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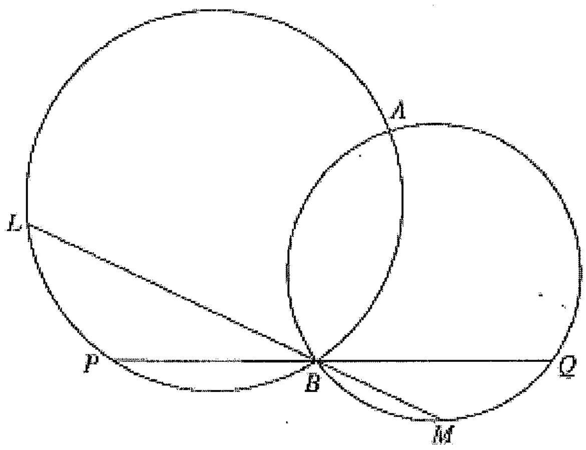
YEAR 12 – EXT.2 MATHS

REVIEW TOPIC (SP1)

CIRCLE GEOMETRY

CEM – Yr 12 – 4U Circle Geometry – Review Paper 1

1. (a)



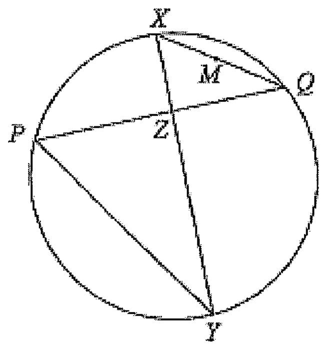
Two circles intersect at A and B.

The lines LM and PQ pass through B, with L and P on one circle and M and Q on the other circle, as shown in the diagram.

Copy or trace this diagram into your writing booklet.

Show that $\angle LAM = \angle PAQ$.

2. Two perpendicular chords PQ and XY of a circle intersect at Z.

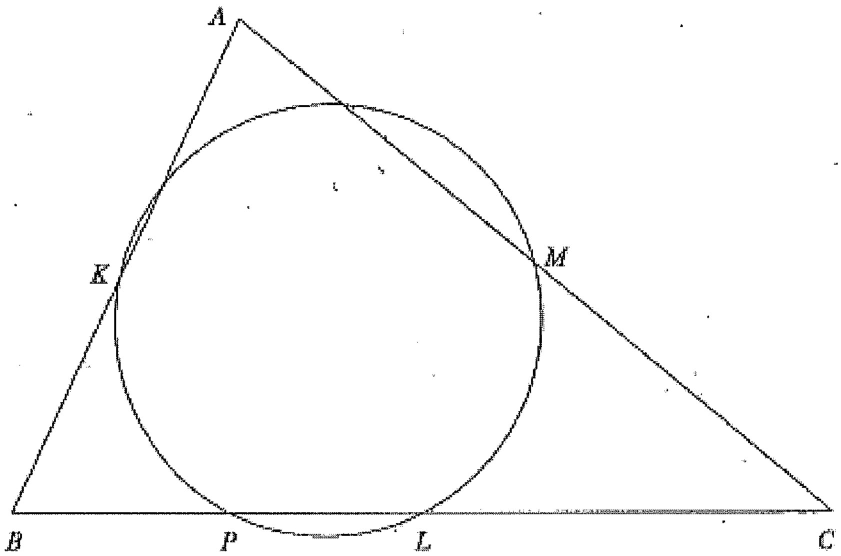


NOT TO SCALE

Copy or trace the diagram into your writing booklet.

If M is the midpoint of the chord QX, prove that MZ produced is perpendicular to the chord PY.

3.



In the acute-angled triangle ABC, K is the midpoint of AB, L is the midpoint of BC and M is the midpoint of CA. The circle through K, L and M also cuts BC at P as shown in the diagram.

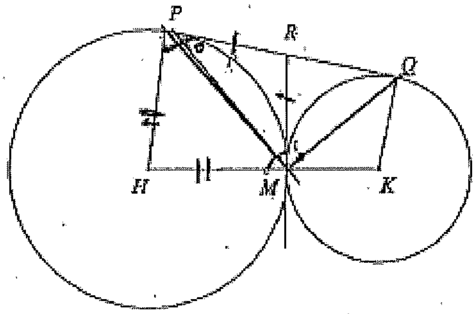
Copy or trace the diagram into your writing booklet.

(i) Prove that $KMLB$ is a parallelogram.

(ii) Prove that $\angle KPB = \angle KML$.

(iii) Prove that $AP \perp BC$.

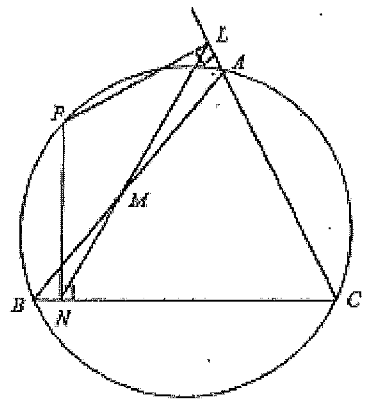
4. Shown are two circles centres H and K which touch at M. PQ and RM are common tangents.



