ASCHAM SCHOOL

FORM 6 MATHEMATICS (2U) TRIAL EXAMINATION

2007

July 2007

Time allowed: 3 hours
Plus 5 minutes reading time

Instructions

1. Attempt all questions.
2. All questions are of equal value.
3. All necessary working should be shown in each question.
4. Marks may be deducted for careless or badly presented work.
5. Standard integrals are provided at the back of the paper.
6. Board approved calculators may be used.
7. Answer each question in a separate writing booklet.
8. Write your student number on each writing booklet.

Question 1

(a) Evaluate, correct to 3 significant figures: $3\sin 0.5$. **2**

(b) Simplify $\frac{x^2-9x+18}{x-3}$. **2**

(c) Solve $|x-2|=3$. **2**

(d) Sketch the graph of $y=x^3+1$, showing intercepts. **2**

(e)

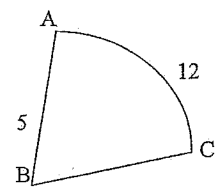


Diagram not to scale.

In the diagram of sector ABC, AC = 12 and AB = 5.

Find:

(i) the size of $\angle ABC$. **1**

(ii) the area of sector ABC. **1**

(f) **2**

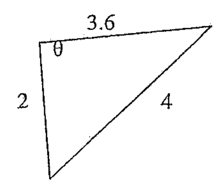


Diagram not to scale.

Find the angle θ, correct to the nearest degree.

Question 2 **Begin a new booklet.**

(a) Differentiate with respect to x: **2**

(i) $y = \sqrt{x^2 - 4}$,

(ii) $3\tan 2x$. **2**

(b) (i) Find $\int \frac{5}{\sqrt{x^2 + 25}}\,dx$, using the table of Standard Integrals. **2**

(ii) Evaluate $\int_0^1 \frac{e^x}{e^x + 1}\,dx$. **3**

(c) Find the equation of the tangent to the curve $y = \ln x - 2$ at the point where $x = e$. Give your answer in general form. **3**

Copyright Ascham School ©

Question 3 **Begin a new booklet.**

(a)

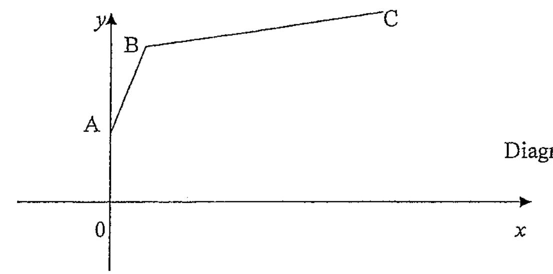


Diagram not to scale.

The points A(0,2), B(1,5) and C(8,6) are three vertices of a kite ABCD.

(i) Find the distance AC. **2**

(ii) Show that the equation of AC is $x - 2y + 4 = 0$. **2**

(iii) Show that the perpendicular distance from B to AC is $\sqrt{5}$ units. **2**

(iv) Find the area of triangle ABC. **1**

(b) Given the quadratic equation $2x^2 - 4x - 5 = 0$, with roots α and β, find:

(i) $\alpha + \beta$ **1**

(ii) $\alpha\beta$ **1**

(iii) $\alpha^2 + \beta^2$ **2**

(iv) whether the roots are rational or irrational, giving reasons. **1**

Question 4 Begin a new booklet

(a)

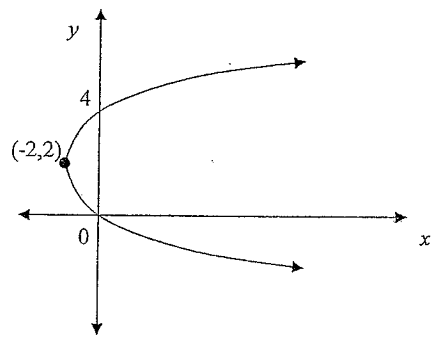


Diagram not to scale.

