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FORTITER ET SUAVITER

Extension I Mathematics

2014 Preliminary Course Task 2

General Instructions

- Working time – 45 minutes
- Write using black or blue pen
- Board-approved calculators may be used
- All necessary working should be shown on every question
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Name: Joanna Gunawan

	Question 1-4	Question 5	Question 6	Total
Deductive Reasoning			/6	/6
Algebra	/3		/9	/12
Calculus	/1	/15		/16
	/4	/15	/15	34

Multiple Choice : Place answers on the answer sheet provided

1. What are the coordinates of the point that divides the interval joining the points $A(-7,5)$ and $B(-1,-7)$ externally in the ratio 1:3?

(A) $(-10,8)$

(B) $(-10,11)$

(C) $(2,8)$

(D) $(2,11)$

2. What is the value of $f'(x)$ if $f(x)=3x^4(4-x)^3$?

(A) $f'(x)=3x^3(4-x)^3(7x-16)$

(B) $f'(x)=3x^3(4-x)^3(-7x+16)$

(C) $f'(x)=3x^3(4-x)^2(7x-16)$

(D) $f'(x)=3x^3(4-x)^2(-7x+16)$

3. The third and seventh terms of a geometric series are 1.25 and 20 respectively. What is the first term?

(A) ± 2

(B) ± 4

(C) $\dfrac{5}{16}$

(D) $\dfrac{5}{1024}$

4. What is the limiting sum of the geometric series

$$x^2 - 2x^3 + 4x^4 - 8x^5 + \ldots\ldots. where\ |x| < 1?$$

(A) $\dfrac{x^2}{1+2x}$

(B) $\dfrac{x^2}{1-2x}$

(C) $\dfrac{1}{1+2x}$

(D) $\dfrac{x^2\left(1-(-2x)^n\right)}{1+2x}$

End of Multiple Choice

Question 5 **(15 Marks) Start a New Page** **Marks**

(a) If $f(x) = 5x^2(3x-1)^4$, find $f'(2)$. **2**

(b) If $y = \dfrac{2x^2}{\sqrt{(x^2-4)}}$, find fully $\dfrac{dy}{dx}$. **2**

(c) Evaluate $\lim\limits_{x \to 2} \dfrac{4-x^2}{x-2}$ **2**

(d) Find the equation of the normal to the curve $y = \dfrac{x-5}{\sqrt{x+1}}$ at $x = 3$. **3**

(e) For what value of b is the line $y = 12x + b$ tangent to $y = x^3$ **3**

(f) A circular metal plate is heated so that its diameter is increasing at a constant rate of 0.005 m/s. At what rate is the area of the circular surface of the plate increasing when its diameter is 6 metres? **3**

(Answer in m^2/s, correct to 2 decimal places.)

Question 6 (15 Marks) **Start a New Page** **Marks**

(a) Use the method of mathematical induction to show **3**

$$1+3+5+......+(2n-1)=n^2 \text{ for all integers } n\geq 1$$

(b) Solve $5^{2x+1}=9$ **2**

(c) Sketch $y=1-\log_5(2x+5)$ clearly indicating any asymptotes and where the curve cuts the x axis. **2**

(d) For what values of x will the infinite geometric series **3**

$$2+\frac{4}{x+5}+\frac{8}{(x+5)^2}+.........$$

have a limiting sum?

(e) Find the least number of terms for which the sum of the series **2**

$30+5+\frac{5}{6}+......$ is greater than 35.99

(f) Use the method of mathematical induction to prove that 4^n+8 is divisible by 6 for $n\geq 1$. **3**

