NAME :

Centre of Excellence in Mathematics
S201 / 414 GARDENERS RD. ROSEBERY 2018
www.cemtuition.com.au

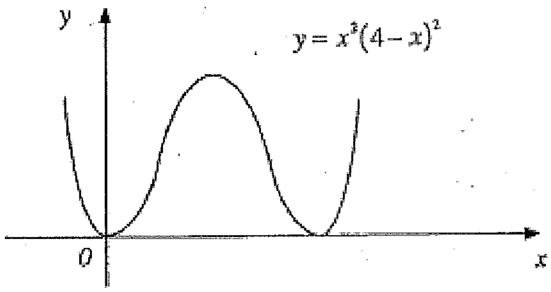
YEAR 12 – MATHS EXT.2

REVIEW TOPIC: GRAPHS

Received on			Check corrections on pages:
Completed on			
Checked by			
	Tutor's Initials	Dated on	

(1) (a) Show that the stationary points of $y = \{f(x)\}^2$ are exactly those points on the curve that have x coordinates which are zeros of either $f(x)$ or $f'(x)$. 2

(b) 4



Use the graph of $y = x(4-x)$ to justify the features shown on the graph above.
Copy the graph of $y = x^2(4-x)^2$ and mark on the coordinate axes the values of x and y at the stationary points.

(2)(a) $P(x)=(x+2)(x-1)(x-3)$

(i) Sketch $y=P(x)$ showing the intercepts on the coordinate axes. 1

(ii) On separate diagrams, sketch the graphs of $y=|P(x)|$, $y=P(|x|)$, $y=\dfrac{1}{P(x)}$

showing the intercepts on the coordinate axes and the equations of any asymptotes. 4

(b) (i) $P(x_1, y_1)$ is a point on the curve $y = e^{-x}$. The tangent to the curve at P passes through the origin. Find the coordinates of P. 3

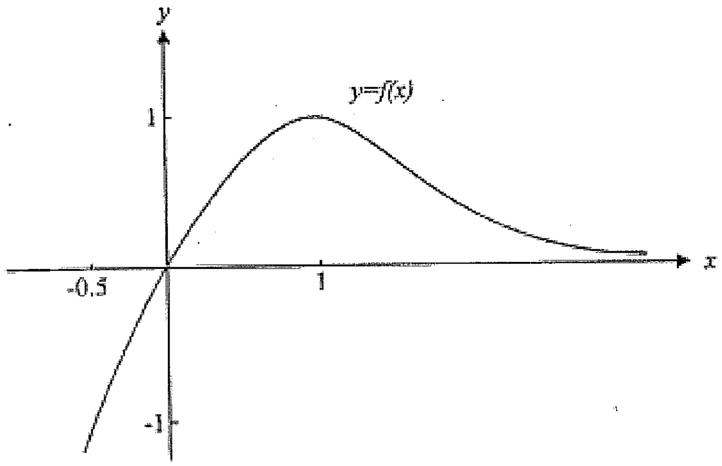
(ii) Find the set of values of the real number k such that the equation $e^{-x} = kx$ has two real and distinct solutions. 2

(c) Consider the function $f(x)=\ln(1+\cos x)$, $-2\pi \le x \le 2\pi$, where $x \ne \pi$, $x \ne -\pi$.

(i) Show that the function f is even and the curve $y=f(x)$ is concave down for all values of x in its domain. 3

(ii) Sketch the graph of the curve $y=f(x)$. 2

(3) (a)



The diagram shows the graph of $y = f(x)$.
Sketch each of the following functions on separate half page sketches.

