ASCHAM.

FORM 5 MATHEMATICS ASSESSMENT TEST NOVEMBER 2003

$1\frac{1}{2}$ hours 74 marks

Please do each question in a separate booklet

Question 1 (17 marks)

a) Differentiate with respect to x and simplify your answers:

i) $5x^3 - 3x + 2$ **(2)**

ii) $\dfrac{x^2 - 3}{x}$ **(2)**

iii) $\dfrac{2}{5\sqrt{x}}$ **(2)**

iv) $6x(3x-2)^5$ **(3)**

factorise your answer fully

v) $\dfrac{6-x}{5+3x}$ **(3)**

b) Find the primitive of $6x^2 - 4x + 3$ **(2)**

c) Evaluate $\int_3^5 (\frac{x^2}{3} - x)dx$ **(3)**

Question 2 (20 marks)

Consider $y = 2x^3 - x^4$

i) Find where the curve cuts the x axis **(2)**

ii) Find the stationary points and determine their nature. **(7)**

iii) Find any points of inflexion. **(2)**

iv) Sketch the curve in the domain $-1 \leq x \leq 3$. **(4)**

v) Find the minimum value of $2x^3 - x^4$ in the given domain. **(1)**

vi) For what values of k will the equation $2x^3 - x^4 = k$ have no solution? Explain your answer. **(2)**

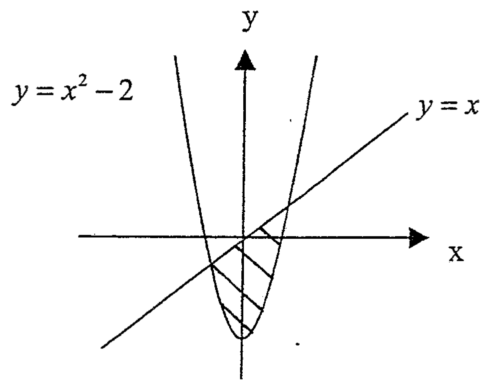
vii) By adding another graph to your diagram, solve the equation $x^4 - 2x^3 + x^2 - 2x = 0$. Do not solve algebraically. **(2)**

Question 3 (19 marks)

a) The gradient function of a curve is given by $\frac{dy}{dx} = 4x^2 - 16$.
For what values of x does the curve increase with downward concavity? **(5)**

b) i) Show that the two graphs below intesect when x = 2 or x = -1. **(2)**

ii) Find the area of the shaded region. **(4)**



c) The area between the curve $y = \sqrt{x}$, the x axis, x = 2 and x = 4, is rotated about the x axis. Find the volume generated. **(4)**

d) If $\frac{d^2y}{dx^2} = 2x - 4$, and there is a minimum turning point on the curve y at the point (-1,3), find y in terms of x. **(4)**

Question 4 (18 marks)

a) Edwina is doing the following question in her mathematics assignment:

The area enclosed by y = f(x) and the x axis is rotated about the ______.
Find the ________.

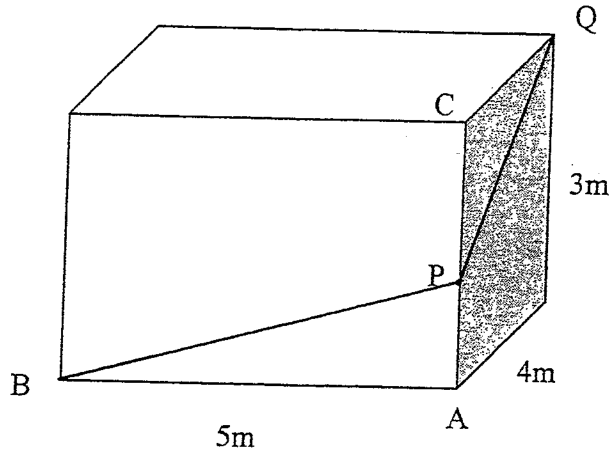
Edwina writes down $\pi \int_0^6 36x^2 - 12x^3 + x^4 dx$.

i) Find the equation of f(x) **(2)**

ii) Draw a diagram showing the graph of y = f(x).
Shade the area that was rotated. **(2)**

iii) Fill in the missing words in Edwina's assignment question. **(2)**

b) Fibre cabling is to be laid in a rectangular room along BP and PQ from the corner B of the floor as shown in the diagram below. P is a point on AC.



