Stephanie Sim

ASCHAM SCHOOL

2000 TRIAL HSC EXAMINATION

MATHEMATICS 3 / 4 UNIT COMMON PAPER

Time allowed: 2 hours

- All questions should be attempted.
- All necessary working must be shown.
- All questions are of equal value.
- Marks may not be awarded for careless or badly arranged work.
- Write your name and the number of the question on each booklet.
- Begin each question in a new booklet.
- Approved calculators may be used.

QUESTION 1

a) Differentiate i) $4\sec^3 x$

ii) $\sin^{-1}\frac{1}{2}x$ (2)

b) Find the primitive of $\dfrac{1}{9x^2+1}$ (2)

c) Find the co-ordinates of the point P which divides the line joining A(-3,4) and B(2,-8) externally in the ratio 2:5. (2)

d) Use the substitiution $x = \cos\theta$ to evaluate

$$\int_{\frac{1}{2}}^{1}\frac{\sqrt{1-x^2}}{x^2}dx$$ (3)

e) If α, β and γ are the roots of the equations $x^3 - 2x + 5 = 0$, find the value of $\alpha^2 + \beta^2 + \gamma^2$ (3)

QUESTION 2

a) $x = 0.8$ is a good approximation to a root of the equation $x^2 = \cos x$.

Use Newton's method once to find a better approximation to the root, giving your answer to 2 decimal places. **(2)**

b) i) Show that $x = 2$ is a zero of $x^3 - 4x^2 + 8$

ii) Hence find all the real zeros of $x^3 - 4x^2 + 8$, leaving your answers in simplified surd form.

iii) Solve for x: $\dfrac{4}{x-2} \le x$ **(6)**

c) Solve the equation (to the nearest degree)
$3\cos x - 4\sin x = 3$ for $0° \le x \le 360°$. **(4)**

QUESTION 3

a) Sketch the graph of $y = 3\cos^{-1}(2x+1)$. **(2)**

b) Solve for x, $0 \le x \le 2\pi$, $2\sin^2 x < \sin x$. **(3)**

c) The acceleration of a particle moving in a straight line is given by $\dfrac{d^2x}{dt^2} = \dfrac{-72}{x^2}$, where x metres is the displacement from the origin after t seconds. Initially the particle is 9 metres to the right of the origin with a velocity of 4 metres per second.

i) Show that the velocity v of the particle in terms of x is $v = \dfrac{12}{\sqrt{x}}$. Explain why v is always positive for the given initial conditions.

ii) Find an expression for t in terms of x.

iii) How many seconds (to the nearest second) does it take for the particle to reach a point 35m to the right of the origin? **(7)**

QUESTION 4

a) i) Show that $\dfrac{d}{dx}(\cos^3 x \sin x) = 4\cos^4 x - 3\cos^2 x$.

ii) Hence show that $\displaystyle\int_0^{\frac{\pi}{4}} \cos^4 x\,dx = \frac{1}{4} + \frac{3\pi}{32}$ **(6)**

b) Prove by mathematical induction that

$$\sum_{r=1}^{n} r2^{r-1} = 1 + (n-1)2^n$$ **(5)**

c) Solve for x if $\cos 2x = \dfrac{1}{2}$ **(1)**

QUESTION 5

a) Find, to the nearest degree, the acute angle between the curves $y = x^2 - 1$ and $y = x(x-1)$ **(2)**

b) Without calculus, draw the graph of $y = \dfrac{x}{(x-1)^2}$, showing asymptotes and intercepts on axes. **(2)**

c) Joyce and Agnes are playing a game. Joyce has 6 cards numbered 1 to 6 and Agnes has 8 cards numbered 7 to 14. Joyce goes first and draws a card, looks at it and replaces it in her pack. If it is even, she wins. If it is odd, it is Agnes' turn to draw a card, look at it and replace it in her pack. If Agnes draws an odd card, she wins. If Agnes draws an even card, she loses her turn and Joyce draws a card and so on.

Find the probability that Joyce wins:

i) in the first draw

ii) in her second draw

iii) in her third draw

iv) in the long run. **(4)**

d) A spherical balloon is being blown up so that its surface area is increasing at the constant rate of 10cm^2/second. Find the rate at which the volume is increasing when r = 5 cm. **(4)**

QUESTION 6

a) i) Show that the equation of the tangent to the parabola $x^2 = 16y$ at any point $P(8t, 4t^2)$ on it is $y = tx - 4t^2$.

ii) Show that the equation of the line l through the focus S of the parabola which is perpendicular to the focal chord through P is $(t^2 - 1)y + 2tx = 4(t^2 - 1)$.

iii) Find the locus of the point of intersection of the line l and the tangent at P. (7)

b) AB and CD are two towers of equal heights. CD is due north of AB. From a point P on the same horizontal plane as the feet B and D of the towers, and bearing due east of the tower AB, the angles of elevation of A and C, the tops of the towers, are 47° and 31° respectively. If the distance between the towers is 88m, find the height of the towers to the nearest metre. (5)