Consider the graph of the equation $(y-2)^2 = 4a(x+2)$ above, passing through (0,0) and (0,4). **2**

(i) Explain why the value of a is ½ . **1**

(ii) Hence state the coordinates of the focus S. **4**

(iii) Find the area bounded by the curve and the y-axis.

(b) Consider the quadratic equation in x, $x^2-(k+2)x+2k=0$.

(i) Find the value(s) of k if the roots are reciprocals of each other. **2**

(ii) Show that the roots are always real for all values of k. **3**

Copyright Ascham School ©

Question 5 Begin a new booklet.

(a) Consider the curve given by the equation $y = 4x^2 + \frac{1}{x}$, for $x > 0$.

(i) Explain why there are no x or y intercepts in this domain. **2**

(ii) Find $\frac{dy}{dx}$ and show there is a stationary point at (½ , 3). **2**

(iii) If $\frac{d^2y}{dx^2} = 8 + \frac{2}{x^3}$, find the nature of the stationary point. **1**

(iv) Sketch the curve for $0 < x \leq 1$. **2**

(b)

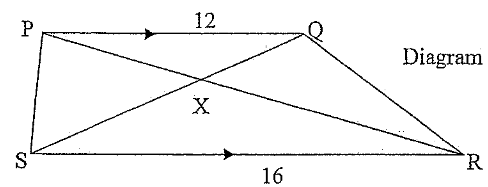


Diagram not to scale.

In the diagram, $PQ = 12$, $SR = 16$ and $PQ \,//\, SR$.

(i) Prove that $\Delta PQX \;|||\; \Delta RSX$. **2**

(ii) If $PX = 7$, hence or otherwise find the length of PR. **3**

Question 6 **Begin a new booklet.**

(a) (i) Find $\frac{d}{dx}(\ln(\sin x))$. **2**

(ii) **3**

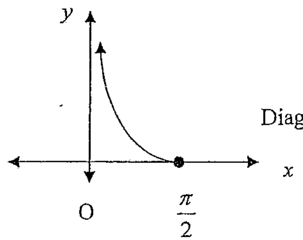


Diagram not to scale.

The graph of $y = \sqrt{\cot x}$ for $0 < x \leq \frac{\pi}{2}$ is sketched.

Using part (i) or otherwise, find the volume generated when the curve between $x = \frac{\pi}{3}$ and $x = \frac{\pi}{2}$ is rotated about the x-axis. Give answer in exact form.

(b) A marathon runner begins to train by running 900 metres on the first day. He then increases his run to 1300 metres on the second day, 1700 metres on the third day, and so on.

(i) Determine how far he will run on the 60^{th} day. **2**

(ii) Find the total number of kilometres he runs over the first 60 days. **2**

(c) (i) State the domain of $y = \log_{10} x$. **1**

(ii) Solve the equation $\log_{10}(x+3) - \log_{10} 2x = 0$. **2**

Question 7 **Begin a new booklet**

(a) **2**

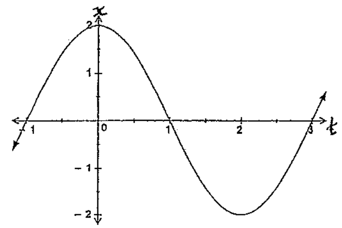


Diagram not to scale.

Find the equation of the curve in the form $x = a\cos nt$, where a and n are constants.

(b) The gradient function of a curve is given by $y' = 3x^2 - 2$. Find the equation of the curve if it passes through the point (2, -1). **2**

(c)

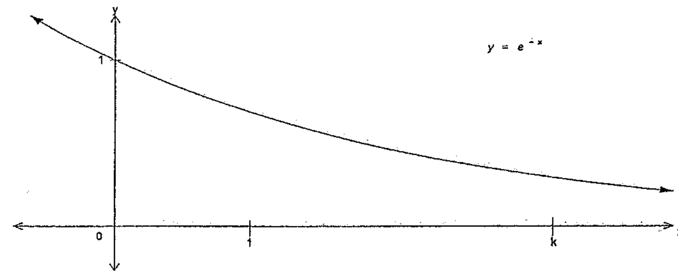


Diagram not to scale.