Not to scale

Let AP = x metres

i) State the length of BP in terms of x. **(1)**

ii) Show that $PQ = \sqrt{25 - 6x + x^2}$ **(2)**

iii) Show that the total length L metres of cabling is given by

$$L = \sqrt{25 - 6x + x^2} + \sqrt{25 + x^2}.$$ **(1)**

iv) Show that $\dfrac{dL}{dx} = \dfrac{x-3}{\sqrt{25 - 6x + x^2}} + \dfrac{x}{\sqrt{25 + x^2}}$ **(2)**

v) Find the value of AP when the total Length L is a minimum. **(6)**

END OF TEST

FORM 5 MATHEMATICS (2UNIT) ASSESSMENT TEST 2003

QUESTION 1 (17 marks)

a) i) $\frac{d}{dx}(5x^3-3x+2) = 15x^2-3$ (2)

ii) $\frac{d}{dx}\left(\frac{x^2-3}{x}\right) = \frac{d}{dx}(x-3x^{-1})$

$= 1+3x^{-2}$

$= 1+\frac{3}{x^2}$ (2)

iii) $\frac{d}{dx}\frac{2}{5\sqrt{x}} = \frac{d}{dx}\frac{2}{5}x^{-1/2}$

$= \frac{-1}{5}x^{-1\frac{1}{2}}$

$= \frac{-1}{5x^{3/2}}$ (2)

(OR $\frac{-1}{5\sqrt{x^3}}$ OR $\frac{-1}{5x\sqrt{x}}$)

iv) $\frac{d}{dx}6x(3x-2)^5$

$= 6x.5(3x-2)^4 3 + (3x-2)^5 6$ ✓

$= 90x(3x-2)^4 + 6(3x-2)^5$

$= 6(3x-2)^4(15x+(3x-2))$ ✓

~~$= 6(3x-2)^4(15x+18x-12)$~~

$= 6(3x-2)^4(18x-2)$ ½ (3)

$= 12(3x-2)^4(9x-1)$ ½

v) $\frac{d}{dx}\frac{6-x}{5+3x} = \frac{(5+3x).-1-(6-x)3}{(5+3x)^2}$ ✓

$= \frac{-5-3x-18+3x}{(5+3x)^2}$ ✓

$= \frac{-23}{(5+3x)^2}$ ✓ (3)

b) $y' = 6x^2-4x+3$

$y = \frac{6x^3}{3} - \frac{4x^2}{2} + 3x + c$ (2)

$= 2x^3-2x^2+3x+c$

c) $\int_3^5\left(\frac{x^2}{3}-x\right)dx = \left[\frac{x^3}{9}-\frac{x^2}{2}\right]_3^5$ ✓

$= \frac{125}{9}-\frac{25}{2}-\left(\frac{27}{9}-\frac{9}{2}\right)$ ✓

$= 2\frac{8}{9}$ ✓ (3)

QUESTION 2 (20)

$y = 2x^3-x^4$

i) $0 = 2x^3-x^4$

$x^3(2-x) = 0$

$x=0$ or $x=2$ (2)

ii) $y' = 6x^2-4x^3$ ✓

$y'' = 12x-12x^2$ ✓

For stat pts $y'=0$

$2x^2(3-2x)=0$

$x=0$ or $x=\frac{3}{2}$

$y=0$ or $y=1.6875$ (or $1\frac{11}{16}$)

$\therefore (0,0)$ ✓ & $(1.5, 1.6875)$ ✓ are stat pts

Consider their nature

When $x=0$ $y''=0$

$\therefore (0,0)$ is a possible horiz pt of inflexion ✓

Test for change in concavity

x	-1	0	$\frac{1}{2}$
y''	-24	0	3

~~½ if tested later.~~ ✓

$\therefore$ change in concavity

$\therefore (0,0)$ is a horiz pt of infl. ✓

When $x=1.5$ $y''=-9<0$ ✓ (7)

$\therefore (1.5, 1.6875)$ is a max t/pt

$1\frac{11}{}$

iii) For possible pts of infl, $y'' = 0$

$12x(1-x) = 0$

$\therefore x = 0, 1$

$y = 0, 1$ ✓

$(0,0)$ is horiz pt of infl (from ii)