QUESTION 7

a) At the Tildesley Tennis Competition, Felicity served a ball from a height of 1.8m above the ground. The ball was hit in a horizontal direction with initial velocity V = 35m/s.
Assume that the equations of motion for the ball in flight are

$$y = -5t^2 + 1.8 \text{ and } x = 35t$$

where the acceleration due to gravity is taken at 10m/s^2.

i) How long does it take for the ball to hit the ground?

ii) How far will the ball travel horizontally before bouncing?

iii) The net is 0.95 metres high and is 14 metres away from where Felicity hit the ball. Will the ball clear the net? Explain. (5)

b) Geometry question on the next page.

b) **DO NOT RE-DRAW THIS DIAGRAM.**

DETACH THIS PAGE AND STAPLE IT IN YOUR BOOK FOR QUESTION 7.

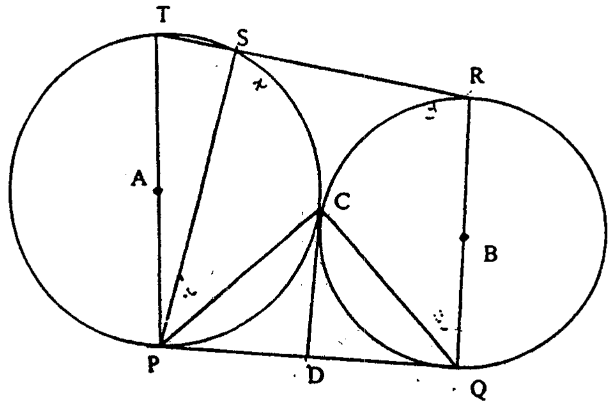
DO THE WRITING IN YOUR ANSWER BOOK NOT ON THIS PAGE.

A circle centre A touches a smaller circle centre B externally at a point C. PQ is a common tangent to the two circles, touching them at P and Q. CD is a common tangent to both circles at C. RT cuts the circle centre A at S.

i) Show that $\angle PCQ = 90°$.

ii) Show that P, C and R are collinear.

iii) Show that P, Q, R and S are concyclic points. (7)



(1)

Stephanie Sim

$\frac{82}{84}$ Excellent work!

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1) A.) i.) $\frac{d}{dx} 4\sec^3 x = 12\sec^2 x \times \frac{-1}{\cos^2 x} \times -\sin x$

$= 12\sec^2 x \tan x \sec x$ ✓

$= 12\tan x \sec^3 x.$

ii.) $\frac{d}{dx} \sin^{-1}\frac{x}{2} = \frac{1}{\sqrt{1-\frac{x^2}{4}}} \times \frac{1}{2} = \frac{2}{\sqrt{4-x^2}} \times \frac{1}{2} = \frac{1}{\sqrt{4-x^2}}$ ✓

B) $\int \frac{1}{9x^2+1}\,dx = \int \frac{1}{9(x^2+\frac{1}{9})}\,dx = \frac{1}{9}\int \frac{1}{x^2+\frac{1}{9}}\,dx$ ✓

$= \frac{1}{9}\tan^{-1}3x + c$ (× 3) $= \frac{1}{3}\tan^{-1}3x + c$

(Must check with standard integrals table)

c) $P = \left(\frac{2(2)-5(-3)}{-3}, \frac{2(-8)-5(4)}{-3}\right)$ ✓

$= \left(\frac{4+15}{-3}, \frac{-16-20}{-3}\right) = \left(\frac{-19}{3}, 12\right)$ ✓

D) $\int_{\frac{1}{2}}^{1} \frac{\sqrt{1-x^2}}{x^2}\,dx$

$x = \cos\theta$

$\frac{dx}{d\theta} = -\sin\theta$ ✓

$dx = -\sin\theta\, d\theta.$

$= \int_{\frac{\pi}{3}}^{0} \frac{\sqrt{1-\cos^2\theta}}{\cos^2\theta} \times -\sin\theta\, d\theta$

$= \int_{\frac{\pi}{3}}^{0} \frac{-\sin^2\theta}{\cos^2\theta}\, d\theta = \int_{\frac{\pi}{3}}^{0} -\tan^2\theta\, d\theta$ ✓

$1+\tan^2\theta = \sec^2\theta$ ✓

$\tan^2\theta = \sec^2\theta - 1$

$= \int_{\frac{\pi}{3}}^{0} -\sec^2\theta + 1\, d\theta$

$= \left(-\tan\theta + \theta\right)_{\frac{\pi}{3}}^{0}$

$= 0 - \left(-\tan\frac{\pi}{3} + \frac{\pi}{3}\right) = \tan\frac{\pi}{3} - \frac{\pi}{3}$