(i) Show that quadrilaterals $HPRM$ and $MRQK$ are cyclic.

(ii) Prove that triangles PRM and MKQ are similar.

5. (a)



ABC is an acute-angled triangle inscribed in a circle, P is a point on the minor arc AB of the circle. PL and PN are the perpendicuars from P to CA (produced) and CB respectively. LN cuts AB at M.

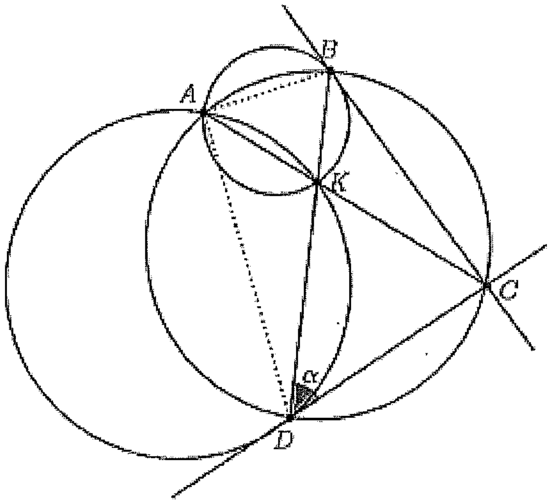
(i) Copy the diagram

(ii) Explain why $PNCL$ is a cyclic quadrilateral. **1**

(iii) Show that $\angle PBM = \angle PNM$. **3**

(iv) Hence show that PM is perpendicular to AB. **3**

6.



In the diagram above, $ABCD$ is a cyclic quadrilateral and diagonals AC and BD intersect at K. Circles AKD and AKB are drawn and it is known that CD is a tangent to circle AKD. Let $\angle BDC = \alpha$.

NOTE: You do not have to copy the diagram above. It has been reproduced for you on a tear-off sheet at the end of this paper. Insert this sheet into your answer booklet.

(i) Prove that $\triangle BCD$ is isosceles.

(ii) Prove that CB is a tangent to circle AKB.

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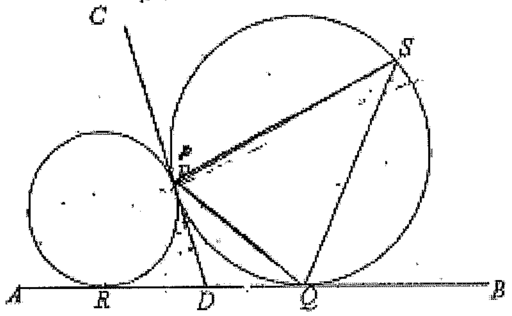
7. AB and CD are tangents to both circles and they intersect at the point D.
The tangent AB touches the smaller circle at R and the larger circle at Q.
The tangent CD touches the two circles at P.

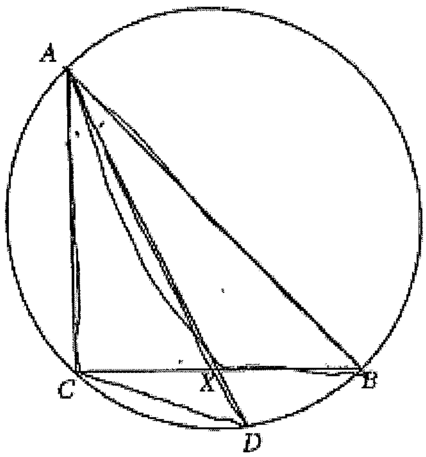


Copy the diagram into your examination booklet.

(i) Prove that $\angle RPQ$ is a right angle.
Hint: assign pronumerals to appropriate angles to help you in your proof.

(ii) Is quadrilateral $RPSQ$ a cyclic quadrilateral? You must support your answer with geometric reasons.

8.



ABC is a triangle inscribed in a circle as shown above. The bisector of $\angle BAC$ meets BC in X and the circle at D.

(i) Prove that $\Delta ABX \, ||| \, \Delta ADC$

(ii) Prove that $AB.AC = AD.AX$

(iii) Prove that $AB.AC = AX^2 + BX.XC$

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Solutions:

1. Let $\angle LAP = \alpha$

$\therefore \angle LBP = \angle LAP$ ($\angle$s in the same segment on

$= \alpha$

$\therefore \angle QBM = \angle LBP$ (Vertically opposite

$= \alpha$

$\therefore \angle QAM = \angle QBM$ ($\angle$s in the same segment on (

$= \alpha$

$\therefore \angle LAP = \angle QAM$

Now $\angle LAM = \angle LAP + \angle PAM$ (from ab

$= \angle QAM + \angle PAM$

$\therefore \angle LAM = \angle PAQ$.

