ASCHAM SCHOOL

FORM 5 PRELIMINARY COURSE EXAMINATION

MATHEMATICS

Time allowed: 3 hours **AUGUST 2002**

- All questions should be attempted.
- All necessary working should be shown.
- Marks may not be awarded for careless or badly presented work.
- Do each question in a different booklet.
- Write your name and your teacher's name on each booklet.
- Clearly label the front of each booklet with the number of the question.
- Approved calculators may be used.

QUESTION 1

a) Solve $2+3x \le 8$ (1)

b) Find the exact value of $\dfrac{\frac{2}{5}+\frac{2}{3}}{1-\frac{4}{15}}$ (2)

c) Factorise completely $8x^3+1$ (1)

d) Show that $\dfrac{4}{2+\sqrt{5}}-\dfrac{1}{9-4\sqrt{5}}$ is rational. (4)

e) Solve the simultaneous equations

$$x-y=1$$

$$2x+y=5$$ (2)

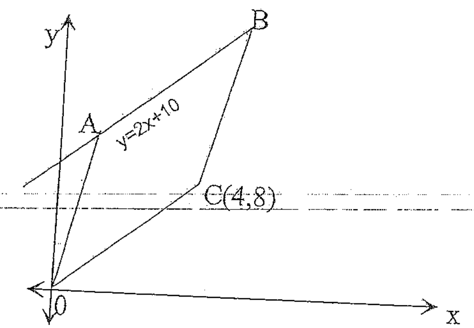
f) Subtract $5x+2y-3$ from $x-7y+9$ (2)

QUESTION 2

a) Mark on a number line the values of x for which $|x+1| \ge 2$ (3)

b) Express $0.\dot{2}\dot{5}$ as a fraction. Show all working. (3)

c) The equation AB is y =2x +10. The point C is (4,8)



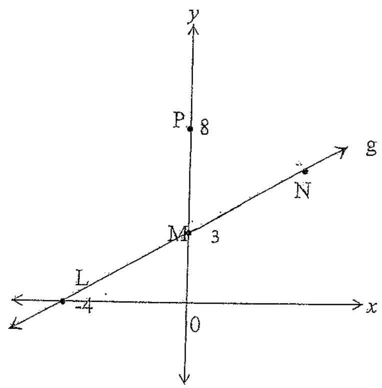
Copy or trace the diagram into your writing booklet.

i) Show that OC and AB are parallel. (2)

ii) State why ∠ABO = ∠BOC (1)

iii) The line OB divides the quadrilateral OABC into two congruent triangles. Prove that OABC is a parallelogram (3)

QUESTION 3



The line g cuts the x axis at L(-4,0) and the y axis at M(0, 3) as shown. N is a point on the line g, and P is the point (0, 8).

Copy the diagram into your writing booklet

a) Find the equation of line g (2)

b) By considering the lengths of ML and MP, show that ΔLMP is isosceles. (2)

c) Find the gradient of line PL. (1)

d) M is the midpoint of the interval LN. Find the coordinates of the point N. (2.5)

e) Show that ∠NPL is a right angle. (2.5)

f) Find the equation of the circle that passes through N, P and L (2)

QUESTION 4

a) Sketch the curve $y = \log_{10}(x+1)$ (2)

b) i) Sketch the graph of $y = x^2 - 6$ and label all intercepts with the axes. (2)

ii) On the same set of axes, carefully sketch the graph of $y = |x|$ (1)

iii) Find the coordinates of the two points where the graphs intersect. (4)

iv) Hence solve the inequality $x^2 - 6 \le |x|$ (1)

c) A function is defined such that $f(x) = 2x^3 - x$.
Is this function odd, even or neither ? Show all working. (2)

QUESTION 5

a) Simplify $\dfrac{16}{2^{3x} \times 8^{1-x}}$ (2)

b) Solve for x.

