b $x = e^{\frac{t}{2}} - t$

i) $v = \frac{1}{2}e^{\frac{t}{2}} - 1$ ✓

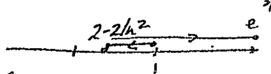
ii) $t=0 \quad v = \frac{1}{2}\times 1 - 1$
$= -\frac{1}{2} < 0$
∴ particle moving to left ✓

iii) $v = 0$
$\frac{1}{2}e^{t/2} = 1$
$e^{t/2} = 2$
$\ln 2 = t/2$
$t = 2\ln 2$
$= 1\cdot 39$ (2dp)
∴ particle comes to rest ✓
after $2\ln 2$ or $1\cdot 39$ sec.

iv) $a = \frac{1}{4}e^{t/2}$ ✓

v) $a > 0$ for all t
∴ after $2\ln 2$ sec when
particle is moving to right & $a > 0$ ✓
particle continues moving to
right & speeds up. ($a>0, v>0$)

vi) $t=0 \quad x = 1$
$t = 2\ln 2 \quad x = 2 - 2\ln 2$ ~~≈ 0.61~~
$t = 3 \quad x = e^{3/2} - 3$



or
$2[1-(2-2\ln 2)] + e^{3/2} - 3 - 1$

Particle's distance $= 1 - (2-2\ln 2) + e^{3/2} - 3 - (2-2\ln 2)$
$= 1 - 2 + 2\ln 2 + e^{3/2} - 3 - 2 + 2\ln 2$
$= -6 + e^{3/2} + 4\ln 2$ ✓✓
$= 1\cdot 254$ units (3dp)

OR
Use definite integrals

7

ASCHAM SCHOOL

FORM 6 2 UNIT MATHEMATICS TRIAL EXAMINATION

2001

July 2001

Time allowed: 3 hours
Plus 5 minutes reading time

Instructions

1. Attempt all questions
2. All questions are of equal value
3. All necessary working should be shown in each question. Marks may be deducted for careless or badly presented work.
4. Standard integrals are provided at the back of the paper.
5. Board approved calculators may be used.
6. Answer each question in a separate writing booklet

QUESTION 1

a) Solve $5-3x \geq 7$ and graph your solution on a number line. (2)

b) Solve the equations $x-2y=3$

$2x+y=1$ (2)

c) Evaluate $\log_e 5$ to 2 decimal places. (1)

d) Solve for x: $|3x+1|=4$ (2)

e) Fnd the exact value of $\cos\frac{5\pi}{6}$ (1)

f) Find p and q if $\frac{6}{2-\sqrt{3}}=p+q\sqrt{3}$ and p and q are integers (2)

g) Sketch $y=x(x-1)$ on the Cartesian plane indicating intercepts on axes and vertex. (2)

b) $mx^2+(m+1)x+1=0$

$\Delta=(m+1)^2-4m$

$=m^2+2m+1-4m$

$=m^2-2m+1$

$=(m-1)^2$ ✓

For real & different roots $\Delta>0$

$(m-1)^2>0$ for all m except $m=1$ $\frac{1}{2}$

∴ roots are real & different for all $m \neq 1$. ✓ (2)

c) $\frac{d}{dx}\left(\frac{\cos x}{1+\sin x}\right)$

$=\frac{(1+\sin x).-\sin x-\cos x.\cos x}{(1+\sin x)^2}$

$=\frac{-\sin x-\sin^2 x-\cos^2 x}{(1+\sin x)^2}$

$=\frac{-\sin x-(\sin^2 x+\cos^2 x)}{(1+\sin x)^2}$

$=\frac{-\sin x-1}{(1+\sin x)^2}$ ✓$1\frac{1}{2}$

$=\frac{-(\sin x+1)}{(1+\sin x)^2}$

$=\frac{-1}{1+\sin x}$

For stat pt. first deriv = 0

But this is impossible

∴ no stationary pt

Now consider endpoints

When $x=0$: $\frac{\cos x}{1+\sin x}=1$ ✓

When $x=\pi$: $\frac{\cos x}{1+\sin x}=-1$ ✓

1 & min value

QUESTION 10

a) Let A_n be amount owing after n years

15% pa = 1.25% p month

$=0.0125$

i) $A_1=400000(1.0125)^6-M$ ✓

ii) $A_2=A_1(1.0125)^{12}-M$

$=[400000(1.0125)^6-M](1.0125)^{12}-M$

$=400\,000(1.0125)^{18}-M(1.0125)^{12}-M$

$=400\,000(1.0125)^{18}-M(1.0125^{12}+1)$ ✓

iii) $A_4=400000(1.0125)^{42}-M(1.0125^{36}+1.0125^{24}+1.0125+1)$ ✓

But $A_4=0$

∴ $400000(1.0125)^{42}=M\left[1.\frac{1.0125^{12\times4}-1}{1.0125^{12}-1}\right]$ ✓

