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Ascham School

Form 5
Preliminary Course Examination

Mathematics

August 2003

Time allowed: 3 hours
You will be allowed 5 minutes of reading time.

Instructions:

- All questions should be attempted
- All necessary working should be shown
- Marks may not be awarded for careless or badly presented work
- Do each question in a separate booklet
- Write your name and your teacher's name on each booklet
- Clearly label the front of each booklet with the number of the question
- Approved calculators may be used.

Question 1 (12 marks)

a) Evaluate to 3 significant figures $\sqrt{(1.3)^4+(3.4)^2}$ (1)

b) Express in scientific notation $\frac{3.78\times10^7}{9\times10^3}$ (1)

c) Simplify: $5\sqrt{3}+\sqrt{20}-2\sqrt{12}+\sqrt{45}$ (2)

d) Solve for x and y if $x+y=6$ and

$2x-y=9$ (2)

e) Factorise: $3x^2+8x-16$ (2)

f) Express $\frac{2\sqrt{2}-3}{2\sqrt{2}+3}$ with a rational denominator. (2)

g) Solve for x: $x^2=4x$ (2)

Question 2 (12 marks) **Start this question in a new booklet**

a) Solve $\frac{x}{3}-\frac{x-1}{4}=1$ (2)

b) Solve $|3x-1|=4$ (2)

c) Find the values of x for which $|1-2x|<5$ (3)

d) Find the exact value of $\sin 225^0$ (1)

e) If $\tan\theta=-\sqrt{3}$ for $0^\circ\le\theta\le360^\circ$, find θ (2)

f) Simplify $\frac{1-\cos^2\theta}{\sin\theta\cos\theta}$ (2)

Question 3 (12 marks) <u>**Start this question in a new booklet**</u>

a) Solve for x: $9^x = \sqrt{27}$ (2)

b) If $\cos\theta = -\frac{1}{3}$ and $\tan\theta < 0$, find the value of $\sin\theta$ (2)

c) Find $\lim_{x\to 3}\frac{x^2-9}{x^2-x-6}$ (2)

d) Simplify $\frac{\log_x 3}{\log_x 9}$ (2)

e) The first three terms of an arithmetic series are 12, 17 and 22.

i) Find the twenty-fifth term of this series. (2)

ii) Find the sum of the first twenty-five terms. (2)

Question 4 (12 marks) <u>**Start this question in a new booklet**</u>

a) For the function $f(x) = 2\sqrt{25-x^2}$ find the domain. (1)

b) For the the curve $y = -x^2 + x + 12$

i) Draw the graph (2)

ii) Find the range if the domain is $0 \le x \le 5$ (2)

c) Given $f(x) = \begin{cases} x^2, & x < 0 \\ 2x, & 0 \le x \le 3 \\ 4, & x > 3 \end{cases}$

(i) Find f(-2) (1)

(ii) Sketch the curve of y = f(x) (3)

d) On a number plane diagram clearly show the region where (3)

$x^2 + y^2 \le 9$ *and* $y > \frac{1}{x}$

Question 5 (12 marks) <u>**Start this question in a new booklet**</u>

A(2,-2), B(-2,-3) and C(0,2) are the vertices of a triangle ABC.

a) Plot the points to form the triangle ABC (1)

b) Find the length of AC and the gradient of AC (3)

c) Show that the equation AC is $2x + y - 2 = 0$ (2)

d) Calculate the perpendicular distance of B from the side AC (2)

e) Find the coordinates of D such that ABCD is a parallelogram. (2)

f) Find the area of the parallelogram ABCD. (2)

Question 6 (12 marks) <u>**Start this question in a new booklet**</u>

a) Find the values of m for which the equation (3)

$x^2 + (m-2)x + 4 = 0$

has no real roots

b) Find the equation of a parabola with focus (1,2) and whose directrix is y = -4 (3)

c) If α and β are the roots of the equation $x^2 + 5x - 8 = 0$

find the value of

i) $\alpha + \beta$ (1)

ii) $\alpha\beta$ (1)

iii) $\frac{1}{\alpha} + \frac{1}{\beta}$ (2)

iv) $\alpha^2 + \beta^2$ (2)

Question 7 (12 marks) <u>Start this question in a new booklet</u>

a) Differentiate

i) $\frac{1}{2}x^3 - x$ (1)

ii) $x^3(1-x)^4$ (2)

iii) $\frac{x^2}{1+x}$ (2)

