Exercise 3.7

1. Use the binomial theorem to expand each of the following.
   (a) $(p + q)^4$
   (b) $(m - n)^7$
   (c) $(2 + k^2)^6$
   (d) $(2a - b)^5$
   (e) $(x + \dfrac{1}{x})^3$
   (f) $(y - \dfrac{1}{2y})^8$

2. In each of the following expansions, find the term as stated.
   (a) $(1 + x)^{10}$, 5th term
   (b) $(2 - 3x)^8$, term in x^2
   (c) $(2a + b)^{12}$, 10th term
   (d) $(p - 3q^2)^7$, term in $p^4 q^6$
   (e) $(x - \dfrac{1}{x})^6$, constant term
   (f) $(x^2 + \dfrac{1}{x})^9$, term in $\dfrac{1}{x^3}$.

3. Expand the expression
   $$(2x + \frac{1}{x^2})^5 + (2x - \frac{1}{x^2})^5,$$
   simplifying the terms.

4. The coefficient of x^3 is four times the coefficient of x^2 in the expansion of $(1 + x)^n$. Find the value of n.

5. In the binomial expansion of $(1 + \dfrac{1}{3}x)^n$, the coefficients of the fourth and fifth terms are equal. Find the value of n.

6. The coefficient of x^5 in the binomial expansion of $(1 + 5x)^8$ is the same as the coefficient of x^4 in the expansion of $(a + 5x)^7$. Find the value of a.

7. If the first three terms in the expansion of $(1 + ax)^n$ in ascending powers of x are $1 - 4x + 7x^2$, find n and a.

8. Find the first four terms of each of the following expansions, in ascending powers of x.
   (a) $(1 + x)^7$
   (b) $(1 + x - x^2)^7$

9. Expand $(1 + 2x + 3x^2)^8$ in ascending powers of x up to and including the term in x^3.

10. Find the first three terms, in ascending powers of x, of the expansion $(1 - 3x)(1 + 2x)^6$.

11. Find the coefficient of the terms in x as indicated, in the following expansions.
    (a) $(1 + x^2)(2 - 3x)^7$, term in x^3
    (b) $(1 - 3x - 2x^2)(1 + x^2)^{20}$, term in x^{20}
    (c) $x(x - \dfrac{2}{x^2})^{12}$, term in x^4
    (d) $(x + \dfrac{1}{x})^2 (1 - x)^5$, term in x^2

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1. (a) $p^4 + 4p^3q + 6p^2q^2 + 4pq^3 + q^4$
   (b) $m^7 - 7m^6n + 21m^5n^2 - 35m^4n^3 + 35m^3n^4 - 21m^2n^5 + 7mn^6 - n^7$
   (c) $64 + 192k^2 + 240k^4 + 160k^6 + 60k^8 + 12k^{10} + k^{12}$
   (d) $32a^5 - 80a^4b + 80a^3b^2 - 40a^2b^3 + 10ab^4 - b^5$
   (e) $x^3 + 3x + \dfrac{3}{x} + \dfrac{1}{x^3}$
   (f) $y^8 - 4y^6 + 7y^4 - 7y^2 + \dfrac{35}{8} - \dfrac{7}{4y^2} + \dfrac{7}{16y^4} - \dfrac{1}{16y^6} + \dfrac{1}{256y^8}$

2. (a) $210x^4$
   (b) $16\,128x^2$
   (c) $1\,760\,a^3b^9$
   (d) $-945p^4q^6$
   (e) -20
   (f) $\dfrac{36}{x^3}$

3. $64x^5 + \dfrac{160}{x} + \dfrac{20}{x^7}$

4. 14

5. 15

6. 2

7. $n = 8$, $a = -\dfrac{1}{2}$

8. (a) $1 + 7x + 21x^2 + 35x^3$
   (b) $1 + 7x + 14x^2 - 7x^3$

9. $1 + 16x + 136x^2 + 784x^3$

10. $1 + 9x + 24x^2$

11. (a) $-16\,464$
    (b) $-151\,164$
    (c) $-1\,760$
    (d) 26