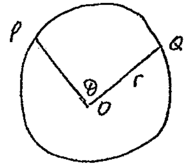
$M=\frac{400\,000(1.0125)^{42}.(1.0125^{12}}{1.0125^{48}-1}$

iv) $M=132882.52$

∴ annual instalment is \$132882.52. ✓ (5)

QUESTION 8

a)



$P = 10cm$

i) $r\theta + 2r = 10$ ✓

$\theta = \frac{10-2r}{r}$

Area $= \frac{1}{2}r^2\theta$

$A = \frac{1}{2}r^2\left(\frac{10-2r}{r}\right)$ ✓

$= \frac{1}{2}r(10-2r)$

$= r(5-r)$ ✓ (3)

$= 5r - r^2$

ii) $A' = 5 - 2r$ ✓

$A'' = -2$

Max A if $A' = 0$ & $A'' < 0$

$5 - 2r = 0$

$r = 2\frac{1}{2}$ $\frac{1}{2}$ (2)

$A'' = -2 < 0$ $\frac{1}{2}$

∴ if radius is $2\frac{1}{2}$cm, area is maximum

b) $M = M_0 e^{-kt}$

i) $\frac{dM}{dt} = M_0 \cdot -k e^{-kt}$

$= -k M_0 e^{-kt}$ (1)

$= -kM.$

ii) $M = 200e^{-kt}$

$M = 150, t = 2:$ $150 = 200e^{-k\times 2}$

$\frac{150}{200} = e^{-2k}$

$\frac{3}{4} = e^{-2k}$

$k = \frac{\ln 3/4}{-2}$ (2)

iii) $t = 10:$ $M = 200 e^{-\frac{\ln\frac{3}{4}}{-2}\times 10}$

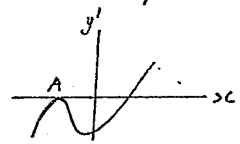
$= 200 e^{\ln 3/4}$

$\doteqdot 47.4609....$ (2)

∴ amount of chemical left after 10 years is 47 g (to n gram)

∴ Amt decomposed is 153g (to n gram)

c)



When x is less than A $y' < 0$

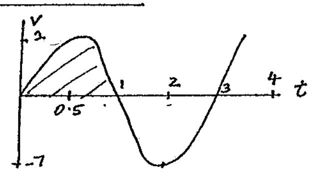
at A $y' = 0$ ✓

to right of A $y' < 0$

∴ A is a horizontal point of inflexion (on a decreasing curve). ✓ (2)

QUESTION 9

a)



i) $v = 2$ m/s in → after 0.5 sec. ✓

ii) $a = 0$ when $t = 2$ ✓

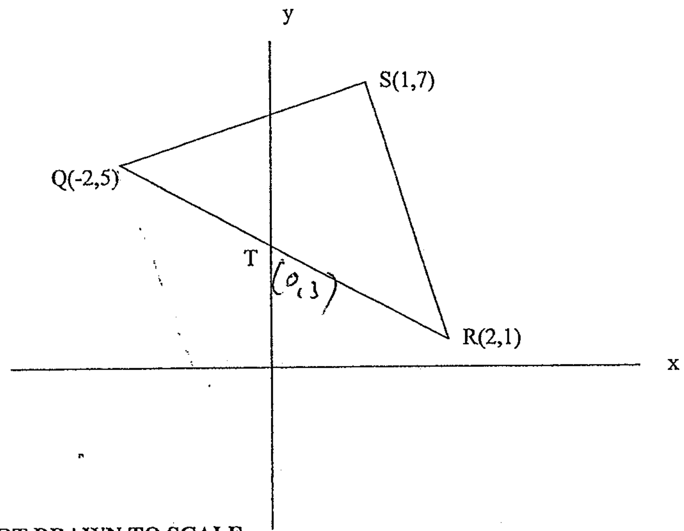
iii) Particle changes direction when $v = 0$ ie. $t = 1$ & $t = 3$ ✓

iv) Shaded area represents the distance travelled by the particle in the first 1 second ✓

v) Speed max ie. 7 m/s when $t = 2$ ✓

(5)