$$3^{2x} - 10(3^x) + 9 = 0$$ (3)

c) Solve $2\log_5 2x = \log_5(x+5)$ (3)

d) The first four terms of an arithmetic series are - 15 - 8 - 1 + 6 .....

i) Find the 30th term

ii) Hence or otherwise find the sum of the first 30 terms of the series (4)

QUESTION 6

a) If α and β are the roots of $5x^2 - 7x - 10 = 0$ find:

i) $\alpha + \beta$ (1)

ii) $\alpha\beta$ (1)

iii) $\dfrac{1}{\alpha^2\beta} + \dfrac{1}{\alpha\beta^2}$ (2)

iv) $\alpha^2 + \beta^2$ (2)

b) Find the values of k for which the quadratic equation,

$$3x^2 + 2x + k = 0$$

has no real roots (2)

c) i) Sketch the parabola P which has focus (2,3) and directrix $y = -2$. On your diagram show the focus, directrix and vertex. (3)

ii) Find the equation of P (1)

QUESTION 7

a) Differentiate:

i) $\dfrac{3}{2}x^2 + 7$ (2)

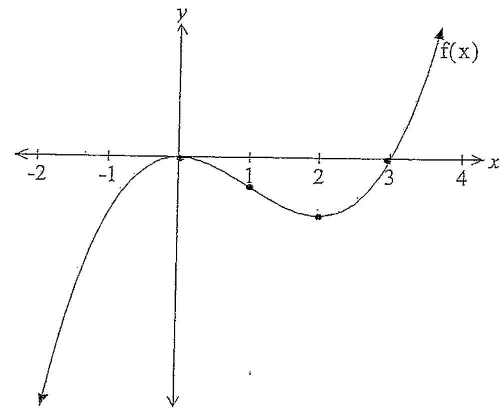
ii) $\dfrac{2}{3x^2}$ (2)

iii) $\dfrac{2x}{1+x^2}$ (2)

iv) $x^2(2x-1)^5$ (2)

v) $y = \sqrt[3]{4 - x^2}$ (2)

b) The graph of the curve $y = f(x)$ is drawn below.
In your answer booklet draw the corresponding gradient function for this curve, clearly indicating all the important values of x.



(2)

QUESTION 8

a) Find, in general form, the equation of the normal to the curve

$$y = x^2 + 3x$$

at the point (1,4) on it. (5)

b) An infinite geometric series has a first term of 8 and a limiting sum of 12. Calculate the common ratio. (2)

c) A ball is dropped from a height of 4 metres onto a hard floor and bounces. After each bounce the maximum height reached by the ball is 75% of the previous maximum height. Thus after it hits the floor it reaches a height of 3 metres before falling again, and after the second bounce it reaches a height of 2.25 metres before falling again.

i) What is the maximum height reached after the third bounce? (1)

ii) What kind of sequence is formed by the successive maximum heights? (1)

iii) What is the total distance travelled by the ball from the time it was first dropped until it eventually comes to rest on the floor? (3)

QUESTION 9

a) If $\sin\theta = \frac{2}{5}$ for $90^\circ \leq \theta \leq 180^\circ$ and $\sin\beta = \frac{1}{2}$ for $0^\circ \leq \beta \leq 90^\circ$

Find the exact value of :

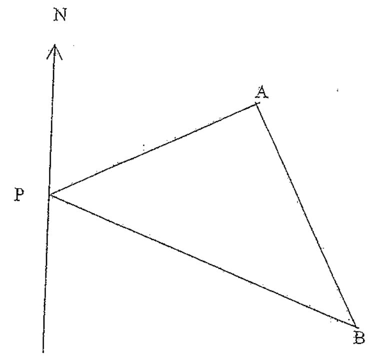
i) $\cos\beta$ (1)

ii) $\sec^2\beta$ (2)

iii) $\tan\theta$ (3)

iv) $\operatorname{cosec}\beta + \tan(90-\theta)$ (2)

b)



Ship A is 20 nautical miles from a port P and is on a bearing of 055°. Ship B is 27 nautical niles from P and is on a bearing of 115°.