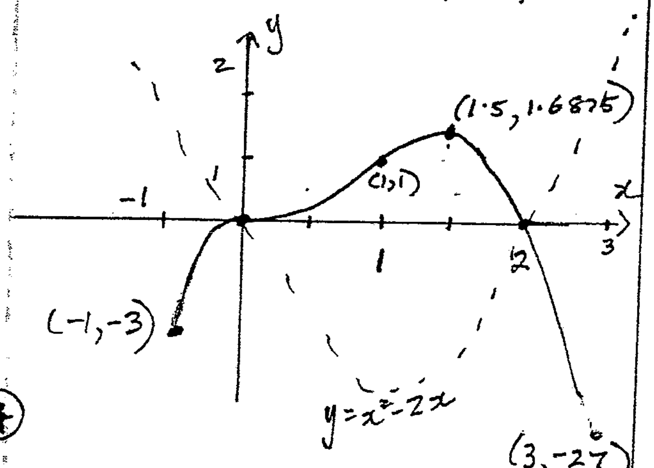
x	$\frac{1}{2}$	1	$1\frac{1}{2}$
y''	3	0	-9

(2) ✓

$\therefore$ change in concavity

$\therefore (1,1)$ is a pt of infl.

iv)



ape
urve ✓
oords ✓✓
ndpts ✓

(4)

v) Min val $= -27$ (1)

vi) $2x^3 - x^4 = k$ ✓

will have no soln if $k > 1.6875$

b/c $y = k$ is horiz line &

if $k > 1.6875$, horiz line ✓

will not cut the graph (2)

vii) $x^4 - 2x^3 + x^2 - 2x = 0$

$x^2 - 2x = 2x^3 - x^4$ ✓

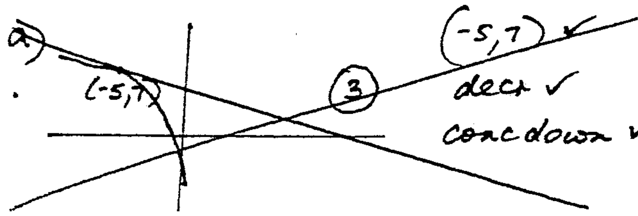
Draw $y = x^2 - 2x$

$y = x(x-2)$

$\therefore$ Solns are $x = 0$, $x = 2$ ✓ (2)

QUESTION 3 [19]

a)



$(-5,7)$ ✓

decr ✓

conc down ✓

(3)

) $\dfrac{dy}{dx} = 4x^2 - 16$

Curve incs when $\dfrac{dy}{dx} > 0$

$4x^2 - 16 > 0$ ✓

$x^2 > 4$

$x < -2$ or $x > 2$ ✓

$\dfrac{d^2y}{dx^2} = 8x$ ✓

$\therefore$ if curve is conc. down $8x < 0$

ie. $x < 0$ ✓

$\therefore$ for $x < -2$, curve increases with downward concavity. ✓ (5)

c)

x	-3	-2	0	4	5
y'	$+$	0	$-$	0	$+$

$\therefore$ when $x = 4$, $y = f(x)$ has rel min b/c curve is decr when x just < 4 & is incs to right of 4.

b) $y = x \quad — (1)$

$y = x^2 - 2 \quad -(2)$

$x^2 - 2 = x$

$x^2 - x - 2 = 0$

$(x+1)(x-2) = 0$ (2)

$x = -1$ or $x = 2$

$\therefore$ graphs intersect when $x = -1$ or 2

OR Sub $x=-1$ in (1) $y = -1$

Sub $x = -1$ in ii) $y = 1^2 - 2 = -1$

Sub $x = 2$ in i) $y = 2$

Sub $x = 2$ in ii) $y = 2^2 - 2 = 2$

$$\text{Area} = \int_{-1}^{2} x - (x^2 - 2)\, dx \ ✓$$

$$= \int_{-1}^{2} x - x^2 + 2 \, dx$$

$$= \left[\frac{x^2}{2} - \frac{x^3}{3} + 2x \right]_{-1}^{2} ✓$$

$$= 2 - \frac{8}{3} + 4 - \left(\frac{1}{2} + \frac{1}{3} - 2\right) ✓$$

$$= 8 - 3 - \frac{1}{2}$$

$$= 4\frac{1}{2}\, u^2 \ ✓$$ (4)

c) $$\text{Volume} = \pi \int_2^4 (\sqrt{x})^2 \, dx \ ✓$$

$$= \pi \int_2^4 x \, dx$$

$$= \pi \left[\frac{x^2}{2} \right]_2^4 ✓$$

$$= \pi \left[\frac{16}{2} - \frac{4}{2} \right] ✓$$

$$= 6\pi \, u^3 \ ✓$$ (4)