QUESTION 2



NOT DRAWN TO SCALE

Draw the diagram in your answer booklet

a)	Show that the equation of the line QR is $x + y - 3 = 0$	(2)
b)	Find the perpendicular distance from S to QR.	(2)
c)	Hence find the area of triangle SQR.	(2)
d)	If T lies on the y axis, show that T is the midpoint of QR.	(2)
e)	Find the co-ordinates of the point P such that QPRS is a parallelogram.	(1)
f)	Find the equation of the circle on QR as diameter	(2)
g)	Shade the region defined by $x + y - 3 \geq 0$ and $x \geq 0$	(1)

QUESTION 3

a) Differentiate and simplify:

i) $\frac{1}{x}$ **(1)**

ii) $\tan(2-3x)$ **(2)**

iii) $\frac{e^{2x}}{3x+2}$ **(3)**

iv) $\sin^2 x$ **(1)**

b) ABCD is a rhombus. $\angle DBC = (x+20)°$ and $\angle ABD = 3x°$. Find x. **(2)**

c) The third term of a geometric sequence is –3 and the eighth term is 96. Find the first term and the common ratio of the sequence. **(3)**

QUESTION 4

a) Find i) $\int \frac{x^2+3}{x}dx$ **(2)**

ii) $\int \frac{x}{x^2+3}dx$ **(2)**

b) Find the largest angle of ΔABC to the nearest minute

if AB = 6cm, BC = 8 cm and AC = 12cm. **(3)**

x^2+x

c) i) Show that the gradient of the tangent to the curve $y = \log[x(x+1)]$ is $\frac{3}{2}$ when x = 1. **(3)**

ii) Hence find the equation of the normal to the curve $y = \log[x(x+1)]$ at the point when x = 1. **(2)**

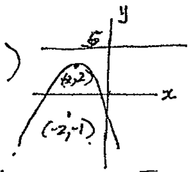
QUESTION 6

a) $(x+2)^2 = -4a(y-2)$

$a = 3$

½ ✓ ½

$\therefore (x+2)^2 = -12(y-2)$ (2)



b) $y = x(x-3)^2$

i) $y' = x\,2(x-3) + (x-3)^2$

$= (x-3)(2x+x-3)$

$= (x-3)(3x-3)$ ✓

$= 3(x-3)(x-1)$

Stationary points at $(3,0)$ ✓

and $(1,4)$ ✓

$y'' = 6x-12$

At $(3,0)$ $y'' = 6 > 0$

$\therefore (3,0)$ a min t/pt ½

At $(1,4)$ $y'' = -6 < 0$

$\therefore (1,4)$ a max t/pt (4) ½

ii) Poss Pt of inflexion at $(2,2)$ ✓

Check change in concavity

x	$2-\varepsilon$ $(1\frac{1}{2})$	2	$2+\varepsilon$ $(2\frac{1}{2})$
y''	-3	0	3

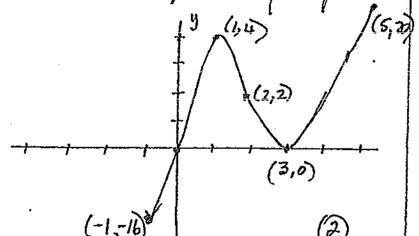
– +

✓ (2)

$\therefore$ change in concavity

$\therefore (2,2)$ a point of inflexion

iii)



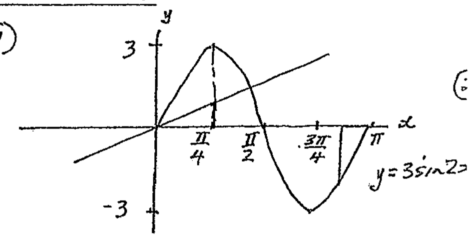
(2)

iv) Max value is 20 (1)

v) Concave down for $-1 < x < 2$ (1)

QUESTION 7

a) i)



ii) Draw $y = x$

When $x = \frac{\pi}{4}$ $y = 0.79$ (1)

There are 2 solutions

✓

iii) Area $= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} 3\sin 2x\,dx + \left|\int_{\frac{\pi}{2}}^{\frac{7\pi}{8}} 3\sin 2x\,dx\right|$