$= \sqrt{3} - \frac{\pi}{3}$ ✓

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E.) Let $P(x) = x^3 - 2x + 5 = 0$

$\alpha+\beta+\gamma = 0 \qquad \alpha\beta+\alpha\gamma+\beta\gamma = -2$ ✓

$\alpha\beta\gamma = -5.$ ✓

$\alpha^2+\beta^2+\gamma^2 = (\alpha+\beta+\gamma)^2 - 2(\alpha\beta+\alpha\gamma+\beta\gamma)$

$= 0 - 2(-2) = \underline{4}$

2) A.) $x^2 = \cos x$

$x^2 - \cos x = 0$

Let $P(x) = x^2 - \cos x = 0$

$P'(x) = 2x + \sin x$ ✓

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$x = 0.8$

$x_1 = x - \dfrac{P(x)}{P'(x)}$ ✓

$x_1 = 0.8 - \dfrac{P(0.8)}{P'(0.8)} = \underline{0.82}$ (2dp) ✓

B) i.) Let $P(x) = x^3 - 4x^2 + 8$

$P(2) = 2^3 - 4(2^2) + 8 = 8 - 16 + 8 = 0$ ✓

$\therefore$ $x = 2$ is a zero of $P(x)$

ii.) If $x=2$ is a zero of $P(x)$, $\therefore$ $(x-2)$ is a factor of $P(x)$

$$
\begin{array}{r}
x^2 - 2x - 4 \\
(x-2)\overline{\big)\, x^3 - 4x^2 + 0x + 8} \\
\underline{x^3 - 2x^2 \qquad\qquad} \\
-2x^2 + 0x \qquad \\
\underline{-2x^2 + 4x \qquad} \\
-4x + 8 \\
\underline{-4x + 8} \\
00
\end{array}
$$
✓

$\therefore$ $P(x) = (x-2)(x^2-2x-4)$

$x = 2$ OR $x^2 - 2x - 4 = 0$

$x = \dfrac{2 \pm \sqrt{4 - 4(-4)}}{2}$

$x = \dfrac{2 \pm \sqrt{20}}{2}$

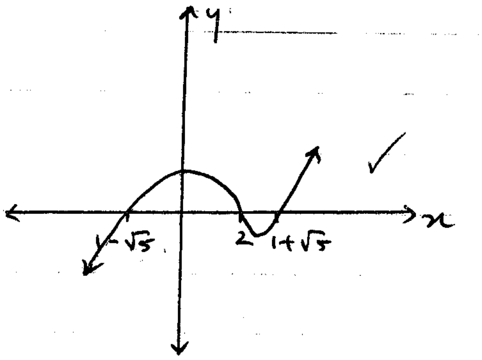
$= \underline{1 \pm \sqrt{5}}$

$\therefore$ the zeroes of $P(x)$ are ✓

$\underline{x = 2, \quad x = 1+\sqrt{5} \text{ and } x = 1-\sqrt{5}}$

iii.) $\cancel{\frac{4}{x-2}\ 1-x1\leq 10}$ $\qquad \frac{4}{x-2} \leq x$ $\qquad$ (multiply throughout by $(x-2)^2$)

$$\frac{4}{\cancel{x-2}} \times (x-2)^2 \leq x\,(x-2)^2$$



$$4(x-2) \leq x\,(x^2-4x+4)$$

$$\cancel{4x} - 8 - x^3 + 4x^2 - \cancel{4x} \leq 0$$

$$4x^2 - 8 - x^3 \leq 0$$

$$x^3 - 4x^2 + 8 \geqslant 0 \quad ✓$$

This is true for $\underline{1-\sqrt{5} \leq x \leq 2}$

$\underline{\text{or} \quad x \geqslant 1+\sqrt{5}}$. ✓

c) $\quad 3\cos x - 4\sin x = 3 \qquad 0 \leq x \leq 360°$

$$3\cos x - 4\sin x = R\cos(x+\alpha)$$

$$= R\,(\cos x\cos\alpha - \sin x\sin\alpha) \quad ✓$$

$3 = R\cos\alpha$ — (1)

$4 = R\sin\alpha$ — (2)