Consider the curve $y = e^{-x}$ shown above.

(i) Show that the area under the curve $y = e^{-x}$ between $x = 0$ and $x = k$ is $1 - \frac{1}{e^k}$ square units. **3**

(ii) As k increases, what value does the area approach? **1**

(d) Use Simpson's Rule with 5 function values to estimate the value of the integral $\int_1^3 \log_{10} x \, dx$. **4**

Question 8 Begin a new booklet.

(a) A common strategy for pensioners to access money is to take out a reverse mortgage on their home. A couple is offered $100,000 from Ditchbank on the following terms:
The loan is to be paid back in forty equal quarterly instalments, the first instalment due at the end of the first quarter. Interest accrues on the balance owing at the rate of 8% per annum, compounded quarterly. Let \$ A_n be the amount owing at the end of the n-th instalment and let \$Q be the size of each instalment.

(i) Show that the amount owing at the end of the first year is **2**

$$A_4 = 100000(1.02)^4 - Q(1 + 1.02 + 1.02^2 + 1.02^3).$$

(ii) Find the value of Q. **3**

(iii) At the end of 5 years, the couple decides to abandon the arrangement. Determine how much they still owe at that stage. **1**

(b) A particle P moves such that the displacement x cm from 0 after t seconds is given by $x = t^3 - 3t, \quad t \geq 0$.

(i) Find the initial velocity. **2**

(ii) Find when the particle is at rest. **2**

(iii) Is the particle speeding up or slowing down at $t = 2$? Give reasons. **2**

Question 9 Begin a new booklet.

(a) Solve the equation $\sin^2 x = \cos^2 x, \ 0 \leq x \leq \pi$. **2**

(b)

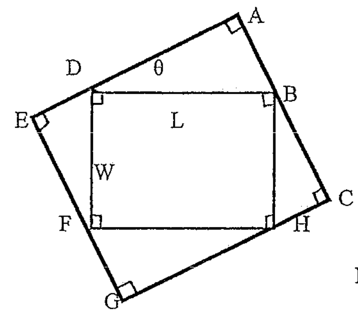


Diagram not to scale.

A larger rectangle, EACG, of length EA and width AC is placed around a smaller rectangle, DBHF, of constant length L and constant width W, as shown in the diagram. The vertices D,B,H, F lie on the perimeter of EACG.

(i) If $\angle ADB = \theta$, explain why $\angle DFE = \angle CBH = \theta$. **1**

(ii) Show that $EA = L\cos\theta + W\sin\theta$. **2**

(iii) Find a similar expression for AC. **1**

(iv) Hence show that the area of EACG is given by **2**

$$A = (L^2 + W^2)\cos\theta\sin\theta + LW.$$

(v) Show that $\dfrac{dA}{d\theta} = (L^2 + W^2)(\cos^2\theta - \sin^2\theta)$. **2**

(vi) Using part (a) or otherwise, show that the maximum area of EACG occurs when $\theta = \dfrac{\pi}{4}$. **2**

Question 10 Begin a new booklet.

(a) Spherical mothballs, each of mass M milligrams, slowly vaporise according to the equation $\frac{dM}{dt} = -4\pi(10-t)^2$, where t is in days. When they are manufactured, they have mass $\frac{4000\pi}{3}$ milligrams.

(i) Find after how many days a mothball ceases to vaporise. 1

(ii) Find M as a function of t. 2

(iii) Find the mass of a mothball after 7 days. 1

(b) The population N of native birds on an island after t years is given by the equation $N = N_0e^{0.03t}$, where N_0 is a constant. There are 8000 native birds on January 1, 2007 (when $t = 0$).

(i) Show that $N_0 = 8000$. 1

(ii) Find the rate at which the population is growing on January 1, 2007. 2

A migrant bird population M settles on the island on January 1, 2007 and begins to compete with the native bird population. The number of migrant birds M after t years is given by $M = 200e^{kt}$, where k is a constant. After 10 years the migrant population has grown to 1000.