$= \frac{-3}{2}\left[\cos 2x\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}} + \left|-\frac{3}{2}\left[\cos 2x\right]_{\frac{\pi}{2}}^{\frac{7\pi}{8}}\right|$

$= -\frac{3}{2}\left[\cos\pi - \cos\frac{\pi}{2}\right] + \left|-\frac{3}{2}\left(\cos\frac{7\pi}{4} - \cos\pi\right.\right.$

$= \frac{-3}{2}(-1-0) + \left|-\frac{3}{2}\left(\frac{1}{\sqrt{2}} - -1\right)\right|$ ✓ ✓

$= \frac{3}{2} + \left|-\frac{3}{2\sqrt{2}} - \frac{3}{2}\right|$

$= \frac{3}{2} + \frac{3}{2\sqrt{2}} + \frac{3}{2}$

$= \frac{6}{2} + \frac{3}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$ ✓ (5)

$= \left(3 + \frac{3\sqrt{2}}{4}\right) u^2$

) $\frac{d^2y}{dx^2} = 6x+2$

$\frac{dy}{dx} = 3x^2+2x+c$ ✓

$\frac{dy}{dx} = 0$ if $x = 1$: $0 = 3+2+c$

$c = -5$ ✓

$\frac{dy}{dx} = 3x^2+2x-5$

$y = x^3+x^2-5x+d$ ✓

$x = 1, -3$: $3 = 1+1-5+d$ (4)

QUESTION 4

a) i) $\int \frac{x^2+3}{x}\,dx = \int\left(x + \frac{3}{x}\right)dx$

$= \frac{x^2}{2} + 3\log_e x + c$ ✓ ✓ (2)

ii) $\int \frac{x}{x^2+3}\,dx = \frac{1}{2}\log_e(x^2+3)+c$ ✓ ✓ (2)

b) Largest angle is $\angle ABC$

$\cos B = \frac{a^2+c^2-b^2}{2ac}$

$= \frac{8^2+6^2-12^2}{2\times 8\times 6}$ ✓

$= -0.458\dot{3}$ ✓

$B = 180 - 62^\circ 43'$ ✓ (3)

$= 117^\circ 17'$ (to n minute)

$\therefore$ largest angle is $117^\circ 17'$ (to n. minute)

c) i) $y = \log[x(x+1)]$

$= \log x + \log(x+1)$ ✓

$y' = \frac{1}{x} + \frac{1}{x+1}$ ✓

When $x=1$ $y' = \frac{1}{1}+\frac{1}{2}$

$= 1\frac{1}{2}$ ✓

$\therefore$ grad of tangent when $x=1$ is $\frac{3}{2}$. (3)

ii) $\therefore$ grad of normal $= -\frac{2}{3}$ ✓

eqn of normal:

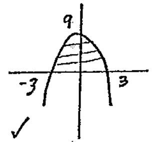
$y - \log 2 = -\frac{2}{3}(x-1)$

$3y - 3\log 2 = -2x+2$ ✓

$2x+3y-3\log 2 - 2 = 0$ (2)

QUESTION 5

a) $y = 9 - x^2$



Volume $= \pi\int_0^9 (9-y)\,dy$ ✓

$= \pi\left[9y - \frac{y^2}{2}\right]_0^9$ ✓

$= \pi\left[81 - \frac{81}{2}\right]$

$= 40.5\pi\ u^3$ ✓ (3)

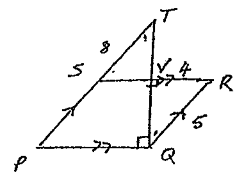
(or $\frac{81\pi}{2}u^3$)

b) Area $\doteqdot \frac{h}{3}\{y_0 + y_4 + 4(y_1+y_3)+2y_2\}$

$= \frac{1}{3}(3.2+3.4+4(2.8+2.5)+2\times 2.3)$ ✓✓

$= 10.8\ m^2$ ✓ (3)

c)



i) In Δs VRQ, QPT

$\angle PQR = \angle QVR = 90^\circ$ (alt $\angle$s, SR∥PQ)

$\angle RQV = \angle STV$ (alt $\angle$s, PT∥QR) ✓✓

$\therefore \Delta VRQ\ |||\ \Delta QPT$ (equiangular)

ii) SP = 5 (opp sides of ∥gram) ✓

$\therefore$ PT = 13 (PS+ST)