$\frac{(2)}{(1)} = \tan\alpha = \frac{4}{3}$; $\alpha = \tan^{-1}\frac{4}{3}$ ✓

$$\alpha = 53°8'$$

$R = \sqrt{a^2+b^2} = \sqrt{9+16} = \sqrt{25} = 5$ ✓

$\therefore\ 3\cos x - 4\sin x = 5\cos(x+53°8') = 3.$

$\cos(x+53°8') = \frac{3}{5}$

S	A ✓
T	C ✓

$\therefore\ x+53°8' = \cos^{-1} 0.6 \quad (53°8' \leq x+53°8' \leq 413°8')$

$x+53°8' = 53°8',\ 306°52',\ 413°8'$

$\therefore\ \underline{x = 0,\ 253°44',\ 360°}$ ✓

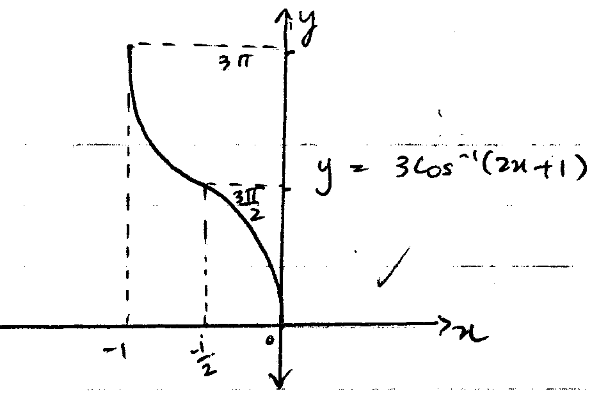
3) A) $y = 3\cos^{-1}(2x+1)$

D = $-1 \le 2x+1 \le 1$

$-2 \le 2x \le 0$

$\underline{-1 \le x \le 0}$ ✓

Range $0 \le y \le 3\pi$



✓

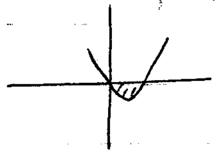
B) $2\sin^2 x < \sin x$

$2\sin^2 x - \sin x < 0$

$\sin x(2\sin x - 1) < 0$

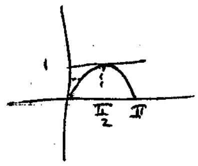
$\sin x = 0$ or $\sin x = \frac{1}{2}$ ✓

$\therefore 0 < \sin x < \frac{1}{2}$.



$\sin x = 0$ when $x = 0, \pi, 2\pi$

$\sin x = \frac{1}{2}$ when $x = \frac{\pi}{6}, \frac{5\pi}{6}$



✓

$\therefore \quad \underline{0 < x < \frac{\pi}{6}}$. ✓ or $\underline{\frac{5\pi}{6} < x < \pi}$

c) $\ddot{x} = -\frac{72}{x^2}$

i.) $\ddot{x} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$; $\frac{1}{2}v^2 = \int -72x^{-2}\,dx$.

$\frac{1}{2}v^2 = \frac{-72x^{-1}}{-1} + c$

$\frac{1}{2}v^2 = \frac{72}{x} + c$ ✓

when $x = 9$, $v = 4$.

$\frac{1}{2} \cdot 16 = \frac{72}{9} + c$; $8 = 8 + c$; $c = 0$

✓

$\therefore \frac{1}{2}v^2 = \frac{72}{x}$.

$v^2 = \frac{144}{x}$; $v = \pm\frac{12}{\sqrt{x}}$. Since $v > 0$ when $x = 9$ and $t = 0$, ✓

$v = \frac{12}{\sqrt{x}}$ m/s

③

ii.) $\frac{dx}{dt} = \frac{12}{\sqrt{x}}$

$\frac{dt}{dx} = \frac{\sqrt{x}}{12}$; $t = \int \frac{\sqrt{x}}{12} dx$

$= \frac{2x^{\frac{3}{2}}}{3} \times \frac{1}{12} + c$

$t = \frac{x^{\frac{3}{2}}}{18} + c$ ✓

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When $t=0$, $x=9$.

$0 = \frac{9^{\frac{3}{2}}}{18} + c$

$0 = \frac{27}{18} + c = \frac{3}{2} + c$; $\underline{c = -\frac{3}{2}}$

$\therefore \underline{t = \frac{x^{\frac{3}{2}}}{18} - \frac{3}{2}}$ ✓

iii.) Find t when $x = 35$. ✓

$t = \frac{35^{\frac{3}{2}}}{18} - \frac{3}{2}$

$t = \frac{\sqrt{42875}}{18} - \frac{3}{2}$ ✓

$=$ <u>10sec (to nearest s)</u>

4) A) i.) R.T.P. $\frac{d}{dx}(\cos^3 x \sin x) = 4\cos^4 x - 3\cos^2 x$.

<u>LHS</u> $\frac{d}{dx}(\cos^3 x \sin x) = \frac{d}{dx}\left[\sin x\,(1-\sin^2 x)\cos x\right]$

