}{3}$

c) $\frac{d^2x}{dt^2} = 6 + e^{-t}$

$\frac{dx}{dt} = 6t - e^{-t} + c$

$t=0$ $\frac{dx}{dt} = -1$: $-1 = 0 - e^0 + c$

$-1 = -1 + c$

$c = 0$

$\frac{dx}{dt} = 6t - e^{-t}$

$x = 3t^2 + e^{-t} + d$

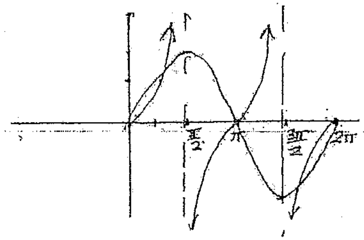
$x=0$ $t=0$: $0 = 0 + e^0 + d$

$d = -1$

$\therefore x = 3t^2 + e^{-t} - 1$

QUESTION 9

a) i)



ii) No of solns is 5

iii) $2\sin x = \tan x$

$2\sin x = \frac{\sin x}{\cos x}$

$2\sin x \cos x = \sin x$

$2\sin x \cos x - \sin x = 0$

$\sin x(2\cos x - 1) = 0$

iv) $\sin x = 0$ or $\cos x = \frac{1}{2}$

$x = 0°, 180°, 360°$, $x = 60°, 300°$

$x = 0, \pi, 2\pi, \frac{\pi}{3}, \frac{5\pi}{3}$

b) $x = 3 + 4t - 5\sqrt{t^2+4}$

i) $v = 4 - 5.\frac{1}{2}(t^2+4)^{-1/2}.2t$

$v = 4 - \frac{5t}{\sqrt{t^2+4}}$

ii) Particle at rest when $v = 0$

$4 = \frac{5t}{\sqrt{t^2+4}}$

$16 = \frac{25t^2}{t^2+4}$

$16t^2 + 64 = 25t^2$

$9t^2 = 64$

$t^2 = \frac{64}{9}$

$t = \pm\frac{8}{3}$

But $t \geq 0$

$\therefore t = 2\frac{2}{3}$

$\therefore$ particle at rest after $2\frac{2}{3}$ seconds

iii) $t=0$ $x = 3 + 0 - 5\sqrt{4}$

$= 3 - 10$

$= -7$

$t = 2\frac{2}{3}$ $x = 3 + 4 \times \frac{8}{3} - 5\sqrt{\frac{64}{9} + 4}$

$= -3$

$t = 4$ $x = 3 + 4 \times 4 - 5\sqrt{16+4}$

$= 3 + 16 - 5\sqrt{20}$

$= 19 - 10\sqrt{5}$

$= -3.36$ (2dp)

$\therefore$ Distance travelled $= 4 + 0.36$

$= 4.36$ m (2dp)

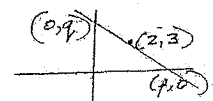
or $-3-(-7) + [-3-(19-10\sqrt{5})]$

$= 4 + (-3 - 19 + 10\sqrt{5})$

$= -18 + 10\sqrt{5}$

QUESTION 10

c)



i) Eqn of line: $y - 0 = -\frac{q}{p}(x - p)$

Sub(2,3): $3 = -\frac{q}{p}(2-p)$

$3p = -2q + qp$

$3p + 2q = pq$ ✓

(2)

ii) Area $= \frac{1}{2} pq$

From $3p + 2q = pq$

$3p - pq = -2q$

$p(3-q) = -2q$

$p = \frac{-2q}{3-q}$ ✓

$\therefore$ Area $= \frac{1}{2} \cdot \frac{-2q}{3-q} \times q$

$= \frac{-q^2}{3-q}$ ✓ (2)

$A = \frac{q^2}{q-3}$

iii) For min A; $A' = 0$ & $A'' > 0$

$A = \frac{q^2}{q-3}$

$\frac{dA}{dq} = \frac{(q-3)2q - q^2 \times 1}{(q-3)^2}$

$= \frac{2q^2 - 6q - q^2}{(q-3)^2}$

$= \frac{q^2 - 6q}{(q-3)^2}$ ✓

$\frac{d^2A}{dq^2} = \frac{(q-3)^2(2q-6) - (q^2-6q)2(q-3)}{(q-3)^4}$

$= \frac{2(q-3)^2(q-3) - 2q(q-6)(q-3)}{(q-3)^4}$

$= \frac{2(q-3)[(q-3)(q-3) - q^2 + 6q]}{(q-3)^4}$

$= \frac{2(q-3)(q^2 - 6q + 9 - q^2 + 6q)}{(q-3)^4}$

$= \frac{2(q-3)(9)}{\ldots}$

Let $\frac{dA}{dq} = 0$

$q^2 - 6q = 0$

$q(q-6) = 0$ ✓

$q = 0$ or 6

But $q \neq 0$ because then line does not go through $(2,3)$

If $q = 6$ $\frac{d^2A}{dq^2} = \frac{2(3)9}{3^4}$

> 0

$\therefore$ if $q = 6$ A is min. ✓

If $q = 6$ $p = \frac{-12}{3-6}$

$= 4$

$\therefore$ min $A = \frac{6^2}{6-3}$ or $\left(\frac{1}{2} 6 \times 4 = 12\right)$

$= \frac{36}{3}$

$= 12$ ✓

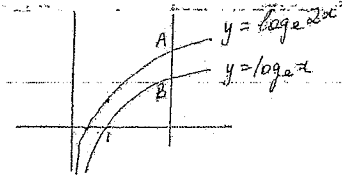
$\therefore$ min area is $12u^2$ (4)

OR

Instead of finding A''
Test for min in A' (or 1)

x	5	6	7
A'	$-\frac{5}{9}$	0	7

$\therefore$ min A if $x = 6$.



$y = \log_e 2x$

$= \log_e 2 + \log_e x$

$y = 0$ on x axis

$\log_e x = -\log_e 2$

$x = 2^{-1}$

$= \frac{1}{2}$

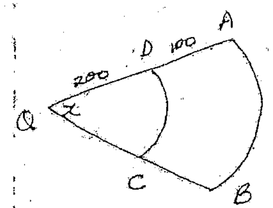
at A & B $x = p$.

$\therefore AB = \log_e 2p - \log_e p$ ✓

$= \log_e 2 + \log_e p - \log_e p$

$= \log_e 2$ ✓ (2)

$\therefore$ distance AB is constant.



Arc $DC = 200x$ metres ✓

Arc $AB = 300x$ metres

Shorter along $ADCB$ if

$100 + 200x + 100 < 300x$ ✓

$-100x < -200$

$x > 2$ ✓ (2)

$\therefore 2 < x < \pi$