$\frac{VR}{QP} = \frac{RQ}{PT} = \frac{VQ}{QT}$ (corr sides of sim Δs) ✓

$\frac{4}{PQ} = \frac{5}{13} = \frac{VQ}{QT}$

$5PQ = 52$ ✓ (6)

$PQ = 10.4$

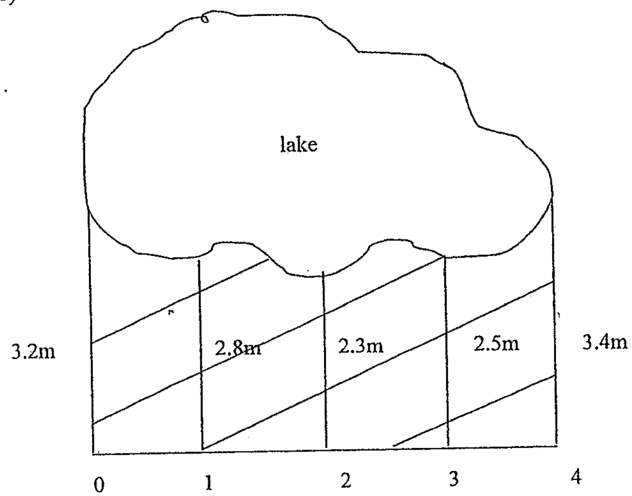
$\therefore SR = 10.4$ (opp sides of ∥gram)

$SV = 10.4 - 4$

QUESTION 5

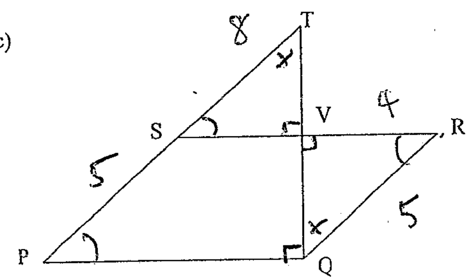
a) Find the volume generated when the area between the curve $y = 9 - x^2$ and the x axis is rotated about the y axis. (3)

b)



The shaded area on the side of a lake is to be grassed. The above plan was drawn showing the measurements.
Using Simpson's Rule, find the approximate area of the grassed section. (3)

c)



PQRS is a parallelogram and PT is a straight line though S.
TQ is perpendicular to PQ.

i) Prove $\Delta VRQ ||| \Delta QPT$

ii) If QR = 5, VR = 4, ST = 8 find TP and QP and hence SV.

(6)

QUESTION 6

a) Find the equation of the parabola with focus (-2,-1) and directrix y = 5. (2)

b) For the curve $y = x(x-3)^2$

i) Find any stationary points and describe their nature. (4)

ii) Find the point of inflexion. (2)

iii) Draw the graph of $y = x(x-3)^2$ for $-1 \le x \le 5$, showing significant points (2)

iv) What is the maximum value of $x(x-3)^2$ for $-1 \le x \le 5$? (1)

v) For what values of x is $y = x(x-3)^2$ concave down for $-1 \le x \le 5$? (1)

QUESTION 7

a) i) Draw the graph of y = 3 sin 2x for $0 \le x \le \pi$ (2)

ii) By adding another line to your graph, find the number of solutions of the equation 3 sin 2x - x = 0 in the interval $0 \le x \le \pi$ (1)

iii) Find the area bounded by the curve y = 3 sin 2x, the x axis, x = $\frac{\pi}{4}$ and x = $\frac{7\pi}{8}$. Give your answer in exact form. (5)

b) If $\frac{d^2y}{dx^2} = 6x + 2$, find the equation of the curve given that it has a minimum turning point at (1,3). (4)

y int of $x + y - 3 = 0$ is $(0,3)$ ✓

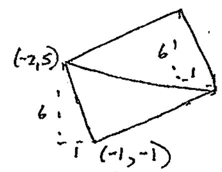
d) Midpoint of QR $= \left(\frac{-2+2}{2}, \frac{5+1}{2}\right)$

$= (0, 3)$ ✓

The pt (0,3) lies on the y axis

∴ T(0,3) is the midpoint of QR. (2)

e)



∴ By comparing gradients P is point (-1,-1)

OR using property that diagonals bisect each other:

T is midpoint of PS

$(0,3) = \left(\frac{x+1}{2}, \frac{y+7}{2}\right)$

$\frac{x+1}{2} = 0$, $\frac{y+7}{2} = 3$

$x + 1 = 0$ $y + 7 = 6$

$x = -1$ $y = -1$

∴ P is (-1,-1) ✓ (1)

f) Eqn of circle:

$x^2 + (y-3)^2 = 8$ ✓ (2)

½ ½

g) Shading on diagram (1)

QUESTION 3

a) i) $\frac{d}{dx}\frac{1}{x} = \frac{d}{dx}x^{-1}$

$= -x^{-2}$

$= \frac{-1}{x^2}$ (1)

ii) $\frac{d}{dx}\tan(2-3x) = \sec^2(2-3x) \times -3$ ✓ ✓

$= -3\sec^2(2-3x)$ (2)

iii) $\frac{d}{dx}\frac{e^{2x}}{3x+2} = \frac{(3x+2)2e^{2x} - e^{2x} \cdot 3}{(3x+2)^2}$ ✓

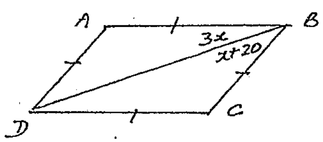
$= \frac{e^{2x}(6x+4-3)}{(3x+2)^2}$ ✓

$= \frac{e^{2x}(6x+1)}{(3x+2)^2}$ ✓ (3)

iv) $\frac{d}{dx}\sin^2 x = 2\sin x\cos x$ (1)

½ ½

b)



$3x = x + 20$ (diag bisects angle of rhombus ✓

$2x = 20$

$x = 10$ ✓ (2)

c) $T_3 = -3$, $T_8 = 96$

$ar^2 = -3$ — ① ½

$ar^7 = 96$ — ② ½

② ÷ ① $r^5 = -32$ ✓

$r = -2$

Sub in ①: $4a = -3$

$a = \frac{-3}{4}$ ✓ (3)

∴ first term is $-3/4$

, common ratio is -2.

ASCHAM

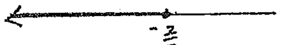
FORM 6 2UNIT EXAM TRIAL 2001

QUESTION 1

a) $5 - 3x \geq 7$

$-3x \geq 2$

$x \leq -\frac{2}{3}$ ✓



✓ ②

b) $x - 2y = 3$ — ①

$2x + y = 1$ — ②

①×2: $2x - 4y = 6$ — ③

② - ③ $5y = -5$

$y = -1$ ✓

Sub in ①: $x + 2 = 3$

$x = 1$ ✓ ②

$\therefore x = 1,\ y = -1$

c) $\log_e 5 \doteqdot 1.6094\ldots$

$= 1.61$ (2dp) ✓ ①

d) $|3x + 1| = 4$

$3x + 1 = 4$ or $3x + 1 = -4$

$3x = 3$ or $3x = -5$

$x = 1$ ✓ or $x = -5/3$ ✓ ②

e) $\cos\frac{5\pi}{6} = \cos 150°$

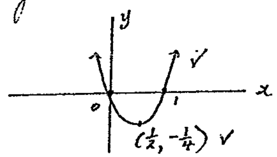
$= -\cos 30°$

$= -\frac{\sqrt{3}}{2}$ ✓ ①

f) $\frac{6}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} = \frac{12 + 6\sqrt{3}}{4-3}$

