NAME :

Centre of Excellence in Mathematics
S201 / 414 GARDENERS RD. ROSEBERY 2018
www.cemtuition.com.au

MOBILE 0412 880 475

PHONE 96663331

YEAR 12 – EXT.2 MATHS

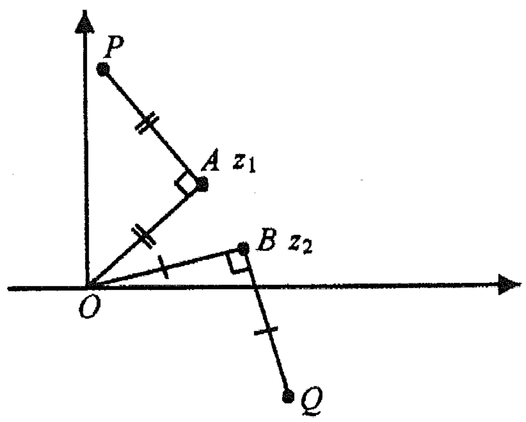
REVIEW TOPIC (SP1)
COMPLEX NUMBERS III

HSC '99

(2) (c) Sketch the region in the Argand diagram where the two inequalities
2

$|z-i| \le 2$ and $0 \le \arg(z+1) \le \frac{\pi}{4}$ both hold.

(2) (e)



The points A and B in the complex plane correspont to complex numbers z_1 and z_2 respectively. Both triangles OAP and OBQ are right-angled isosceles triangles.

(i) Explain why P corresponds to the complex number $(1+i)\,z_1$.

(ii) Let M be the midpoint of PQ. What complex number corresponds to M?

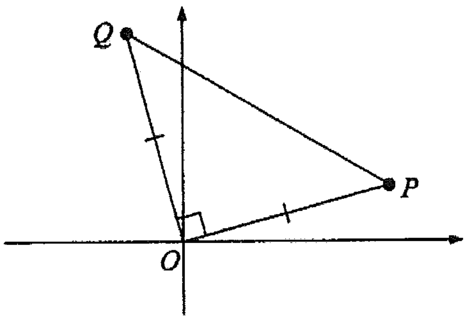
$$\boxed{\frac{z_1(1+i)+z_2(1-i)}{2}}$$

HSC '98

(2) (c) Sketch the region in the complex plane where the inequalities **2**

$|z-2+i|\le 2$ and $\mathrm{Im}(z)\ge 0$ both hold.

(d) **1**



The points P and Q in the complex plane correspond to the complex numbers z and w respectively. The triangle OPQ is isosceles and $\angle POQ$ is a right angle.

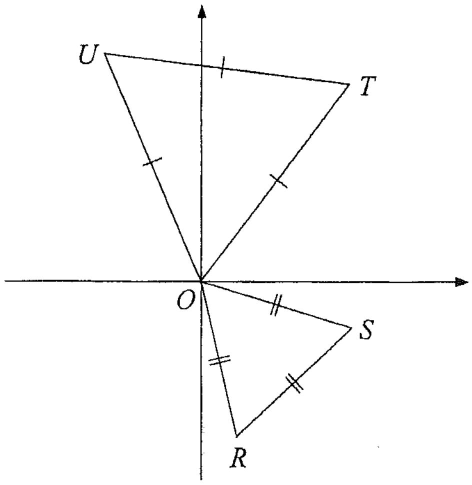
Show that $z^2 + w^2 = 0$.

HSC '97

(2) (c) sketch the region where the inequalities **3**

$|z-3+i| \le 5$ and $|z+1| \le |z-1|$ both hold.

(7) (b) **5**



The diagram shows points O, R, S, T, and U in the complex plane. These points correspond to the complex numbers 0, r, s, t, and u respectively. The triangles ORS and OTU are equilateral. Let $\omega = \cos\frac{\pi}{3} + i\sin\frac{\pi}{3}$.

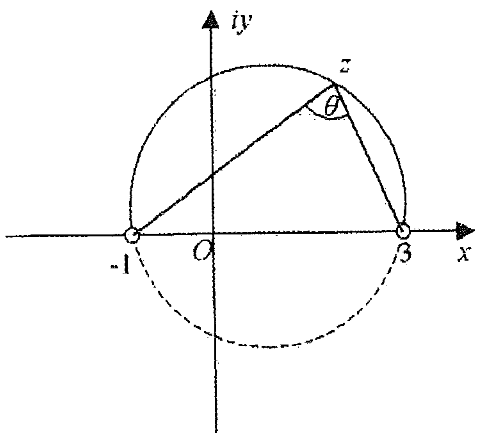
(i) Explain why $u = \omega t$.

(ii) Find the complex number r in terms of s.

(iii) Using complex numbers, show that the lengths of RT and SU are equal.

HSC '96

(2) (d) **4**



The diagram shows the locus of points z in the complex plane such that

$$\arg(z-3)-\arg(z+1)=\frac{\pi}{3}.$$

This locus is part of a circle. The angle between the line -1 to z and From 3 to z is θ, as shown.

(i) Explain why $\theta=\dfrac{\pi}{3}$.

(ii) Find the centre of the circle.

$\left(1,\dfrac{2}{\sqrt{3}}\right)$

HSC '95

(2) (c) Sketch the locus of z satisfying: **5**

(i) $\arg(z-4)=\frac{3\pi}{4}$;

(ii) $\operatorname{Im} z = |z|$.

