1 Simplify:

(a) $\left(x^{\frac{1}{2}}+x^{-\frac{1}{2}}\right)^2$

(b) $2\log_a 2+\log_a 15-\log_a 12$

(c) $3x^{\frac{1}{2}}y^{-2}\div\dfrac{\sqrt{x}}{\sqrt[3]{y^2}}$

2 If $x=12.4$, find the value of:

$$8^{\frac{4x}{3}}\div\log_{10}\left(\frac{5x-3}{x^e}\right)$$

giving your answer correct to two significant figures.

3 Solve for x:

(a) $\sqrt[3]{27^x}=1$

(b) $2^{x-3}=14.9$ *(answer to one decimal place)*

(c) $\log_x 7=\dfrac{\log_2 7}{\log_2 5}$.

4 Consider the function $y=1-e^x$.

(a) Use the first derivative to prove that as x increases, the value of y decreases.

(b) Use the second derivative to show that the curve is always concave down.

(c) State the natural domain and range of the function.

(d) What is the equation of the asymptote to this curve?

5 Find the gradient of the curve $y = \dfrac{e^{-3x}}{x^3 + 4}$ at the point where $x = 0.23$.

6 Show that the curve $y = \log_e(\log_e x)$ has no turning points.

7 Show that $\dfrac{1}{x+3} - \dfrac{1}{x+4} \equiv \dfrac{1}{x^2 + 7x + 12}$

Hence evaluate $\displaystyle\int_{2.5}^{2.7} \frac{dx}{x^2 + 7x + 12}$

8 Evaluate, correct to two decimal places:

(a) $\displaystyle\int_{\frac{1}{2}}^{1} \frac{x^2}{x^3 + 3}\, dx$

(b) $\displaystyle\int_{0}^{1} \frac{e^x - e^{-x}}{e^x + e^{-x}}\, dx$

9 Find the area under the curve $y = e^{2x}$ between $x = 0$ and $x = 2$.

ASSIGNMENT 12

(1) (a) $\left(x^{\frac{1}{2}}\right)^2 + 2\left(x^{\frac{1}{2}}\right)\left(x^{-\frac{1}{2}}\right) + \left(x^{-1/2}\right)^2$

$= x^1 + 2x^0 + x^{-1}$

$= x + 2 + \frac{1}{x}$

(b) $2\log_a 2 + \log_a\left(\frac{15}{12}\right)$

$= \log_a 2^2 + \log_a\left(\frac{5}{4}\right)$

$= \log_a\left(4 \times \frac{5}{4}\right) = \log_a 5$

(c) $3\cancel{x^{\frac{1}{2}}}\,y^{-2} \times \frac{y^{\frac{2}{3}}}{\cancel{x^{1/2}}}$

$= 3y^{-\frac{4}{3}} = \frac{3}{y^{4/3}} = \frac{3}{\sqrt[3]{y^4}}$

(2) $8^{\frac{4}{3}\times 12.4} \div \log_{10}\left(\frac{5\times 12.4 - 3}{1}\right)$ then calculator work

$= 8.5327\ldots \times 10^{14} \div \log_{10}(59)$

$= \underline{\dfrac{4.8\times 10^{14}}{\cancel{4.4\times 10}}}$ (to 2 s.f.)

(3) (a) $\left(\sqrt[3]{27^x}\right)^3 = 1^3$

$27^x = 1$ or $27^x = 27^0$

$(3^3)^x = 3^0$ $\qquad \underline{x = 0}$

$\therefore 3x = 0$

$\underline{x = 0}$

(b) $2^{x-3} = 14.9$

$\therefore \log_{10} 2^{(x-3)} = \log_{10} 14.9$

$\therefore (x-3)\log_{10} 2 = \log_{10} 14.9$

$\therefore x = \frac{\log_{10} 14.9}{\log_{10} 2} + 3 = \underline{6.9}$ (to 1 d.p)

Assignment 12

(3) (c) $\log_x 7 = \frac{\log_2 7}{\log_2 x}$ (by change of base)

$= \frac{\log_2 7}{\log_2 5}$

$\therefore \underline{x = 5}$

(4) (a) $y = 1 - e^x$

$\frac{dy}{dx} = -e^x$ as $x \to \infty$, $-e^x \to -\infty$

$\therefore y \to -\infty$ decreases.