(iii) Find the value of the growth constant k, and show $k \approx 0.1609$. 2

(iv) Find the number of years it takes for the migrant population to overtake the native population. 3

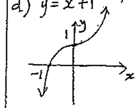
ASCHAM MATHEMATICS (2UNIT) TRIAL EXAM SOLUTIONS + CRITERIA MARKING SCHEME 2007 ✓=1

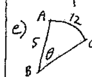
1 a) $3\sin 0.5 = 1.43827...$ ✓
$= 1.44$ ✓ (3 sig fig) (2)
1 mark for giving 3 digits 0.0262

b) $\frac{x^2-9x+18}{x-3} = \frac{(x-6)(x-3)}{x-3}$ ✓ $(x \neq 3)$
$= x-6$ ✓ (2)

c) $|x-2| = 3$
$x-2=3$ or $x-2=-3$
$x=5$ ✓ or $x=-1$ ✓ (2)

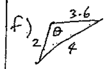
d) $y = x^3+1$ (2)



e) 
i) $l = r\theta$
$12 = 5\theta$
$\theta = \frac{12}{5}$ ✓ (1)
$= 2.4$ rad

ii) Area $= \frac{1}{2}r^2\theta$
$= \frac{1}{2}5^2.\frac{12}{5}$
$= 30\ u^2$ (1)

Award (ii) 1 mark if subbed into correct formula.

f) 
$\cos\theta = \frac{3.6^2+2^2-4^2}{2.2.3.6}$ ✓ (2)
$\theta = 86°$ ✓ (nearest °)

1 mark for correct or consistent use of cosine rule to find any angle.

2 a) i) $y = \sqrt{x^2-4}$
$y = (x^2-4)^{\frac{1}{2}}$ ✓
$y' = \frac{1}{2}(x^2-4)^{-\frac{1}{2}}2x$ ✓ (2)
$= \frac{x}{\sqrt{x^2-4}}$

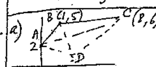
ii) $\frac{d}{dx}(3\tan 2x) = 3.2\sec^2 2x$ ✓ ✓ (2)
$= 6\sec^2 2x$

2 b) i) $\int \frac{5\,dx}{\sqrt{x^2+25}} = 5\ln(x+\sqrt{x^2+25})$ ✓ ✓ (2)

ii) $\int_0^1 \frac{e^x}{e^x+1}dx = [\ln(e^x+1)]_0^1$ ✓
$= \ln(e^1+1) - \ln(e^0+1)$ ✓
$= \ln(e+1) - \ln 2$ ✓ (3)
$= \ln\left(\frac{e+1}{2}\right)$ or $0.62011...$

Award 1 for F(1) − F(0) if F(x) is ...

c) $y = \ln x - 2$
$y' = \frac{1}{x}$ ✓
$x = e$ $y' = \frac{1}{e}$, $y = \ln e - 2$
$= 1-2$
$= -1$ ✓ (3)
$\therefore y+1 = \frac{1}{e}(x-e)$ ✓
$ey + e = x - e$
$\therefore x - ey - 2e = 0$

3. a) 
i) $AC = \sqrt{(8-0)^2+(6-2)^2}$ ✓
$= \sqrt{80}$ ✓ (2)
$= 4\sqrt{5}$

ii) $m_{AC} = \frac{6-2}{8-0}$
$= \frac{1}{2}$ ✓
$\therefore y-2 = \frac{1}{2}(x-0)$ ✓ (2)
$2y-4 = x$
$\therefore x - 2y + 4 = 0$.

iii) $d = \left|\frac{1.1-2.5+4}{\sqrt{1^2+2^2}}\right|$ ✓ (2)
$= \left|\frac{-5}{\sqrt{5}}\right|$ ✓
$= \sqrt{5}$

0 if wrong formula.

iv) Area $= \frac{1}{2}AC.d$ ✓ (1)
$= \frac{1}{2}.4\sqrt{5}\sqrt{5} = 10\,u^2$.