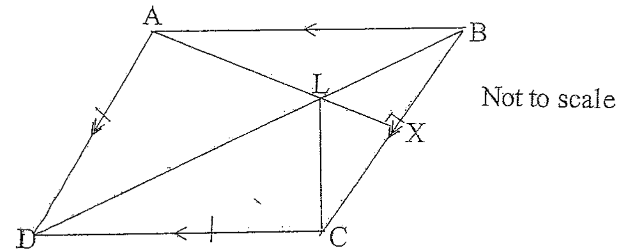
i) Copy the diagram into your answer booklet, indicating all the given information. (1)

ii) Show that ∠APB = 60° (1)

iii) Use the cosine rule to determine the distance between the two ships, giving your answer correct to 3 significant figures. (2)

QUESTION 10

a)



ABCD is a rhombus, AX is perpendicular to BC and intersects BD at L

i) Copy the diagram into your answer booklet and state why ∠ADB = ∠CDB. (1)

ii) Prove that the triangles ALD and CLD are congruent. (2)

iii) Show that ∠DAL is a right angle. (1)

iv) Hence or otherwise find the size of ∠LCD (1)

b) i) Show that $m^2 - 2m + 9 = (m-1)^2 + 8$.
What can you say about the expression $m^2 - 2m + 9$ for all real values of m ? (2)

ii) Hence prove that the equation $x^2 - (m+1)x + (m-2) = 0$ has real and different roots for all real values of m (2)

c) If the quadratic equation $x^2 - 2px - q^3 = 0$ has one root the square of the other, show that $p = \frac{q(q-1)}{2}$ (3)

End of Examination

FORM 5 2UNIT MATHEMATICS AUGUST EXAM 2002

Question 1

a) $3x \le 6$
$x \le 2$

b) $\frac{16}{15} \div \frac{11}{15} = 1\frac{5}{11}$

c) $(2x+1)(4x^2-2x+1)$

d) $\frac{4}{2+\sqrt5} \times \frac{2-\sqrt5}{2-\sqrt5} - \frac{1}{9-4\sqrt5} \times \frac{9+4\sqrt5}{9+4\sqrt5}$

$= \frac{8-4\sqrt5}{-1} - \frac{9+4\sqrt5}{1}$

$= -8+4\sqrt5-9-4\sqrt5$

$= 17$ which is rational

e) $x-y=1$ — ①
$2x+y=5$ — ②
① + ② $3x=6$
$x=2$
$y=1$

f) $x-7y+9-(5x+2y-3)$
$= x-7y+9-5x-2y+3$
$= -4x-9y+12$

QUESTION 2

a) $|x+1| \ge 2$
$x+1 \ge 2$ or $x+1 \le -2$
$x \ge 1$ or $x \le -3$

b) Let $x = 0{\cdot}2525\ldots$
$100x = 25{\cdot}25\ldots$
$x = 0{\cdot}25\ldots$
$99x = 25$
$x = \frac{25}{99}$
$\therefore 0{\cdot}\dot2\dot5 = \frac{25}{99}$

c) i) $M_{OC} = \frac{8-0}{4-0} = 2$
grad $AB = 2$
$\therefore AB \parallel OC$ (same grad)
ii) $\angle ABO = \angle BOC$ (alt ∠s, $AB \parallel OC$)
iii) $AB = OC$ (corr sides of cong Δs)
$AB \parallel OC$ (proved above)
$\therefore OABC$ is a llgram (1 pr opp sides $= \,\&\, \parallel$)