$= \frac{d}{dx}\left(\frac{\sin 2x}{2}(1-\sin^2 x)\right)$

$= \frac{1}{2}\frac{d}{dx} \sin 2x(\cos^2 x)$ ✓

$= \frac{1}{2}(\cos^2 x \cdot 2\cos 2x + \sin 2x \cdot 2\cos x \cdot -\sin x) = \frac{1}{2}(2\cos^2 x(2\cos^2 x - 1) -$
$2\sin x\cos x \times 2\sin x\cos x$

$$= \frac{1}{2}\left(4\cos^4 x - 2\cos^2 x - 4\sin^2 x\cos^2 x\right)$$

$$= \frac{1}{2}\left(4\cos^4 x - 2\cos^2 x - 4\cos^2 x\,(1-\cos^2 x)\right)$$

$$= \frac{1}{2}\left(4\cos^4 x - 2\cos^2 x - 4\cos^2 x + 4\cos^4 x\right)$$

$$= \frac{1}{2}\left(8\cos^4 x - 6\cos^2 x\right) = 4\cos^4 x - 3\cos^2 x = \underline{RHS}$$

ii.) $\displaystyle\int_0^{\frac{\pi}{4}} \cos^4 x\, dx = \frac{1}{4} + \frac{3\pi}{32}.$

show that:

we know that $\displaystyle\int 4\cos^4 x - 3\cos^2 x\, dx = \cos^3 x\sin x + c.$

$$\int 4\cos^4 x\, dx = \cos^3 x\sin x + \int 3\cos^2 x\, dx + c.$$

$$\therefore \int \cos^4 x\, dx = \frac{\cos^3 x\sin x}{4} + \frac{1}{4}\int 3\cos^2 x\, dx + c$$

$$\int_0^{\frac{\pi}{4}} \cos^4 x\, dx = \left(\frac{\cos^3 x\sin x}{4}\right)_0^{\frac{\pi}{4}} + \frac{3}{8}\int_0^{\frac{\pi}{4}} 1+\cos 2x\, dx.$$

$$= \frac{\left(\cos\frac{\pi}{4}\right)^3 \sin\frac{\pi}{4}}{4} + \frac{3}{8}\left(x + \frac{1}{2}\sin 2x\right)_0^{\frac{\pi}{4}}$$

$$= \frac{\left(\frac{1}{\sqrt{2}}\right)^4}{4} + \frac{3}{8}\left(\frac{\pi}{4} + \frac{1}{2}\sin\frac{\pi}{2}\right)$$

$$= \frac{1}{16} + \frac{3}{8}\left(\frac{\pi}{4} + \frac{1}{2}\right)$$

$$= \frac{1}{16} + \frac{3\pi}{32} + \frac{3}{16} = \underline{\underline{\frac{1}{4} + \frac{3\pi}{32}}}$$

$$\therefore \int_0^{\frac{\pi}{4}} \cos^4 x\, dx = \frac{1}{4} + \frac{3\pi}{32}$$

R.T.P. $1\times 2^0 + 2\times 2^1 + 3\times 2^2 + 4\times 2^3 + \dots\ n\times 2^{n-1} = 1+(n-1)2^n$

B) Step 1

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let $n=1$

LHS $n\times 2^{n-1} = 1\times 2^0 = 1$

RHS $1+(n-1)2^n = 1+0 = 1 = \text{LHS}$. $\therefore$ true for $n=1$.

Step 2

Assume true for $n=k$.

$1\times 2^0 + 2\times 2^1 + 3\times 2^2 + \dots\ k\times 2^{k-1} = 1+(k-1)2^k$

R.T.P. also true for $n=k+1$

$1\times 2^0 + 2\times 2^1 + \dots\ k\times 2^{k-1} + (k+1)2^k = 1+k\cdot 2^{k+1}$

LHS $1\times 2^0 + 2\times 2^1 + \dots\ k\times 2^{k-1} + (k+1)2^k = 1+(k-1)2^k + (k+1)2^k$ (from assumption)

$= 1+2^k(k-1+k+1)$

$= 1+2^k(2k)$

$= 1+2^k\times 2\times k = 1+2^{k+1}\cdot k = \underline{\text{RHS}}$

Step 3

If true for $n=k$ and $n=k+1$ and also true for $n=1$, then it is true for $n=1+1=$ $n=2+1=3$ and so on. $\therefore$ by POMI, it is true for all integers n.

c) $\cos 2x = \frac{1}{2}$

$2x = \cos^{-1}\left(\frac{1}{2}\right)$

$2x = 2n\pi \pm \left(\frac{\pi}{3}\right)$

$x = n\pi \pm \frac{\pi}{6}$.

5) A.) $y = x^2 - 1$

$\frac{dy}{dx} = 2x$ — m_1

$y = x^2 - x$

$\frac{dy}{dx} = 2x - 1$ — m_2

$\tan\theta = \left|\frac{2x - (2x-1)}{1 + 2x(2x-1)}\right|$

$= \left|\frac{2x - 2x + 1}{1 + 4x^2 - 2x}\right|$

$\tan\theta = \left|\frac{1}{4x^2 - 2x + 1}\right|$

To find where the two curves intersect, solve $y = x^2 - 1$ and $y = x^2 - x$ simult.

$x^2 - 1 = x^2 - x$.