Award if us... their AC fo...

ASCHAM MATHEMATICS (2UNIT) TRIAL EXAM SOLUTIONS + CRITERIA MARKING SCHEME 2007 ✓=1mark

3 b) $2x^2-4x-5=0$
i) $\alpha+\beta = \frac{-b}{a}$
$= \frac{4}{2}$ (1)
$= 2$

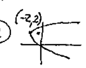
ii) $\alpha\beta = \frac{c}{a}$
$= \frac{-5}{2}$ (1)

iii) $\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$ ✓
$= 2^2 - 2\left(\frac{-5}{2}\right)$ ✓ (2)
$= 9$

iv) rational roots if Δ = square
$\Delta = b^2-4ac$
$= (-4)^2 - 4.2.(-5)$ (1)
$= 56$ ✓
irrational since 56 is not a square.

award 1 mark if say irrational with some valid justification.

i) $(y-2)^2 = 4a(x+2)$ sub $(0,0)$.
$(0-2)^2 = 4a(0+2)$
$4 = 8a$ (2)
$\therefore a = \frac{1}{2}$



Give 1 mark if sub in a point (0,0) or (0,4).

ii) $S(-1\frac{1}{2}, 2)$ (1)

iii) Equation $(y-2)^2 = 4\left(\frac{1}{2}\right)(x+2)$
$\therefore (y-2)^2 = 2(x+2)$
$\therefore x = \frac{(y-2)^2}{2} - 2$ ✓
$\therefore$ Area $= \left|\int_0^4 \frac{(y-2)^2}{2} - 2\,dy\right|$ ✓
$= \left|\left[\frac{(y-2)^3}{6} - 2y\right]_0^4\right|$ ✓

4 iii) Cont'd $= \left|\left[\frac{(4-2)^3}{6} - 2(4)\right] - \left[\frac{(0-2)^3}{6} - 2(0)\right]\right|$ ✓
$= \left|\left(\frac{2^3}{6} - 8\right) - \left(\frac{(-2)^3}{6} - 0\right)\right|$
$= \frac{16}{3}\ u^2$ (4)

Don't penalise if omit | | signs.

b) $x^2 - (k+2)x + 2k = 0$
i) Roots are reciprocals $\therefore \alpha\beta = 1 \therefore \frac{c}{a} = 1$ ✓
$\therefore \frac{2k}{1} = 1$ (2)
$\therefore k = \frac{1}{2}$

ii) Real roots $\Delta \geq 0$ ✓
$\Delta = (-(k+2))^2 - 4.1.2k$ ✓
$= k^2+4k+4-8k$ (3)
$= k^2-4k+4$
$= (k-2)^2$
≥ 0 for all k since a square. ✓
$\therefore$ eq'n has real roots for all k.

5. a) $y = 4x^2 + \frac{1}{x}$, $x > 0$.
i) When $x=0$, $y = 4(0)^2 + \frac{1}{0}$ undefined. ✓ (but $x=0$ not in domain anyway)
When $y=0$ $0 = 4x^2 + \frac{1}{x}$
$-\frac{1}{x} = 4x^2$ (2)
$\therefore -1 = 4x^3$
$\therefore x^3 = -\frac{1}{4}$ ✓
$\therefore x = \sqrt[3]{\frac{-1}{4}} < 0$ $\therefore$ outside domain.

Must show working.

5 a) ii) $y = 4x^2 + x^{-1}$

$y' = 8x - x^{-2}$ ✓

$y' = 8x - \frac{1}{x^2}$ (2)

When $x = \frac{1}{2}$ $y' = 8(\frac{1}{2}) - \frac{1}{(\frac{1}{2})^2}$

$= 4 - 4$

$= 0$ ✓

∴ Stat. pt when $x = \frac{1}{2}$.