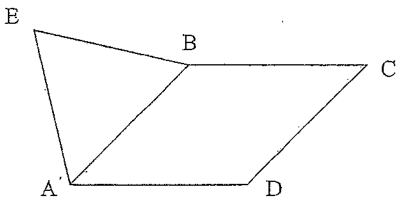
iv) $\frac{x^2+2x}{\sqrt{x}}$ (2)

v) $\frac{1}{3x^2}$ (2)

b) Find the equation of the normal to the curve $y = x^2 + \frac{5}{x} - 2$ at the point where x = 1. (3)

Question 8 (12 marks) <u>Start this question in a new booklet</u>

a) (the figure below is not to scale)



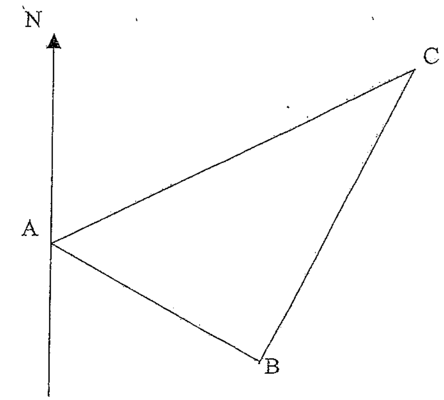
ABCD is a rhombus with $\angle BDC = 48°$. ABE is an equilateral triangle.

(i) In your examination booklet draw a neat sketch showing this information

(ii) Find the size of $\angle EAD$, giving reasons for your answer (4)

(iii)Find the size of $\angle EDA$, giving reasons for your answer (2)

b)



Not to scale

Two geologists on a large flat mining claim drive 20 km from point A on a bearing of 150^0 T to point B. They then drive 40 km on a bearing of 020^0 T to point C.

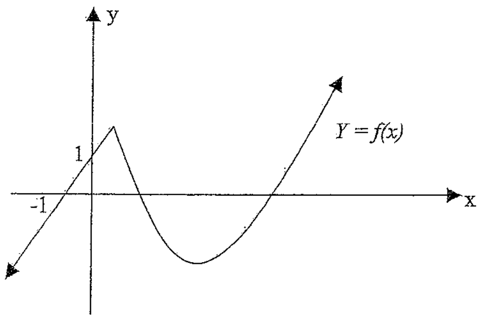
i) Copy the above diagram into your examination booklet and fill in the data (2)

ii) Show that $\angle ABC = 50^0$ (2)

iii) Find the distance of point C from point A to the nearest kilometre. (2)

Question 9 (12 marks) <u>Start this question in a new booklet</u>

a) The graph of the curve $y = f(x)$ *is* drawn below. Copy the diagram into your answer booklet and draw the graph of the corresponding gradient function showing clearly all the important features. (2)



b) A man wishes to form a rectangular enclosure using his existing fence on one side. He has 20 metres of fencing material available to form the other three sides which are x metres, x metres and y metres respectively.

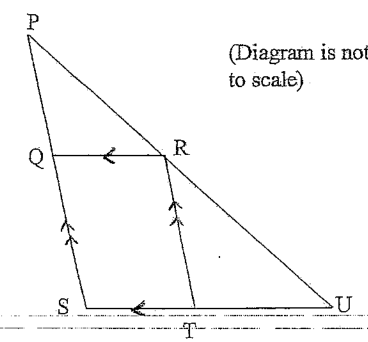
i) Show that the area of the enclosure he can form is given by $2x(10-x)$ square metres. (2)

ii) What are the dimensions of the largest area he can fence? (2)

c)

In the triangle PSU, QR ll SU, SP ll TR, ST = 7.5 cm, PQ = 10 cm, PR = 12 cm and UT = 15 cm.