QUESTION 3

a) $m_g = \frac{3}{4}$ y int $= 3$
$\therefore$ eqn of g: $y = \frac{3}{4}x+3$

b) $d_{ML} = \sqrt{(-4-0)^2+(0-3)^2}$
$= \sqrt{16+9}$
$= 5$

$d_{MP} = 5$
$\therefore \Delta MPL$ is isosceles (2 equal sides)

c) $m_{PL} = \frac{8}{4} = 2$

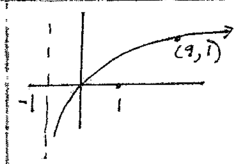
d) Let N be (x, y)
$\frac{-4+x}{2} = 0$ $\frac{0+y}{2} = 3$
$x = 4$ $y = 6$
$\therefore N = (4, 6)$

e) $m_{PN} = \frac{8-6}{0-4}$
$= -\frac{1}{2}$
$m_{PN} \times m_{PL} = -\frac{1}{2} \times 2$
$= -1$
$\therefore PN \perp PL$
$\therefore \angle NPL = 90°$

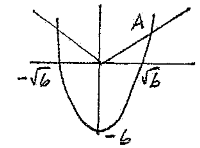
f) M centre, ML radius
$x^2+(y-3)^2 = 5^2$
$x^2+(y-3)^2 = 25$

QUESTION 4

a)



b) i)



ii)

At A:

iii) $y = x$ — ①
$y = x^2-6$ — ②
$x^2-6 = x$
$x^2-x-6 = 0$
$(x-3)(x+2) = 0$
$x = 3$ or $x = -2$
$\therefore$ at A $(3,3)$

Similarly at B:
$y = -x$ & $y = x^2-6$
$\Rightarrow B(-3,3)$

iv) $-3 \le x \le 3$

c) $f(x) = 2x^3-x$
$f(-x) = 2(-x)^3-(-x)$
$= -2x^3+x$
$= -f(x)$
$\therefore$ function is odd

QUESTION 5

a) $\frac{2^4}{2^{3x} \times 2^{3-3x}} = \frac{2^4}{2^3} = 2$

b) Let $3^x = u$
$u^2-10u+9=0$
$(u-9)(u-1)=0$
$u = 9$ or $u = 1$
$\therefore 3^x = 9$ or $3^x = 1$
$x = 2$ or $x = 0$

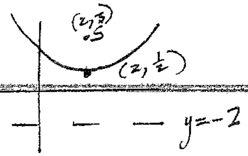
c) $\log_5 (2x)^2 = \log_5 (x+5)$
$4x^2 = x+5$
$4x^2-x-5 = 0$
$(4x-5)(x+1) = 0$
$x = 5/4$ or $x = -1$
But can't have log of neg no
$\therefore x = \frac{5}{4}$

d) i) $T_{30} = -15+29\times7$
$= 188$
ii) $S_{20} = \frac{n}{2}(a+l)$
$= 15(-15+188)$
$= 2595$

QUESTION 6

a) i) $\alpha+\beta = 7/5$
ii) $\alpha\beta = -2$
iii) $\frac{1}{\alpha^2\beta}+\frac{1}{\alpha\beta^2} = \frac{\beta+\alpha}{\alpha^2\beta^2}$
$= \frac{7/5}{(-2)^2}$
$= \frac{7}{20}$
iv) $\alpha^2+\beta^2 = (\alpha+\beta)^2-2\alpha\beta$
$= \left(\frac{7}{5}\right)^2 - 2\times-2$
$= 5\frac{24}{25}$

b) No real roots if $\Delta < 0$
$4-12k < 0$
$-12k < -4$
$k > \frac{1}{3}$



c) i)

ii) $(x-2)^2 = 4 \times 2\frac{1}{2}(y - \frac{1}{2})$

$(x-2)^2 = 10(y-\frac{1}{2})$

QUESTION 7

i) $y' = 3x$

ii) $y' = -\frac{4}{3}x^{-3} = \frac{-4}{x^3}$

iii) $y' = \frac{2(1+x^2) - 2x \times 2x}{(1+x^2)^2}$

$= \frac{2-2x^2}{(1+x^2)^2}$

iv) $y' = x^2\, 5(2x-1)^4 \times 2 + (2x-1)^5\, 2x$

$= 10x^2(2x-1)^4 + 2x(2x-1)^5$

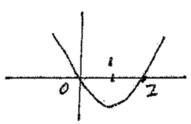
$= 2x(2x-1)^4(5x + 2x - 1)$

$= 2x(2x-1)^4(7x-1)$

v) $y' = \frac{1}{3}(4-x^2)^{-2/3} \times -2x$

$= \frac{-2x}{3(4-x^2)^{2/3}}$

b)