$x = \frac{1}{2}$ $y = 4(\frac{1}{2})^2 + \frac{1}{(\frac{1}{2})}$

$= 1 + 2$

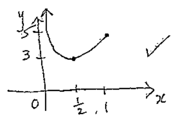
$= 3$

∴ Stat pt at $(\frac{1}{2}, 3)$.

iii) When $x = \frac{1}{2}$ $y'' = 8 + \frac{2}{(\frac{1}{2})^3}$ (1)

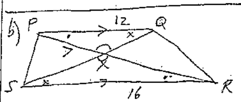
> 0 ✓

Min stat pt at $(\frac{1}{2}, 3)$

iv) 

$x = 1$ $y = 4(1)^2 + \frac{1}{1}$

$= 5$ ✓ (2)

5 b) 

5 b) cont'd In ΔPQX + ΔRSX, (2)

(i) 1. $\angle PXQ = \angle SXR$ (vertically opposite ∠s equal) ✓

2. $\angle PQX = \angle QSR$ (alternate ∠s equal, PQ∥SR) ✓

∴ $\Delta PQX ||| \Delta RSX$ (equiangular or 2∠ test) ✓

(ii) $\frac{PQ}{RS} = \frac{PX}{RX} = \frac{QX}{SX}$ (matching sides of similar Δs in propn) ✓

$\frac{12}{16} = \frac{7}{RX}$ ∴ $RX = \frac{7}{12} \times 16$

$= \frac{28}{3}$ or $9\frac{1}{3}$ ✓ (3)

∴ $PR = PX + XR$

$= 7 + 9\frac{1}{3}$

$= 16\frac{1}{3}$

Award 3 marks if get to RX= with reason.

6 a) i) $\frac{d}{dx}(\ln(\sin x)) = \frac{\cos x}{\sin x}$ ✓ ✓ (2)

$= \cot x$

ii) $y = \sqrt{\cot x}$

$y^2 = \cot x$

$V = \pi \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \cot x \, dx$ ✓ (3)

$= \pi [\ln(\sin x)]_{\frac{\pi}{3}}^{\frac{\pi}{2}}$

$= \pi [\ln(\sin(\frac{\pi}{2})) - \ln(\sin(\frac{\pi}{3}))]$ ✓

$= \pi [\ln 1 - \ln(\frac{\sqrt{3}}{2})]$

$= \pi (\ln 2 - \frac{1}{2} \ln 3)$ ✓

b) 900, 1300, 1700, ... AP $a = 900$

$d = 400$

ASCHAM MATHEMATICS (2UNIT) TRIAL EXAM SOLUTIONS + CRITERIA MARKING SCHEME. 2007 ✓ = 1 mark

8 cont'd b) i) $x = t^3 - 3t$

$\dot{x} = 3t^2 - 3$

$t = 0$ $\dot{x} = 3(0)^2 - 3$ (2)

$= -3$ cm/s

∴ moving to left at 3 cm/s

ii) rest when $\dot{x} = 0 = 3t^2 - 3$

$3t^2 = 3$ (2)

$t^2 = 1$

$t = \pm 1$ but $t > 0$

∴ $t = 1$ particle is at rest

iii) $t = 2$ $\dot{x} = 3(2)^2 - 3$ $\ddot{x} = 6t$

$= 9$ $= 6(2)$

vel > 0 $= 12$

accel > 0

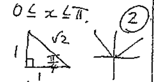
Speeding up since v and a are same sign. (2)

9. a) $\sin^2 x = \cos^2 x$ $0 \le x \le \pi$. (2)

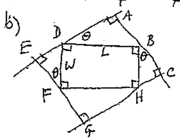
$\tan^2 x = 1$

$\tan x = \pm 1$ ✓

$x = \frac{\pi}{4}$ or $\frac{3\pi}{4}$ ✓



1 mark if omit $\frac{3\pi}{4}$.

b) 

9 b) cont'd i) EDA and ABC are straight lines (1)

∴ ∠EDF + ∠ABD are complements of θ.