(Diagram is not to scale)



i) Prove Δ PQR is similar to ΔPSU (2)

ii) Hence find the length of SQ (2)

c) For what values of x does the series $x + x(3+x) + x(3+x)^2 + \ldots\ldots$ have a limiting sum? (2)

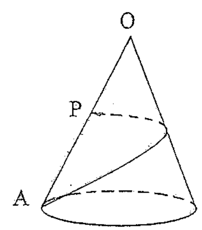
Question 10 (12 marks) <u>Start this question in a new booklet</u>

a) Find the locus of points P such that PA is perpendicular to PB where A is the point (-1,4) and B is the point (2,3). Describe the locus fully. (4)

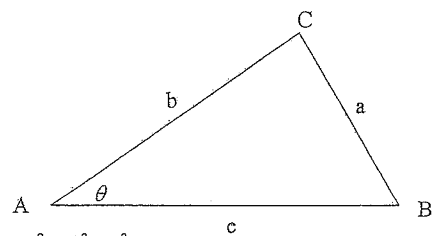
b) A thin sheet of smooth metal is in the form of a sector of a circle with OB and OA as bounding radii each of length 10 cm, and angle AOB is 60^0.

i) Find the exact length of the arc AB (2)

ii) The sheet is now bent to form a right circular cone by welding the bounding radii OA, OB together. On the surface of this cone a string is pulled tight starting with one end fixed at the point A and passing once round the cone to the other end P which is at the midpoint of OA (see diagram). Find the exact length of this string. (2)



c) Using the triangle below, find an expression for *cos θ* or a value for *cos θ* (if it exists) for each of the set of conditions below. Give clear explanations.



i) $a^2 = b^2 + c^2$ (1)

ii) $a = b + c$ (1)

iii) $a > b + c$ (1)

iv) $a < b + c$ (1)

END OF EXAM

FORM 5 MATHEMATICS (2 UNIT) AUGUST EXAM 2003

QUESTION 1 ①

a) $\sqrt{(1\cdot3)^4+(3\cdot4)^2} = 3\cdot80\ (3\ sf)$

b) $\dfrac{3\cdot78\times10^7}{9\times10^3} = 4\cdot2\times10^3$ ①

c) $5\sqrt3+\sqrt{20}-2\sqrt{12}+\sqrt{45}$
$= 5\sqrt3+2\sqrt5-4\sqrt3+3\sqrt5$ ✓
$= \sqrt3+5\sqrt5$ ✓ ②

d) $x+y=6 \quad -(1)$
$2x-y=9 \quad -(2)$
(1)+(2) $3x=15$
$x=5$ ✓
sub in (1) $y=1$ ✓ ②

e) $3x^2+8x-16=(3x-4)(x+4)$ ✓ ✓ ②

f) $\dfrac{2\sqrt2-3}{2\sqrt2+3}\times\dfrac{2\sqrt2-3}{2\sqrt2-3}$ ✓
$= \dfrac{8-12\sqrt2+9}{8-9}$
$= -17+12\sqrt2$ ✓ ②

g) $x^2=4x$
$x^2-4x=0$
$x(x-4)=0$ ✓
$x=0$ or $x=4$ ✓ ②

QUESTION 2

a) $\dfrac{x}{3}-\dfrac{x-1}{4}=1$
$4x-3(x-1)=12$
$4x-3x+3=12$ ✓
$x=9$ ✓ ②

b) $|3x-1|=4$
$3x-1=4$ or $3x-1=-4$
$3x=5$ or $3x=-3$
$x=\frac{5}{3}$ or -1 ✓ ✓ ②

c) $|1-2x|<5$
$-5<1-2x<5$ ✓
$-6<-2x<4$ ✓
$3>x>-2$ ✓ ③
$-2<x<3$

d) $\sin225^\circ=-\sin45^\circ$
$=-\frac{1}{\sqrt2}$ ①

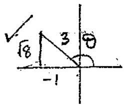
e) $\tan\theta=-\sqrt3$
(acute $\theta=60^\circ$)
$\theta=120^\circ$ or 300° ✓ ✓ ②

f) $\dfrac{1-\cos^2\theta}{\sin\theta\cos\theta}=\dfrac{\sin^2\theta}{\sin\theta\cos\theta}$ ✓
$=\dfrac{\sin\theta}{\cos\theta}$
$=\tan\theta$ ✓ ②

QUESTION 3

a) $9^x=\sqrt{27}$
$3^{2x}=3^{3/2}$ ✓
$2x=3/2$
$x=\frac{3}{4}$ ✓ ②

b) $\cos\theta=-\frac{1}{3}$ $\tan\theta<0$ ✓



$\sin\theta=\dfrac{\sqrt8}{3}$ ✓
$\left(\text{or } \dfrac{2\sqrt2}{3}\right)$ ②
0·94

c) $\lim_{x\to3}\dfrac{x^2-9}{x^2-x-6}=\lim_{x\to3}\dfrac{(x-3)(x+3)}{(x-3)(x+2)}$ ✓
$=\frac{6}{5}$ ✓ ②