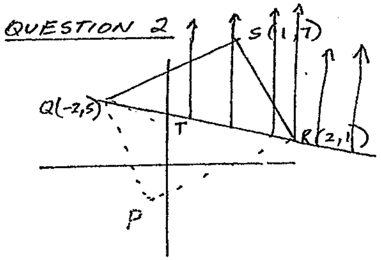
✓ $= 12 + 6\sqrt{3}$ ✓ ②

g) $y = x(x-1)$



②

QUESTION 2



a) $m_{QR} = \frac{1-5}{2+2}$

$= \frac{-4}{4}$ ✓

$= -1$

Eqn QR: $y - 1 = -(x - 2)$

$y - 1 = -x + 2$

$x + y - 1 - 2 = 0$ ✓ ②

$x + y - 3 = 0$

b) $pd = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2+b^2}}$

$= \frac{|1 + 7 - 3|}{\sqrt{1^2+1^2}}$ ✓

$= \frac{5}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$ ✓ ②

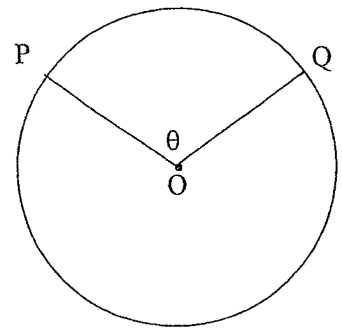
$= \frac{5\sqrt{2}}{2}$

c) Length of $QR = \sqrt{4^2 + (-4)^2}$ ✓

$= \sqrt{32}$

Area $= \frac{1}{2} 4\sqrt{2} \times 5\frac{\sqrt{2}}{2} = 10u^2$ ✓ ②

QUESTION 8



a) Arc PQ subtends an angle of θ radians at the centre.
The radius is r cm and the perimeter of the minor sector is 10 cm.

i) Show that the area of the sector is $A = 5r - r^2$ (3)

ii) Hence find the radius of the sector of maximum area when the perimeter of the sector is 10 cm. (2)

b) The amount M grams of a chemical is given by

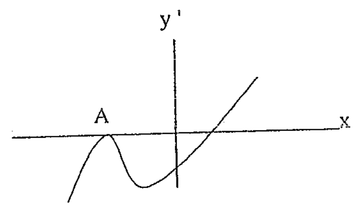
$M = M_0 e^{-kt}$ where M_0 and k are positive constants and time t is measured in years.

i) Show that M satisfies the equation $\frac{dM}{dt} = -kM$ (1)

ii) Find k (in exact form) if 200 grams of the chemical decomposes to 150 grams at the end of 2 years. (2)

iii) Find the amount of the chemical which has decomposed by the end of 10 years (to the nearest gram.) (2)

c)



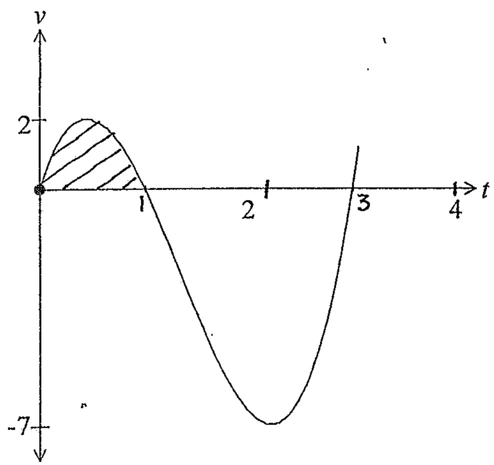
The graph shows the gradient function $y = f'(x)$.

Find the nature of the stationary point of $y = f(x)$ corresponding to A.
Explain.

(2)

QUESTION 9

a)



not drawn to scale

The graph represents the velocity (in m/s) of a particle after t seconds.
The particle is moving in a straight line.

i) What is the velocity of the particle after 0.5 seconds? (1)

ii) What is the acceleration of the particle after 2 seconds? (1)

iii) At what time(s) does the particle change direction? (1)

iv) What does the shaded area on the graph represent? (1)

v) When is the speed of the particle maximum? $(0 \leq t \leq 3)$ (1)

b) Find the values of m for which the equation

$mx^2 + (m+1)x + 1 = 0$ has real and different roots? (2)

c) Find the maximum and minimum values of $\dfrac{\cos x}{1+\sin x}$ for $0 \leq x \leq \pi$. (5)

QUESTION 10

a) Jacqui borrowed \$400 000 from the bank on 1 January 1998. She was not charged interest for the first 6 months. Thereafter she was charged interest of 15% per annum compounded monthly.

She agreed to repay the loan by equal annual instalments of M dollars on 31 December of each year. The loan was to be repaid by the end of 2001.

i) How much did she owe on 31 December 1998 after making her first payment? (1)

ii) How much did she owe on 31 December 1999 after making her second payment? (1)

iii) Show that $M = \dfrac{400000(1.0125)^{42}(1.0125^{12}-1)}{1.0125^{48}-1}$ (2)

iv) Hence find her annual instalment. (1)

b) A particle P is moving in a straight line along the x axis. Its position at time t seconds is given by the equation $x = e^{\frac{t}{2}} - t$.

i) Find an expression for the velocity of the particle at time t. (1)

ii) In what direction is the particle moving initially? (1)

iii) When does the particle come to rest? (1)

iv) Find the acceleration of the particle at time t. (1)

v) What comment can you make about the acceleration and how does it affect the future motion of the particle. (1)

vi) Find the distance travelled by the particle in the first 3 seconds. (2)

END OF EXAM