d) $$\frac{d^2y}{dx^2} = 2x - 4$$

$$\frac{dy}{dx} = \frac{2x^2}{2} - 4x + c$$

$$= x^2 - 4x + c \ ✓$$

$\frac{dy}{dx} = 0$ when $x = -1$

$0 = 1 + 4 + c$

$\therefore c = -5$ ✓

$$\therefore \frac{dy}{dx} = x^2 - 4x - 5$$

$$y = \frac{x^3}{3} - \frac{4x^2}{2} - 5x + d$$

$$y = \frac{x^3}{3} - 2x^2 - 5x + d \ ✓$$

Sub $(-1, 3)$

$$3 = -\frac{1}{3} - 2 + 5 + d$$

$d = \frac{1}{3}$ ✓ (4)

$$\therefore y = \frac{x^3}{3} - 2x^2 - 5x + \frac{1}{3}$$

QUESTION 4 [18]

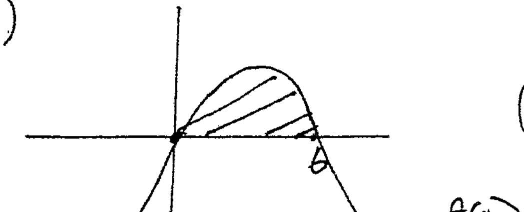
i) $36x^2 - 12x^3 + x^4$

$= x^2(36 - 12x + x^2)$

$= x^2(6 - x)^2$ ✓ (2)

$\therefore f(x) = x(6-x)$ ✓

ii)



(2)

iii) x axis ✓

volume ✓ (2)

b) i) $BP^2 = 5^2 + x^2$ (Pythag)

$BP = \sqrt{25+x^2}$ (1)

ii) $PC = 3 - x$

$CQ = 4$

$\therefore PQ^2 = (3-x)^2 + 4^2$ (Pythag)

$= 9 - 6x + x^2 + 16$

$= 25 - 6x + x^2$ (2)

$\therefore PQ = \sqrt{25-6x+x^2}$

iii) $L = BP + PQ$ (1)

$= \sqrt{25+x^2} + \sqrt{25-6x+x^2}$

iv) $\frac{dL}{dx} = \frac{1}{2}(25+x^2)^{-\frac{1}{2}}.2x + \frac{1}{2}(25-6x+x^2)^{-\frac{1}{2}}.(-6+2x)$ ✓

$= \frac{x}{\sqrt{25+x^2}} + \frac{\cancel{2}(x-3)}{\cancel{2}\sqrt{25-6x+x^2}}$

$= \frac{x}{\sqrt{25+x^2}} + \frac{x-3}{\sqrt{25-6x+x^2}}$ ✓ (2)

v) For min L, L' = 0 ✓

$\frac{x}{\sqrt{25+x^2}} + \frac{x-3}{\sqrt{25-6x+x^2}} = 0$

$\frac{x\sqrt{25-6x+x^2} + (x-3)\sqrt{25+x^2}}{\sqrt{25+x^2}\sqrt{25-6x+x^2}} = 0$

$x\sqrt{25-6x+x^2} + (x-3)\sqrt{25+x^2} = 0$

$x\sqrt{25-6x+x^2} = -(x-3)\sqrt{25+x^2}$

$x^2(25-6x+x^2) = (x-3)^2(25+x^2)$

$25x^2 - 6x^3 + x^4 = (x^2-6x+9)(25+x^2)$

$\cancel{25x^2} - \cancel{6x^3} + \cancel{x^4} = \cancel{25x^2} + \cancel{x^4} - 150x - \cancel{6x^3} + 225 + 9x^2$

$9x^2 - 150x + 225 = 0$ ✓

$x = \frac{150 \pm \sqrt{150^2 - 4 \times 9 \times 225}}{18}$

$= 15$ or $1\frac{2}{3}$ ✓

But $x < 3$

$\therefore x = 1\frac{2}{3}$ ✓

To check if $x = 3$ gives min L

x	1	$1\frac{2}{3}$	2
L'	—	0	+

✓ ✓

$x = 1 \quad L' = \frac{-2}{\sqrt{20}} + \frac{1}{\sqrt{26}}$

$\doteqdot -0.25\ldots$

$x = 2 \quad L' = \frac{-1}{\sqrt{17}} + \frac{2}{\sqrt{29}}$

$\doteqdot 0.1288\ldots$

$\therefore$ When $x = 1\frac{2}{3}$ L is min

$\therefore$ When AP is $1\frac{2}{3}$ m, the length L is minimum. (6)