(b) $\frac{d^2y}{dx} = -e^x < 0$ since $e^x > 0$

$\therefore$ concave downwards

(c) D : All real x R : $y < 1$

(d) $y = 1$ is the equation of the horizontal asymptote.

(5) $y = \frac{e^{-3x}}{x^3 + 4}$ $u = e^{-3x}$ $v = x^3 + 4$

$u' = -3e^{-3x}$ $v' = 3x^2$

$\therefore y' = \frac{-3(x^3+4)e^{-3x} - e^{-3x}.3x^2}{(x^3+4)^2}$

$= \frac{3e^{-3x}\left[-x^3 - 4 - x^2\right]}{(x^3+4)^2}$

At $x = 0.23$, $\frac{dy}{dx} = \frac{-6.1168\ldots}{16.097\ldots}$

$= \underline{-0.38}$ (to 2 d.p)

Assignment 12

(6) $y = \ln(\ln x) = \ln u$ Let $u = \ln x$

$u' = \frac{1}{x}$

$y' = \frac{1}{u} \times u'$

$= \frac{1}{\ln x} \times \frac{1}{x}$

$= \frac{1}{x \ln x} \neq 0$ for any x values.

$\therefore$ No stationary points exist. Hence no turning points.

(7) LHS $= \frac{(x+4)-(x+3)}{(x+3)(x+4)} = \frac{x+4-x-3}{x^2+4x+3x+12}$

$= \frac{1}{x^2+7x+12} =$ R.H.S.

$\therefore \int_{\frac{5}{2}}^{2.7} \frac{dx}{(x^2+7x+12)} dx = \int_{2.5}^{2.7} \frac{1}{x+3} - \frac{1}{x+4} \, dx$

$= \Big[\ln(x+3) - \ln(x+4)\Big]_{2.5}^{2.7}$

$= \Big[\ln\left(\frac{x+3}{x+4}\right)\Big]_{2.5}^{2.7}$

$= \ln\left(\frac{2.7+3}{2.7+4}\right) - \ln\left(\frac{2.5+3}{2.5+4}\right)$

$= \underline{5.41 \times 10^{-3}}$ (to 2 d.p.)

(8) (a) $\frac{1}{3}\int_{\frac{1}{2}}^{1} \frac{3x^2}{x^3+3} dx = \frac{1}{3}\Big[\ln(x^3+3)\Big]_{\frac{1}{2}}^{1}$

$= \frac{1}{3}\Big[\ln(1^3+3) - \ln\left(\left(\tfrac{1}{2}\right)^3+3\right)\Big]$

$= \frac{1}{3}\Big[\ln\left(\frac{4}{\frac{25}{8}}\right)\Big]$

$= \frac{1}{3}\Big[\ln\frac{32}{25}\Big] = \underline{0.08}$ (to 2 d.p.)

Assignment 12

(b) $\displaystyle\int_0^1 \frac{e^x - e^{-x}}{e^x + e^{-x}}\,dx$ $\quad$ (Note : $\displaystyle\int \frac{f'(x)}{f(x)}\,dx = \ln f(x) + c$)

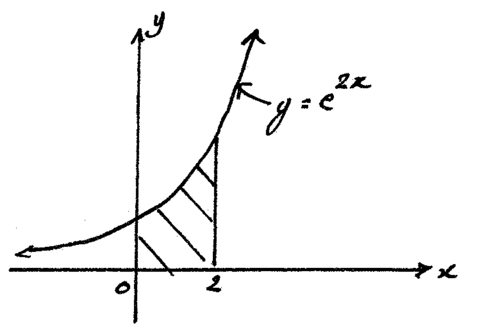
$= \left[\ln(e^x + e^{-x})\right]_0^1$

$= \left[\ln(e^1 + e^{-1}) - \ln(e^0 + e^0)\right]$

$= \ln(3.086\ldots) - \ln 2$

$= \underline{0.43}$ (to 2 d.p.)

(9) Shaded area $= \displaystyle\int_0^2 e^{2x}\,dx$



$= \left[\frac{1}{2}e^{2x}\right]_0^2$

$= \frac{1}{2}\left[e^4 - e^0\right]$

$= \underline{\frac{1}{2}\left[e^4 - 1\right]} \doteqdot \underline{26.8 \text{ units}^2}$ (to 1 d.p.)