$\cancel{x^2} - 1 - \cancel{x^2} + x = 0$

$x - 1 = 0$

$x = 1$

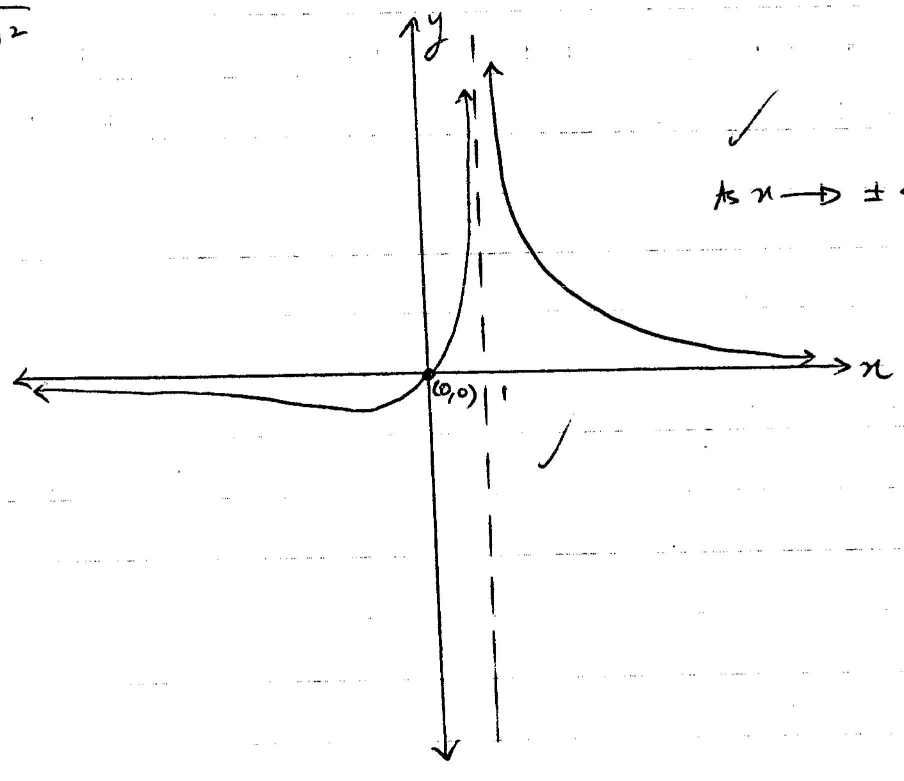
$\therefore$ they intersect at $x = 1$

$\tan\theta = \left|\frac{1}{4 - 2 + 1}\right| = \left|\frac{1}{3}\right|$ (θ is acute. $\therefore \tan\theta > 0$)

$\tan\theta = \frac{1}{3}$

$\theta = 18°26'$

B) $y = \frac{x}{(x-1)^2}$



As $x \to \pm\infty$, $y \to 0$

(5)

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c) i.) P(Joyce wins first draw) = P(even)

$= \frac{3}{6} = \frac{1}{2}$ ✓

ii.) P(J wins 2nd draw) = P(J loses 1st) × P(A loses 1st) × P(J wins 2nd)

$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$ ✓

iii.) P(J wins 3rd draw) = $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$ ✓

$= \frac{1}{32}$

iv.) P(J wins) = P(A wins)

∴ P(J wins in long run) = $\frac{1}{2} + \frac{1}{8} + \frac{1}{32} + \ldots$ ⇒ $S_\infty = \frac{a}{1-r}$

$= \frac{\frac{1}{2}}{1 - \frac{1}{4}} = \frac{2}{3}$

D) $\frac{dA}{dt} = 10cm^2/s$. Find $\frac{dV}{dt}$ when $r = 5cm$.

$A = 4\pi r^2$

$\frac{dA}{dr} = 8\pi r$. $\frac{dr}{dt} = \frac{dr}{dA} \times \frac{dA}{dt} = \frac{1}{8\pi r} \times 10$ ✓

$= \frac{5}{4\pi r}$ ∴ $\frac{dr}{dt} = \frac{5}{4\pi r}$.

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dr} = 4\pi r^2$ ✓

$\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt} = \cancel{4\pi r^2} \times \frac{5}{\cancel{4\pi r}} = 5r$ ✓

$\frac{dV}{dt} = 5r$ $(r = 5)$

∴ $\frac{dV}{dt} = 25cm^3/s$ ✓

6) A.) i.) $x^2 = 16y$; $y = \frac{x^2}{16}$; $y' = \frac{x}{8}$.