∴ ∠DFE + ∠CBH are both θ. (Complements)

ii) $EA = ED + DA$ $ED = W\sin\theta$ from ΔEDF ✓

+ $DA = L\cos\theta$ from ΔDAB (2) ✓

∴ $EA = W\sin\theta + L\cos\theta$ as required.

iii) $AC = AB + BC$ (1)

$= L\sin\theta + W\cos\theta$.

iv) Area $EACG = EA \times AC$ (2)

$= (W\sin\theta + L\cos\theta)(L\sin\theta + W\cos\theta)$

$= WL\sin^2\theta + W^2\sin\theta\cos\theta + L^2\cos\theta\sin\theta + LW\cos^2\theta$ ✓

$= LW(\sin^2\theta + \cos^2\theta) + (W^2 + L^2)\sin\theta\cos\theta$ — must expand correctly

$\doteqdot (L^2 + W^2)\cos\theta\sin\theta + LW$ ∵ $(\sin^2\theta + \cos^2\theta = 1)$ ✓ for 1m

v) $\frac{dA}{d\theta} = u'v + v'u$ ✓ let $u = (L^2 + W^2)\cos\theta$ — 1m for use of product rule

$u' = -(L^2 + W^2)\sin\theta$

$= -(L^2 + W^2)\sin\theta . \sin\theta$ $v = \sin\theta$

$+ \cos\theta . (L^2 + W^2)\cos\theta$ $v' = \cos\theta$ ✓

$= (L^2 + W^2)(\cos^2\theta - \sin^2\theta)$ (2)

vi) Max area of EACG occurs when $A' = 0$

$0 = (L^2 + W^2)(\cos^2\theta - \sin^2\theta)$ ✓ (2)

∴ $\cos^2\theta = \sin^2\theta$

From (a) $\theta = \frac{\pi}{4}$ or $\frac{3\pi}{4}$ but $0 < \theta < \frac{\pi}{2}$

since θ is in a ∟Δ ∴ $\theta = \frac{\pi}{4}$. Must test for 2m

Test:

θ	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$
A'	+	0	−

✓ ∴ Max Area when $\theta = \frac{\pi}{4}$.

ASCHAM MATHEMATICS (2UNIT) TRIAL EXAM SOLUTIONS + CRITERIA MARKING SCHEME 2007 ✓ = 1 mark

6 b) cont'd i) $T_{60} = 900 + (60-1)400$ (2)

$= 24500$

ii) $S_{60} = \frac{60}{2}(900 + 24500)$ ✓

$= 762000$ m ✓ (2)

$= 762$ km

c) i) Domain of $y = \log_{10} x$ is $x > 0$. (1)

ii) $\log_{10}(x+3) - \log_{10} 2x = 0$

$\log_{10} \frac{x+3}{2x} = 0$

$10^0 = \frac{x+3}{2x}$ ✓ (2)

$2x = x + 3$

$x = 3$ ✓

no marks for incorrect use of log laws.

7. a) $x = a\cos nt$ Period $= 4 = \frac{2\pi}{n}$

∴ $x = 2\cos\frac{\pi}{2}t$ ✓ (2) $n = \frac{2\pi}{4} = \frac{\pi}{2}$ ✓

$a = 2$

b) $y' = 3x^2 - 2$

$y = x^3 - 2x + C$

Sub $(2, -1)$ $-1 = 2^3 - 2(2) + C$ (2)

$C = -5$

∴ $y = x^3 - 2x - 5$

c) i) Area $= \int_0^k e^{-x} dx$ (3)

$= [-e^{-x}]_0^k$ $= -\frac{1}{e^k} + 1$

$= -e^{-k} - -e^0$ $= 1 - \frac{1}{e^k}$

7 c) ii) As $k \to \infty$, Area $= 1 - 0$ (1)

$= 1$ unit^2.

d) $\int_1^3 \log_{10} x \, dx$

x	y	Weight
1	$\log_{10} 1$	1
1.5	$\log 1.5$	4
2	$\log 2$	2
2.5	$\log 2.5$	4
3	$\log 3$	1 ✓