End of Exam

C

Multiple choice

1. Ratio $-1:3$

$x = \frac{3(-7)+(-1)(-1)}{3+(-1)}$

$= \frac{-21+1}{2}$

$= \frac{-20}{2}$

$= -10$

$y = \frac{3(5)+(-1)(-7)}{2}$

$= \frac{15+7}{2}$

$= \frac{22}{2}$

$= 11$

$\therefore$ point $= (-10, 11)$

$\therefore$ Answer $= B$

3

4. limiting sum of geometric series

$S = \frac{a}{1-r}$ $\qquad \frac{-2x^3}{x^2} = -2x$

$= \frac{x^2}{1-(-2x)}$

$= \frac{x^2}{1+2x}$

$\therefore A$

2. ~~[illegible]~~

~~let $v = 3x^4$~~

~~$\frac{dv}{dx} = 12x^3$~~

~~let $u = (4-x)^3$~~

~~$\frac{du}{dx} = 3(-1)^2$~~

~~$=$~~

$f'(x) = 12x^3(4-x)^3 + 3x^4 \cdot 3(4-x)^2 \cdot -1$

$= 3x^3(4-x)^2[4(4-x) + x \cdot (-3)]$

$= 3x^3(4-x)^2(16-7x)$

answer $= D$

3. ~~$\boxed{1.25 \quad \_\_ \quad \_\_ \quad \_\_ \quad 20}$~~

~~$T_1 = 1.25$~~

~~$T_5 = 20$~~

~~$T_n = ar^n$~~

~~$T_5 = 1.25 \times 5$~~

answer $= C$

Question 5

a) $f(x) = 5x^2(3x-1)^4$

let $u = 5x^2$ $\qquad$ let $v = (3x-1)^4$

$u' = 10x$ ✓ $\qquad v' = 4(3x-1)^3 \times 3$

$= 12(3x-1)^3$ ✓

$f'(x) = 60x^2(3x-1)^3 + 10x(3x-1)^4$

$= 10x(3x-1)^3(6x + (3x-1))$

$= 10x(3x-1)^3(6x+3x-1)$

$= 10x(9x-1)(3x-1)^3$ ✓

$f'(2) = 10(2)(9(2)-1)(3(2)-1)^3$

$= 20 \times 17 \times 125$

$= 42{,}500$ ✓

b) $y = \frac{2x^2}{\sqrt{(x^2-4)}}$

let $u = 2x^2$ $\qquad$ let $v = (x^2-4)^{\frac{1}{2}}$

$u' = 4x$ ✓ $\qquad = \frac{1}{2}(x^2-4)^{-\frac{1}{2}} \times 2x$

$= \frac{x}{\sqrt{(x^2-4)}}$ ✓

$y' = \frac{4x(x^2-4)^{\frac{1}{2}} - 2x^2 \times \frac{x}{\sqrt{(x^2-4)}}}{x^2-4}$

$= \frac{4x\sqrt{(x^2-4)} - \frac{2x^3}{\sqrt{(x^2-4)}}}{x^2-4}$ ✓

$= \frac{4x(x^2-4) - 2x^3}{\sqrt{x^2-4}} \times \frac{1}{x^2-4}$

$= \frac{4x^3 - 16x - 2x^3}{\sqrt{(x^2-4)^3}}$ ✓

$= \frac{2x^3 - 16x}{\sqrt{(x^2-4)^3}}$

$= \frac{2x(x^2-8)}{\sqrt{(x^2-4)^3}}$ ✓

c) $\lim_{x \to 2} \frac{4-x^2}{x-2} = \lim_{x \to 2} \frac{(2-x)(2+x)}{(x-2)}$

$= \lim_{x \to 2} \frac{-(x-2)(2+x)}{(x-2)}$

$= \lim_{x \to 2} -(2+x)$

$= \lim_{x \to 2} -2-x$

$= -2-(2)$

$= -4$

(5) (d) $y = \dfrac{x-5}{\sqrt{x+1}}$ at $x=3$, $y = \dfrac{3-5}{\sqrt{4}}$ $= -1$

$$y' = \frac{\sqrt{x+1}\cdot 1 - (x-5)\frac{1}{2}(x+1)^{-\frac{1}{2}}}{x+1}$$

$$= \frac{\sqrt{x+1} - \dfrac{x-5}{2\sqrt{x+1}}}{x+1}$$

At $x=3$

$$m_T = \frac{2 - \dfrac{-\cancel{2}^{1}}{\cancel{2}\cdot 2}}{4}$$

$$= \frac{5}{8}$$

$$m_N = -\frac{8}{5}$$

$\therefore$ Eq^n. of the normal is $y+1 = -\frac{8}{5}(x-3)$

$$5y+3 = -8x+24$$

$$\underline{8x+5y-21=0}$$

(e) $3x^2 = 12$ When $x=2$, $y=8$

$x^2 = 4$ $\therefore$ $8 = 24+b$

$x = \pm 2$ $b = -16$

When $x=-2$, $y=-8$

$-8 = -24+b$

$\underline{b=16}$

(5) (f) Given $\frac{dD}{dt} = 0.005$ m/s where D = diameter

Let A = Area of circular plate $= \pi r^2$

$$= \pi\left(\frac{D}{2}\right)^2$$

$$= \frac{\pi}{4}D^2$$

$$\therefore \frac{dA}{dD} = \frac{\pi D}{2}$$

$$\therefore \frac{dA}{dt} = \frac{dA}{dD} \times \frac{dD}{dt}$$

$$= \frac{\pi}{2}D \times 0.005$$

e) $= \frac{\pi}{2} \times 6 \times 0.005$

$= \underline{0.05 \text{ (to 2 d.p) } m^2/s}$

QUESTION 6.