2. **Construction:** Join XQ, PY and MR

To prove: $MR \perp PY$

Proof: Let $\angle YPQ = \alpha$

$\therefore \angle YXQ = \alpha$ (angles on the same arc QY)

$\therefore \angle XQZ = 90 - \alpha$ (given $XY \perp PQ$)

Since $\angle XZQ$ is a right angle, XQ is the diameter and M is the centre of a circle passing through X, Z and Q

$\therefore ZM = MQ$ (equal radii)

$\therefore \angle MZQ = \angle XQZ = 90 - \alpha$ (isosceles triangle)

$\therefore \angle PZR = 90 - \alpha$ (vertically opposite angles)

$\therefore \angle PRZ = 90^\circ$ (angle sum of triangle)

ie $MR \perp PY$ as required

3. i) In triangles ABC and AKM,

$$\frac{AB}{AC} = \frac{2AK}{2AM} = \frac{AK}{AM}$$

and the included angle $\angle A$ is common.

$\therefore \Delta ABC ||| \Delta AKM$ (corresponding sides in proportion and included $\angle$ equal).

$\angle AKM = \angle ABC$ (corresponding $\angle$s in similar Δs)

$\therefore KM \parallel BL$ (corresponding $\angle$s are equal).

The scale ratio $\frac{AK}{AB} = \frac{1}{2}$ (K is the midpoint of AB).

$\therefore KM = \frac{1}{2}BC$.

But $BL = \frac{1}{2}BC$ (L is the midpoint of BC).

$\therefore KM = BL$.

$\therefore KMLB$ is a parallelogram (one pair of opposite sides are equal and parallel).

ii) $KMLP$ is a cyclic quadrilateral.

$\angle KPB = \angle KML$ (exterior $\angle$ of a cyclic quadrilateral is equal to the opposite interior $\angle$).

4. i) $\angle HMR = 90^\circ$ ✓ (tangent to circle and radius of circle touch at 90°)

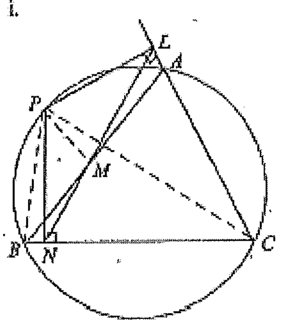
$\angle RMK = 90^\circ$ ✓ (tangent ~~an~~ to circle and radius of circle touch at 90°)

$\angle RQK = 90^\circ$ (" ")

$\angle HPR = 90^\circ$ ✓ (" ")

∴ HPRM and MRQK are cyclic as have pair of opposite angles which are supplementary

5. i.



ii. Opposite angles at N and L are supplementary.
Hence $PNCL$ is a cyclic quadrilateral

iii. Construct PB and PC

$\angle PBM = \angle PBA$ (*B, M, A are collinear*)

$\angle PBA = \angle PCA$ (*$\angle$'s at circumference of circle BPAC standing on same arc PA are equal*)

$\angle PCA = \angle PCL$ (*C, A, L are collinear*)

But $PNCL$ is a cyclic quadrilateral

$\therefore \angle PCL = \angle PNL$ (*$\angle$'s at circumference of circle PNCL standing on same arc PL are equal*)

Also $\angle PNL = \angle PNM$ (*N, M, L are collinear*)

$\therefore \angle PBM = \angle PNM$

iv. Construct PM. Then $PBNM$ is a cyclic quadrilateral (*equal $\angle$'s subtended by PM at B, N on same side of PM*)

$\therefore \angle PMB = \angle PNB$ (*$\angle$'s at circumference of circle PBNM standing on same arc PB are equal*)

$\therefore \angle PMB = 90^\circ$ and $PM \perp AB$.

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6. (i) $\angle DAK = \alpha$ (alt segment th) in circle AKD ✓

$\angle DBC = \alpha$ (angles standing on the same arc DC) ✓

$CD = CB$ (sides opposite equal angles)

(ii) $\angle BAC = \alpha$ (angles standing on the same arc BC) ✓

$\therefore \angle KBC = \angle KAB$

$\therefore$ BC is a tangent to circle ABK by converse to the alt. segment Th ✓

7. let $\angle DRP = \alpha$

$PD = RD$ (tangents from external point D are equal)

$\therefore \Delta RDP$ is isosceles

$\angle DRP = \angle DPR = \alpha$ (equal base angles of isosceles triangle)

let $\angle DQP = \beta$

$PD = DQ$ (tangents from external point D are equal)

$\therefore \Delta DPQ$ is isosceles

$\angle DPQ = \angle DQP = \beta$ (equal base angles of isosceles triangle)

$\angle RPQ = \alpha + \beta$

$2\alpha + 2\beta = 180°$ (angle sum of ΔRPQ)

$\therefore \alpha + \beta = 90°$

8.

(i) Prove $\Delta ABX \parallel\!\mid \Delta ADC$

In ΔABX and ΔADC:

AD is bisector of $\angle BAC$ (data)

$\therefore \angle CAX = \angle BAX$.

$\angle AXC = \angle ABX$ (angles in same arc =)

$\therefore \angle ACX = \angle AXB$ (angle sum of Δ)

ie $\Delta ABX \parallel\!\mid \Delta ADC$ (equiangular)

(b) old

(ii) Prove $AB \cdot AC = AD \cdot AX$

Since $\Delta ABX \parallel\!\mid \Delta ADC$ from (i), then corresponding sides are in proportion.

ie $\frac{AB}{AX} = \frac{AD}{AC}$

$\therefore AB \cdot AC = AD \cdot AX$