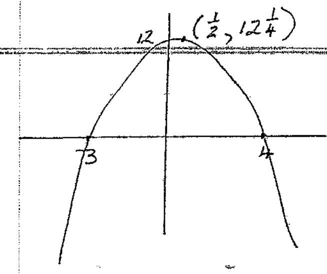
d) $\dfrac{\log_x3}{\log_x9}=\dfrac{\log_x3}{2\log_x3}$ ✓
$=\frac{1}{2}$ ✓ ②

e) 12, 17, 22
i) $T_{25}=12+24\times5$ ✓
$=132$ ✓ ②
ii) $S_{25}=\frac{25}{2}(12+132)$ ✓
$=1800$ ✓ ②

QUESTION 4

a) $f(x)=2\sqrt{25-x^2}$
D: $-5\le x\le5$ ①

b) i) $y=-x^2+x+12$
$=-(x-4)(x+3)$

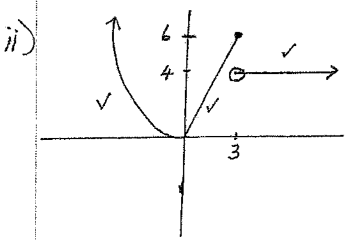


②

ii) $x=5$ $y=-25+5+12$
$=-8$
∴ R: $-8\le y\le12\frac{1}{4}$ ✓✓ ②

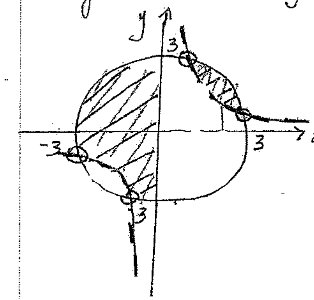
c) i) $f(-2)=4$ ①

ii)



③

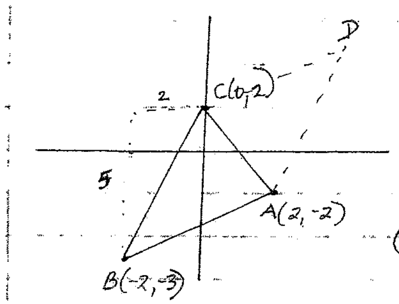
d) $x^2+y^2\le9$ and $y>\frac{1}{x}$



③

QUESTION 5

a)



(1)

b) $AC = \sqrt{4+16}$

$= \sqrt{20}$ (2)

$m_{AC} = \frac{4}{-2}$

$= -2$ (1)

c) $y - y_1 = m(x - x_1)$

$y - 2 = -2(x - 0)$ ✓

or $y+2 = -2(x-2)$

$y - 2 = -2x$

$2x + y - 2 = 0$ ✓ (2)

d) $pd = \frac{|-2\times2 + -3\times1 - 2|}{\sqrt{2^2+1^2}}$ ✓

$= \frac{|-9|}{\sqrt5}$

$= \frac{9}{\sqrt5}$ ✓ (2)

e) $D(4, 3)$ ✓ ✓ (2)

f) Area of $\Delta ABC = \frac12 \sqrt{20} \times \frac{9}{\sqrt5}$

$= \frac12 . 2 \times 9$ ✓

$= 9u^2$

$\therefore$ area of $\parallel$gram $= 18u^2$ ✓ (2)

QUESTION 6

a) $x^2 + (m-2)x + 4 = 0$

$\Delta = (m-2)^2 - 4\times4$

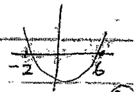
$= m^2 - 4m + 4 - 16$ ✓

$= m^2 - 4m - 12$

For non real roots $\Delta < 0$ ✓

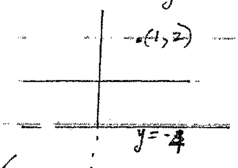
$(m-6)(m+2) < 0$

$-2 < m < 6$ ✓



(3)

b) Focus $(1, 2)$ Direc. $y = -3$



Vertex $= (1, -1)$ ✓ $a = 3$ ✓

$(x-1)^2 = 4\times3(y+1)$

$(x-1)^2 = 12(y+1)$ ✓ (3)

c) $x^2 + 5x - 8 = 0$

i) $\alpha + \beta = -5$ (1)

ii) $\alpha\beta = -8$ (1)

iii) $\frac1\alpha + \frac1\beta = \frac{\beta+\alpha}{\alpha\beta}$ ✓

$= \frac{-5}{-8}$

$= \frac58$ ✓ (2)

iv) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$ ✓

$= 25 - 2\times -8$

$= 41$ ✓ (2)