a) $1+3+5+\ldots+(2n-1) = n^2$ for all integers $n \geq 1$

1. Prove true for $n=1$

LHS : $(2(1)-1)$

$= 2-1$

$= 1$

RHS : $1^2 = 1$

$\therefore$ LHS = RHS

$\therefore$ true for $n=1$

2. assume true for $n=k$

$\therefore$ $1+3+5+\ldots+(2k-1) = k^2$

3. prove true for $n=k+1$

RTP: $1+3+5+\ldots+(2k-1)+2(k+1)-1) = (k+1)^2$

LHS: $1+3+5+\ldots+(2k-1)+(2k+1)$

$= k^2 + 2k+1$

$= (k+1)^2$

$=$ RHS

$\therefore$ Through the principle of mathematic induction since true for $n=1$ and $n = k+1 = 2, 3$ etc. It will be true for all values $n \geq 1$

(6)

b) $5^{2x+1} = 9$

$5^{2x} . 5 = 9$

$5^{2x} = \frac{9}{5}$

$\cancel{\frac{\log 9}{\log 5} = 2x}$

$\therefore \cancel{x = 0.18 \ (2dp)}$

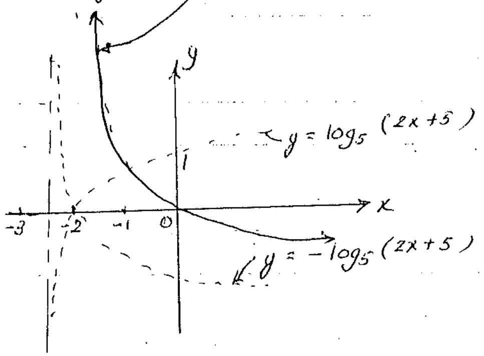
$\log_{10} 5^{2x} = \log_{10}\left(\frac{9}{5}\right)$

$2x = \frac{\log_{10}\left(\frac{9}{5}\right)}{\log_{10} 5}$

$x = 0.18$ (2 d.p).

~~d)~~

(6) (c) $y = 1 - \log_5(2x+5)$



$D: 2x+5 > 0$

$2x > -5$

$x > -\frac{5}{2}$

Also when

$2x+5 = 1$

$2x = -4$

$x = -2$

When $y = 0$

$\log_5(2x+5) = 1$

$2x+5 = 5$

$2x = 0$

$x = 0$

e) r

(d) $r = \frac{\frac{4}{x+5}}{2}$

$= \frac{2}{x+5}$

For limiting sum to exist

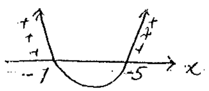
$-1 < r < 1$

$\underbrace{-1 < \frac{2}{x+5} < 1}_{*}$

Also $*$ $-1(x+5)^2 < 2(x+5)$

$2(x+5) + (x+5)^2 > 0$

$(x+5)(x+7) > 0$



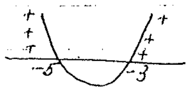
$x < -7$ or $x > -5$

$2(x+5) < (x+5)^2$

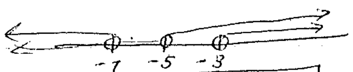
$(x+5)^2 - 2(x+5) > 0$

$(x+5)[(x+5)-2] > 0$

$(x+5)(x+3) > 0$



$x < -5$ or $x > -3$



$\boxed{x < -7 \quad \text{or} \quad x > -3}$

(6) e) $r = 6^{-1}$

$\frac{a(1-r^n)}{1-r} > 35.99$

$\frac{30(1-r^n)}{1-\frac{1}{6}}$

$\frac{30\left(1-\frac{1}{6}^n\right)}{\frac{5}{6}}$

$= \frac{180\left(1-\frac{1}{6}^n\right)}{5} > 35.99$

$= 180\left(1-\frac{1}{6}^n\right) > 179.95 \Rightarrow 1-\left(\frac{1}{6}\right)^n > \frac{179.95}{180}$

$\cancel{180 - 30^n > 179.95}$

$\cancel{n = 1}$

$\cancel{1 - 0.99972 > \left(\frac{1}{6}\right)^n}$

$\frac{\log(\quad)}{\log\left(\frac{1}{6}\right)} < n$

Note: $\log\frac{1}{6} < 0$

$\therefore n > 4.57 \approx 5$

f) $4^n + 8$ divisible by 6.

1. Prove true for $n = 1$

$4(1) + 8$

$= 4 + 8$

$= 12$

$= 6 \times 2$

$\therefore$ divisible by 6

$\therefore$ true for $n = 1$

2. assume true for $n = k$

$4^k + 8 = 6M$

$4^k = 6M - 8$

3. Prove true for $n = k+1$

RTP. ~~$4^{(k+1)}$ +8 is divisible by 6.~~

$4^{(k+1)} + 8$ is divisible by 6

$= 4^k \times 4 + 8$

$= 4(6M - 8) + 8$

$= 24M - 32 + 8$

$= 24M - 24$

$= 6(4M - 4)$

$\therefore$ divisible by 6

$\therefore$ through the principle of mathematical induction

since true for n=1 and true for n=k, n=k+1