HSC '93

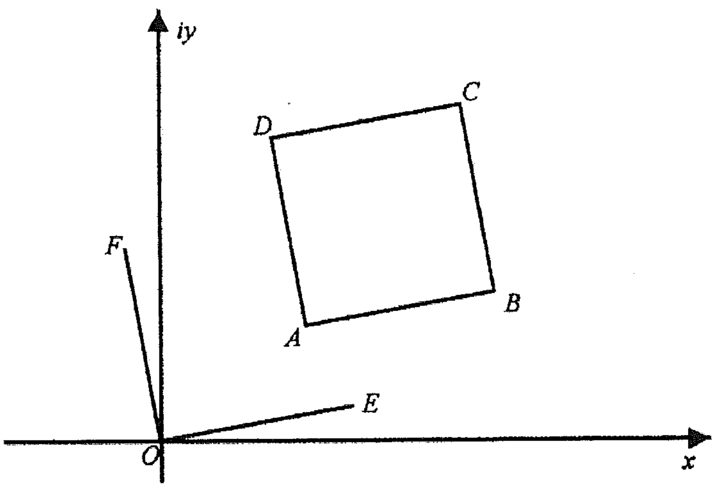
(2) (e) Let P, Q and R represent the complex numbers ω_1, ω_2 and ω_3 respectively. What geometrical properties characterize triangle PQR if $\omega_2-\omega_1=i(\omega_3-\omega_1)$? Give reasons for your answer.

HSC '91

(2) (b) Plot on an Argand diagram the points P, Q and R which correspond to the complex numbers $2i$, $\sqrt{3}-i$, and $-\sqrt{3}-i$, respectively.

Prove that P, Q, and R are the vertices of an equilateral triangle.

(d)



In the Argand diagram, $ABCD$ is a square, and OE and OF are parallel and equal to $AB$and AD respectively. The vertices A and B correspond to the complex numbers w_1 and w_2 respectively.

(a) Explain why the point E corresponds to $w_2 - w_1$.

(b) What complex number corresponds to the point F ?

$$\boxed{i(w_2 - w_1)}$$

(c) What complex number corresponds to the vertex D ?

$$\boxed{(1-i)w_1 + iw_2}$$

HSC 05

(2)(c) Sketch the region on the Argand diagram where the inequalities **3**

$$|z-\bar{z}|<2 \quad \text{and} \quad |z-1|\geq 1$$

hold simultaneously.

HSC '96

(2) (b) On an Argand diagram, shade the region specified by both the conditions **2**

$$\text{Re}(z)\leq 4 \quad \text{and} \quad |z-4+5i|\leq 3.$$

HSC '94

(2) (c) (i) On the same diagram, draw a neat sketch of the locus specified by each of the following:

(α) $\quad |z-(3+2i)|=2$ $\qquad$ (β) $\quad |z+3|=|z-5|$.

(ii) Hence write down all values of z which satisfy simultaneously.

$$|z-(3+2i)|=2 \text{ and } |z+3|=|z-5|.$$

$\boxed{z=1+2i}$

(iii) Use the diagram in (i) to determine the values of k for which the simultaneous equations below have exactly one solution for z

$$|z-(3+2i)|=2 \text{ and } |z-2i|=k$$

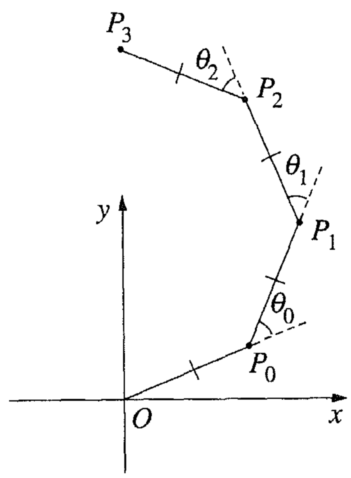
$\boxed{k=1 \text{ or } 5}$

HSC 02

*(7) (b) Suppose $0 < \alpha,\ \beta < \frac{\pi}{2}$ and define complex numbers z_n by

$$z_n = \cos(\alpha + n\beta) + i\sin(\alpha + n\beta)$$

for $n = 0, 1, 2, 3, 4$. The points P_0, P_1, P_2 and P_3 are the points in the Argand diagram that correspond to the complex numbers z_0, $z_0 + z_1$, $z_0 + z_1 + z_2$ and $z_0 + z_1 + z_2 + z_3$ respectively. The angles θ_0, θ_1 and θ_2 are the external angles at P_0, P_1 and P_2 as shown in the diagram below.



(i) Using vector addition, explain why **2**

$$\theta_0 = \theta_1 = \theta_2 = \beta.$$

(ii) Show that $\angle P_0OP_1 = \angle P_0P_2P_1$, and explain why $OP_0P_1P_2$ is a cyclic quadrilateral. **2**

(iii) Show that $P_0P_1P_2P_3$ is a cyclic quadrilateral, and explain why the points O, P_0, P_1, P_2 and P_3 are concyclic. **2**

(iv) Suppose that $z_0 + z_1 + z_2 + z_3 + z_4 = 0$. Show that **2**

$$\beta = \frac{2\pi}{5}.$$