QUESTION 7

a) i) $\frac{d}{dx}\left(\frac12 x^3 - x\right) = \frac32 x^2 - 1$ (1)

ii) $\frac{d}{dx} x^3(1-x)^4 = x^3.4(1-x)^3.-1 + (1-x)^4 3x^2$ ✓

$= x^2(1-x)^3(-4x + 3 - 3x)$

$= x^2(1-x)^3(3-7x)$ ✓ (2)

iii) $\frac{d}{dx}\frac{x^2}{1+x} = \frac{(1+x)2x - x^2}{(1+x)^2}$ ✓

$\frac{2x + 2x^2 - x^2}{(1+x)^2}$

$= \frac{2x + x^2}{(1+x)^2}$ ✓ (2)

iv) $\frac{d}{dx}\frac{x^2+2x}{\sqrt x} = \frac{d}{dx}\left(x^{1\frac12} + 2x^{\frac12}\right)$ ✓

$= \frac32 x^{\frac12} + x^{-\frac12}$

$= \frac{3\sqrt x}{2} + \frac{1}{\sqrt x}$ ✓ (2)

OR

$\frac{d}{dx}\frac{x^2+2x}{\sqrt x} = \frac{\sqrt x(2x+2) - (x^2+2x)\frac12 x^{-\frac12}}{x}$

etc.

v) $\frac{d}{dx}\left(\frac{1}{3x^2}\right) = \frac{d}{dx}\frac13 x^{-2}$ ✓

$= \frac{-2}{3}x^{-3}$

$= \frac{-2}{3x^3}$ ✓ (2)

b) $y = x^2 + \frac5x - 2$

$y' = 2x - \frac{5}{x^2}$ ½

$x = 1$ $y' = 2 - 5$; $y = 4$ ½

$= -3$ ½

grad of tangent $= -3$

grad of normal $= \frac13$ ½

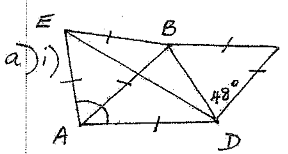
$y - 4 = \frac13(x - 1)$

$3y - 12 = x - 1$

$x - 3y + 11 = 0$ ✓ (3)

QUESTION 8

a) i)



ii) $AB = BE = AE$ (equilat Δ)

$AB = BC = CD = AD$ (rhombus)

$\angle BDA = 48°$ (diag bisects $\angle$ of rhombus) ✓

$\therefore \angle BAD = 180 - 96$ (coint $\angle$s $AB \parallel CD$) ✓

$= 84°$

$\angle EAB = 60°$ ($\angle$ of equilat Δ) ✓

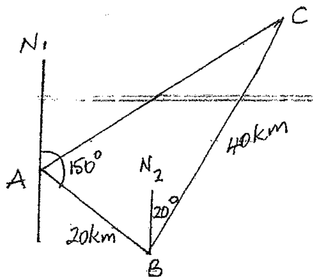
$\therefore \angle EAD = 144°$ ✓ (4)

iii) ΔAED is isos ($AE = AD$ b/c both $= AB$) ✓

$\therefore \angle EDA = \frac{180 - 144}{2}$ ($\angle$ sum of isos Δ)

$= 18°$ ✓ (2)

b) i)



(2)

ii) $\angle N_2BA = 30°$ (co-int ∠s $N_1A \parallel N_2B$)

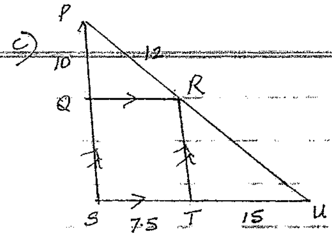
$\therefore \angle ABC = 30° + 20°$
$= 50°$ (2)

iii) $AC^2 = 20^2 + 40^2 - 2 \times 20 \times 40 \cos 50$
$= 971.5398\ldots$

$AC = 31.16\ldots$

$\therefore$ distance of C from A is 31 km (to n km) (2)

c)



i) In Δs PQR, PSU

∠P is common

∠PQR = ∠PSU (corr ∠s, QR ∥ SU) (2)