$= \frac{0.5}{3}[\log 1 + \log 3$ ✓

$+ 4(\log 1.5 + 2.5)$

$+ 2\log 2]$

$= 0.56255...$ ✓ (4)

8. a) $n = 40$ $r = 8\%$pa

$= 0.02$ pq quarter. ✓

i) $A_1 = 100000(1.02)^1 - Q$ (2)

$A_2 = (100000(1.02)^1 - Q)1.02 - Q$ ✓

$= 100000(1.02)^2 - Q(1.02 + 1)$

$A_3 = 100000(1.02)^3 - Q(1.02^2 + 1.02 + 1)$

∴ $A_4 = 100000(1.02)^4 - Q(1.02^3 + 1.02^2 + 1.02 + 1)$ ✓

ii) $A_{40} = 100000(1.02)^{40} - Q(1.02^{39} + 1.02^{38} + ... + 1)$ ✓

but $A_{40} = 0$

∴ $Q\left(\frac{1(1.02^{40} - 1)}{1.02 - 1}\right) = 100000(1.02)^{40}$ ✓ (3)

∴ $Q = \frac{100000(1.02)^{40}}{\frac{1((1.02)^{40} - 1)}{0.02}}$

$\doteqdot 3656$

iii) 5 years $A_{20} = 100000(1.02)^{20} - 3655.57\left(\frac{1(1.02^{20} - 1)}{1.02 - 1}\right)$ ✓

Owing $\doteqdot \$59774$. (1)

ASCHAM MATHEMATICS (2UNIT) TRIAL EXAM SOLUTIONS + CRITERIA MARKING SCHEME 2007 ✓ = 1 mark

10 a) $\frac{dM}{dt} = -4\pi(10 - t)^2$

i) Vaporising ceases when $\frac{dM}{dt} = 0 = -4\pi(10 - t)^2$

∴ $t = 10$ days (1)

ii) $M = \int -4\pi(10 - t)^2 dt$

$M = -4\pi\frac{(10 - t)^3}{3 \times -1} + C$ ✓

$M = +\frac{4\pi}{3}(10 - t)^3 + C$

$t = 0$ $M = \frac{4000\pi}{3}$ (2)

$\frac{4000\pi}{3} = \frac{4\pi}{3}(10 - 0)^3 + C$

$\frac{4000\pi}{3} = \frac{4000\pi}{3} + C$

$C = 0$

∴ $M = \frac{4\pi}{3}(10 - t)^3$ ✓

1m if neglected C.

iii) $t = 7$ $M = \frac{4\pi}{3}(10 - 7)^3$ (1)

$= \frac{4\pi}{3} \times 27$

$= 36\pi$ mg

b) i) $N = N_0 e^{0.03t}$

$t = 0$ $N = 8000 = N_0 e^{0.03 \times 0}$ (1)

$8000 = N_0 \times e^0$

∴ $N_0 = 8000$. ∴ $N = 8000e^{0.03t}$.

ii) $\frac{dN}{dt} = 0.03 \times 8000e^{0.03t}$ ✓ (2)

$t = 0$ $\frac{dN}{dt} = 0.03 \times 8000e^0 = 240$ b/y ✓

10 b) iii) $M = 200e^{kt}$

$t = 10$ $M = 1000 = 200e^{k \times 10}$

$\frac{1000}{200} = e^{10k}$ (2)

$5 = e^{10k}$

$\ln 5 = 10k$

$k = \frac{1}{10}\ln 5$

$\doteqdot 0.16094$

iv) $t = ?$ when $M \ge N$?

$200e^{0.1609t} \ge 8000e^{0.03t}$ ✓

$e^{0.1609t} \ge 40e^{0.03t}$

$\frac{e^{0.1609t}}{e^{0.03t}} \ge 40$

$e^{0.1609t - 0.03t} \ge 40$

$e^{0.1309t} \ge 40$ ✓

$0.1309t \ge \ln 40$ (3)

∴ $t \ge \frac{\ln 40}{0.1309}$

∴ $t \ge 28.18...$ ✓

∴ It takes about 29 years.