At $P(8t, 4t^2)$, grad. of tgt $= \frac{8t}{8} = t$. ✓

Eqn of tgt is = $(y - 4t^2) = t(x - 8t)$

$y = tx - 8t^2 + 4t^2$ ✓

$\underline{y = tx - 4t^2}$

ii.) Focal length; $4A = 16$

$\underline{A = 4}$ ∴ since vertex of parabola is $(0,0)$,

Focus $= (\underline{0, 4})$

∴ $S = (0, 4)$

grad of SP $= \frac{4t^2 - 4}{8t} = \underline{\frac{t^2 - 1}{2t}}$ ✓

Since SP $\perp$ l, grad. of $l = -\frac{2t}{t^2 - 1}$ $(m_1 m_2 = -1)$

Eqt of l is =

$(y - 4) = \frac{-2t}{t^2 - 1}(x - 0)$ ✓

$(t^2 - 1)(y - 4) = -2tx$

$y(t^2 - 1) - 4(t^2 - 1) + 2tx = 0$ ✓

∴ $\underline{y(t^2 - 1) + 2tx = 4(t^2 - 1)}$

iii.) Point of intersection of line l and tgt at P is given by

$tx - 4t^2 = \frac{4(t^2 - 1) - 2tx}{t^2 - 1}$

$(t^2 - 1)(tx - 4t^2) = 4(t^2 - 1) - 2tx$.

$(t^2 - 1)(tx - 4t^2 - 4) = -2tx$.

$\underline{t^3x} - 4t^4 - \underline{tx} + 4 + \underline{2tx} = 0$

$x(t^3 - t + 2t) - 4t^4 + 4 = 0$.

$x = \frac{4t^4 - 4}{t^3 - t + 2t}$. ; $y = t\left(\frac{4t^4 - 4}{t^3 - t + 2t}\right) - 4t^2$

(6)

iii.) Eqt of tgt at P is $y = tx - 4t^2$

$y + 4t^2 - tx = 0$

$4t^2 - tx + y = 0$

$$t = \frac{x \pm \sqrt{x^2 - 4(4)(y)}}{8} = \frac{x \pm \sqrt{x^2 - 16y}}{8}.$$

Since P lies on parabola, $x^2 = 16y$, $x^2 - 16y = 0$.

$$t = \frac{x \pm \sqrt{16y - 16y}}{8}$$

$t = \dfrac{x}{8}$. (sub into l) ✓

Eqt of l is = $(t^2-1)y + 2tx = 4(t^2-1)$

$(t^2-1)(y-4) + 2tx = 0$.

$\left(\dfrac{x^2}{64} - 1\right)(y-4) + 2x\left(\dfrac{x}{8}\right) = 0$. ✓

$$\left(\frac{x^2-64}{64}\right)(y-4) + \frac{x^2}{4} = 0$$

$$y - 4 = \frac{-\frac{x^2}{4}}{\frac{x^2-64}{64}} \quad ; \quad y - 4 = \frac{-x^2}{\cancel{4}} \times \frac{\cancel{64}\,16}{x^2-64}$$

$$y - 4 = -\frac{16x^2}{x^2-64}$$

$$y = 4 - \frac{16x^2}{x^2-64}. \checkmark$$

$$y = \frac{4x^2 - 256 - 16x^2}{x^2-64}$$

$$y = \frac{-12x^2 - 256}{x^2-64} \checkmark$$

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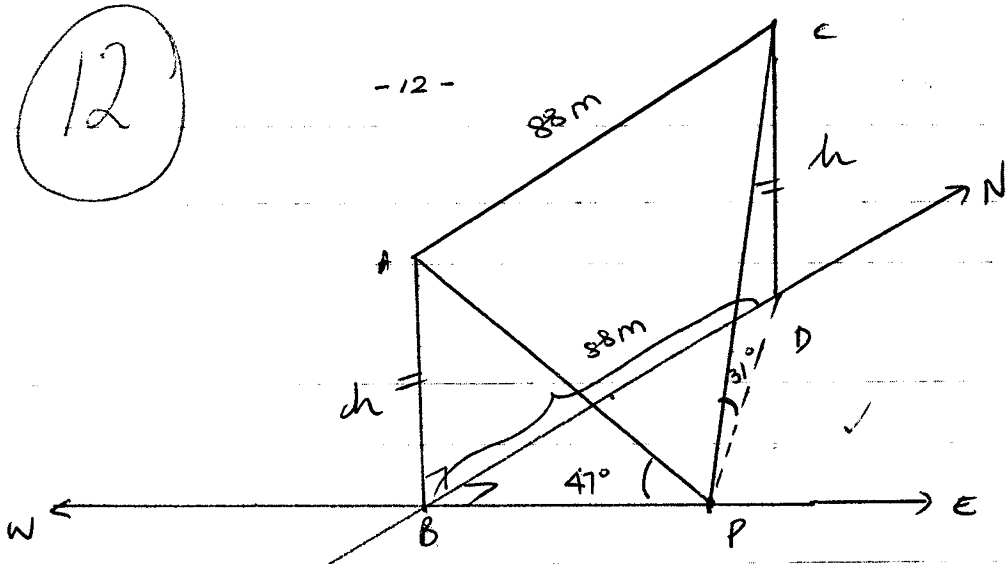
B)



Let $AB = CD = h$.