$\therefore \Delta PQR \parallel\parallel \Delta PSU$ (equiangular)

ii) $\therefore \frac{PQ}{PS} = \frac{QR}{SU} = \frac{PR}{PU}$ (corr sides sim ∆s)

$\frac{10}{PS} = \frac{QR}{22.5} = \frac{12}{PU}$

$\frac{10}{PS} = \frac{7.5}{22.5}$

$225 = 7.5 PS$

$PS = 30$

$\therefore QS = 20$ cm (2)

d) $x,\ x(3+x),\ x(3+x)^2 \ldots\ldots$

For limiting sum $|r| < 1$

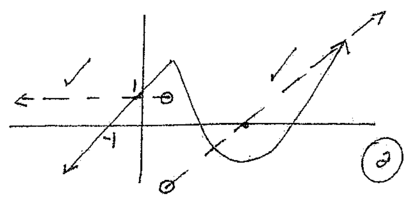
i.e. $|3+x| < 1$

$-1 < 3+x < 1$

$-4 < x < -2$ (2)

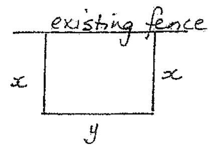
QUESTION 9

a)



(2)

b) i)



$2x + y = 20$

$y = 20 - 2x$

Area $= x(20 - 2x)$
$= 2x(10 - x)$ (2)

ii) $A = 20x - 2x^2$

Axis of Sym $x = 5$

$\therefore x = 5$ m, $y = 10$ m are dimensions of rect of max (2)

QUESTION 10

a) Let P(x, y) be pt of locus

$m_{PA} = \frac{y-4}{x+1}$ ½

$m_{PB} = \frac{y-3}{x-2}$ ½

$m_{PA} \times m_{PB} = -1$

$\frac{y-4}{x+1} \times \frac{y-3}{x-2} = -1$

$\frac{y^2 - 7y + 12}{x^2 - x - 2} = -1$

$y^2 - 7y + 12 = -x^2 + x + 2$

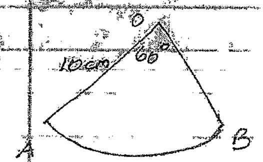
$x^2 - x + y^2 - 7y = -10$

$x^2 - x + (\frac{1}{2})^2 + y^2 - 7y + (\frac{7}{2})^2 = -10 + \frac{1}{4} + \frac{49}{4}$

$(x - \frac{1}{2})^2 + (y - 3\frac{1}{2})^2 = 2\frac{1}{2}$

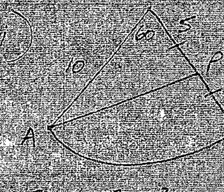
$\therefore$ locus is a circle centre $(\frac{1}{2}, 3\frac{1}{2})$ and radius $\sqrt{2.5}$ (4)

b)



i) Arc AB $= \frac{60}{360} \times 2 \times \pi \times 10$
$= \frac{1}{6} \times 20\pi$
$= \frac{10\pi}{3}$ cm. (2)

ii)



$AP^2 = 10^2 + 5^2 - 2 \times 10 \times 5 \cos 60$
$= 100 + 25 - 50$
$= 75$

$AP = \sqrt{75}$
$= 5\sqrt{3}$ cm. (2)

c) i) $a^2 = b^2 + c^2$

$\angle A = 90°$ (Pythag)

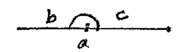
$\cos\theta = \cos 90°$
$= 0$ (1)

ii) $a = b + c$

No ∆ - just a straight line

$\therefore \angle A = 180°$

$\therefore \cos\theta = \cos 180°$
$= -1$



iii) $a > b + c$

∆ABC does not exist (any 2 sides together must be greater than 3rd side)

$\therefore$ no value for $\cos\theta$. (1)

iv) $a < b + c$

$\cos\theta = \frac{b^2 + c^2 - a^2}{2bc}$ (1)

where $-1 < \cos\theta < 1$

because $0° < \theta < 180°$.