(i) $y = f(-x)$ **1**

(ii) $y = |f(x)|$ **1**

(iii) $y = \dfrac{1}{f(x)}$ **2**

(iv) $y = e^{f(x)}$ **2**

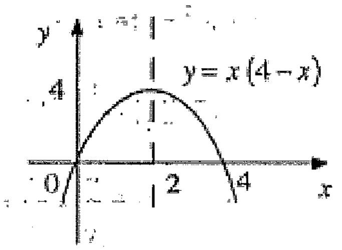
SOLUTIONS

(1) (a) $\frac{d}{dx}\{f(x)\}^2 = 2f(x) \cdot f'(x)$

Stationary points on $y = \{f(x)\}^2$ occur when

$\frac{dy}{dx} = 0$, that is when $f(x) = 0$ or $f'(x) = 0$.

(b)



Parabola, with axis of symmetry
$x = 2$ and vertex $(2, 4)$

$y = \{f(x)\}^2$ where $f(x) = x(4 - x)$

$f(x) = 0 \Rightarrow x = 0, x = 4$

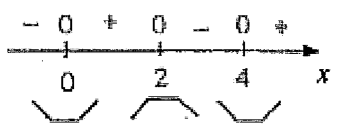
$f'(x) = 0 \Rightarrow x = 2$

Hence $y = x^2(4 - x)^2$ has stationary points at $(0, 0)$, $(4, 0)$, $(2, 16)$, from (a) .

$\frac{d}{dx}\{f(x)\}^2 > 0$ when $f(x)$, $f'(x)$ have same sign

$\frac{d}{dx}\{f(x)\}^2 < 0$ when $f(x)$, $f'(x)$ have opposite sign

sign of $\frac{dy}{dx}$



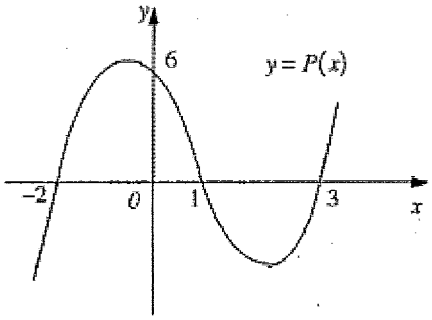
Hence stationary points have nature shown in the diagram.

(2)

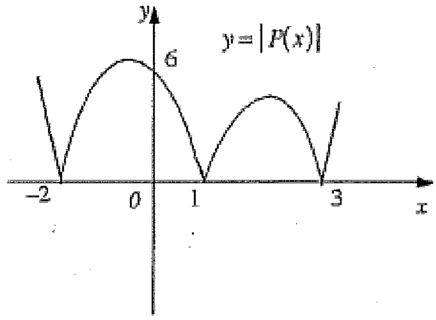
(a)

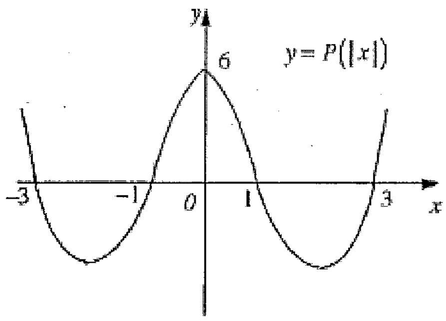
Answer $P(x) = (x+2)(x-1)(x-3)$

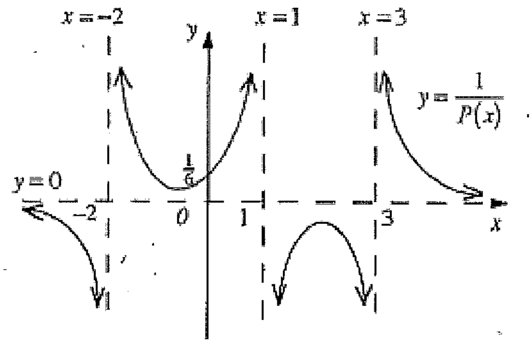
(i)



(ii)