In ΔABP, $\tan 43 = \dfrac{BP}{h}$; $BP = h\tan 43$

In ΔCPD, $\tan 59 = \dfrac{PD}{h}$; $PD = h\tan 59$

In ΔBDP, $BD^2 + BP^2 = DP^2$ (pythag. theorem).

$$88^2 + h^2\tan^2 43 = h^2 \tan^2 59.$$

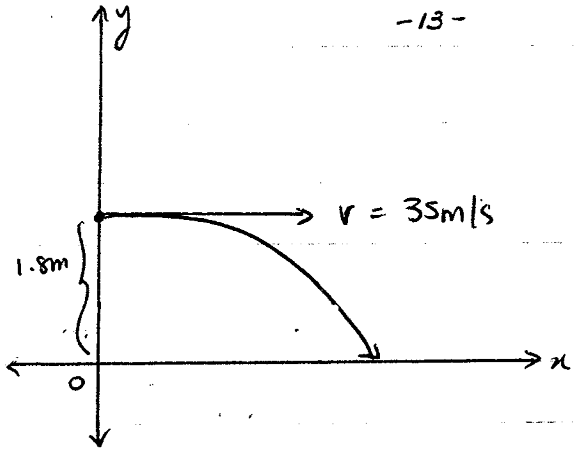
$$88^2 = h^2(\tan^2 59 - \tan^2 43)$$

$$h^2 = \frac{88^2}{\tan^2 59 - \tan^2 43}$$

$h = 63.8$ m ($h > 0$ because it is a positive length)

$\therefore$ $h = 64$m (to nearest m)

7) A)



Given: $y = -5t^2 + 1.8$ $x = 35t$.

$\dot{y} = -10t$ $\dot{x} = 35$.

i.) Find t when $y = 0$.

$1.8 - 5t^2 = 0$ ✓

$5t^2 = 1.8$

$t^2 = 0.36$

$t = 0.6 \quad (t > 0)$ ✓

$\therefore$ it takes 0.6s for ball to hit ground.

ii.) when $t = 0.6$, $x = 35(0.6)$

$= 21\text{m}$ ✓

$\therefore$ the ball travels 21m before bouncing.

iii.) When $x = 14$, $35t = 14$

$t = \frac{14}{35} = \frac{2}{5}\text{ s}$ ✓

when $t = \frac{2}{7}\text{s}$, $y = -5\left(\frac{4}{25}\right) + 1.8$

$= 1\text{ m} > 0.95\text{m}$. ✓

$\therefore$ the ball WILL clear the net, by 0.05m

12

B) i.) $DC = DP = DQ$ (tgts from exterior pt are equal)

$\therefore$ $\triangle PDC$ and $\triangle CDQ$ are isos $\triangle$.

Let $\angle CPD = \alpha$. $\angle CPD = \angle PCD = \alpha$ (base $\angle$ of isos $\triangle$ equal)

$\angle CDQ = \cancel{180}\ 2\alpha$ (exterior $\angle$ equals sum of two interior opp $\angle$

$\angle DCQ = \dfrac{180° - 2\alpha}{2}$ ($\angle$ sum $\triangle = 180°$, base $\angle$ of isos $\triangle$ equal)

$= 90 - \alpha$

$\angle DCQ + \angle PCD = 90 - \alpha + \alpha = 90°$

$\therefore \angle PCQ = 90°$

ii.) $\angle PCQ = 90°$ (already proven)

$\angle QCR = 90°$ ($\angle$ in semicircle with radius RQ $= 90°$)

$\angle PCR = \angle PCQ + \angle QCR$

$= 90° + 90° = 180°$

$\therefore$ PCR is a straight line.

$\therefore$ P, C, and R are collinear

iii.) $\angle CPD = \alpha$.

$\therefore$ ~~$\angle APD$~~ $\angle APC = 90 - \alpha$ ($\angle APD = 90°$, $\angle$ made by radius and tgt $= 90°$)

$\angle TCP = 90°$ ($\angle$ in semicircle $= 90°$)

$\therefore \angle PTC = 180° - (90 - \alpha) - 90°$ ($\angle$ sum $\triangle = 180°$)

$= \alpha$

$\angle PTT = \angle PSC = \alpha$

Since $\angle \cancel{A}PC = 90 - \alpha$, $\angle TSC = 180 - (90 - \alpha)$ (opp $\angle$ of cyclic quad PTSC are supp)

$= 90 + \alpha$

$\therefore \angle CSR = 180° - (90 + \alpha)$ (straight line $= 180°$)

$= 90 - \alpha$

$\angle PSR = \angle PSC + \angle CSR$

$= \alpha + 90 - \alpha = 90°$

$\therefore \angle PSR = 90°$

$\angle PSR + \angle PQR = 90° + 90° = 180°$

$\therefore$ PQRS is a cyclic quad (opp $\angle$ of cyclic quad are supp)

$\therefore$ P, Q, R and S are concyclic points.