QUESTION 8

a) $y' = 2x + 3$

When $x = 1$ grad of tan $= 5$

$\therefore$ grad of normal $= -\frac{1}{5}$

eqn: $y - 4 = -\frac{1}{5}(x-1)$

$5y - 20 = -x + 1$

$x + 5y - 21 = 0$

b) $12 = \frac{8}{1-r}$

$12 - 12r = 8$

$12r = 4$

$r = \frac{1}{3}$

c) i) Ht after 3rd bounce $= \left(\frac{3}{4}\right)^3 \times 4$

$= \frac{27}{16}$ m

ii) Geometric sequence

iii) Total distance =

$4 + \frac{3}{4} \times 4 \times 2 + \left(\frac{3}{4}\right)^2 \times 4 \times 2 + \ldots$

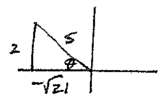
$= 4 + \frac{3}{4} \times 8 + \left(\frac{3}{4}\right)^2 \times 8 + \ldots$

$= 4 + \frac{\frac{3}{4} \times 8}{1 - 3/4}$

$= 28$ m.

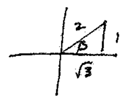
QUESTION 9

a) i) $\cos\beta = \frac{\sqrt{3}}{2}$



ii) $\sec^2\beta = \frac{1}{\cos^2\beta}$

$= \frac{4}{3}$



iii) $\tan\theta = \frac{-2}{\sqrt{21}}$

iv) $\operatorname{cosec}\beta + \tan(90 - \theta)$

$= \frac{1}{\sin\beta} + \cot\theta$

$= 2 + \frac{-\sqrt{21}}{2} = \frac{4 - \sqrt{21}}{2}$

b) ii) $\angle APB = \angle NPB - \angle NPA$

$= 115 - 55$

$= 60°$

iii) $AB^2 = 20^2 + 27^2 - 2 \times 20 \times 27 \times \cos 60$

$AB = \sqrt{589}$

$= 24.3$ nm (3 sf)

QUESTION 10

a) i) Diags of rhombus bisect ∠s at vertex

ii) In ΔS ALD, CLD

∠ADL = ∠LDC (in i)

DL common

AD = DC (sides of rhombus)

∴ ΔALD ≡ ΔCLD (SAS)

iii) ∠DAL = 90° (alt ∠s AD ∥ BC)

iv) ∠LCD = 90° (corr ∠s in cong Δs)

b) i) $m^2 - 2m + 9 = m^2 - 2m + 1 + 8$

$= (m-1)^2 + 8$

$(m-1)^2 \geq 0$ for all m (perf square)

$\therefore (m-1)^2 + 8 > 0$ for all m

$\therefore m^2 - 2m + 9 > 0$ for all m

ii) $x^2 - (m+1)x + (m-2) = 0$

$\Delta = [-(m+1)]^2 - 4 \times 1 \times (m-2)$

$= m^2 + 2m + 1 - 4m + 8$

$= m^2 - 2m + 9$

> 0 for all m (from i)

$\therefore$ roots are real + different for all real m.

c) $x^2 - 2px - q^3 = 0$

Let roots be α & α^2

$\alpha + \alpha^2 = 2p$ — ①

$\alpha^3 = -q^3$ — ②

$\therefore \alpha = -q$

Sub in ①

$-q + q^2 = 2p$

$p = \frac{q^2 - q}{2}$

$= \frac{q(q-1)}{2}$