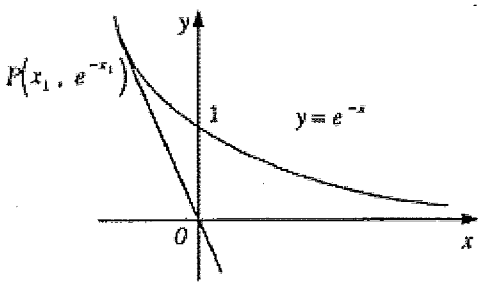






(b)

Answer



(i) $P(x_1\,,\ e^{-x_1})$

$y = e^{-x}$ $\quad$ $grad.\ OP = \dfrac{e^{-x_1}}{x_1}$

$\dfrac{dy}{dx} = -e^{-x}$ $\quad$ $grad.\ tangent\ at\ P = -e^{-x_1}$

Since OP is tangent at P, $\quad \dfrac{e^{-x_1}}{x_1} = -e^{-x_1}$

$\therefore\ (x_1+1)e^{-x_1} = 0$

$\therefore\ x_1 = -1\,,\quad P(-1\,,\ e)$

(ii) $y = -e\,x$ is tangent to the curve $y = e^{-x}$ at $P(-1\,,\ e)$, and intersects the curve at no other point.
By inspection of the graph, for $-e < k \le 0$, $y = kx$ has no points of intersection with the curve.
for $k > 0$, $y = kx$ has exactly one point of intersection with the curve.
Since $y = e^{-x}$ is steeper than any linear function of x as $x \to -\infty$, lines $y = kx$, $k < -e$, will intersect the curve in two distinct points.
Hence $e^{-x} = kx$ has two real and distinct solutions for $\{k:\ k < -e\}$.

(c)

Answer

(i)

$f(x) = \ln(1+\cos x)$

$f(-x) = \ln\{1+\cos(-x)\} = \ln(1+\cos x) = f(x)$

Hence f is an even function.

$$f'(x) = \frac{-\sin x}{1+\cos x}$$

$$f''(x) = -\frac{\cos x(1+\cos x) - \sin x(-\sin x)}{(1+\cos x)^2}$$

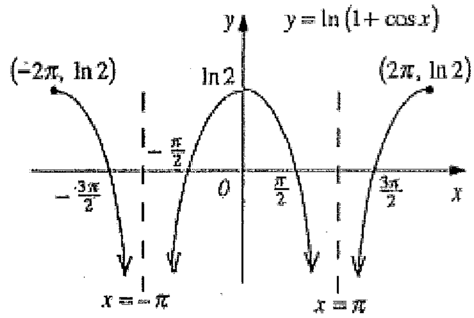
$$= -\frac{\cos x + \cos^2 x + \sin^2 x}{(1+\cos x)^2}$$

$$= -\frac{\cos x + 1}{(1+\cos x)^2}$$

$\therefore f''(x) = \dfrac{-1}{1+\cos x} < 0$ (since $1+\cos x > 0,\ x \neq \pm\pi$)

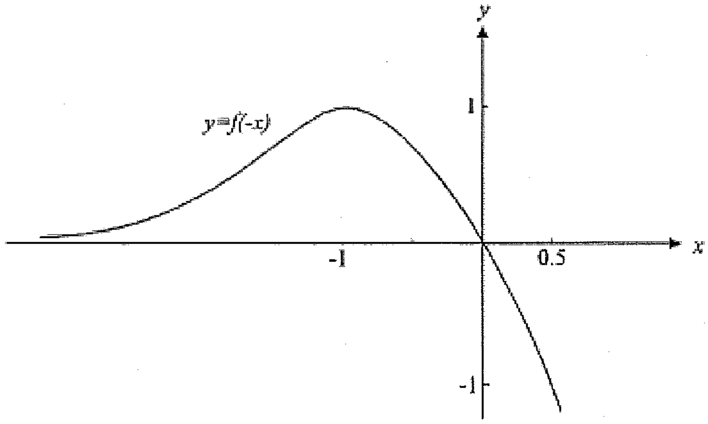
Hence curve is concave down throughout its domain.

(ii)



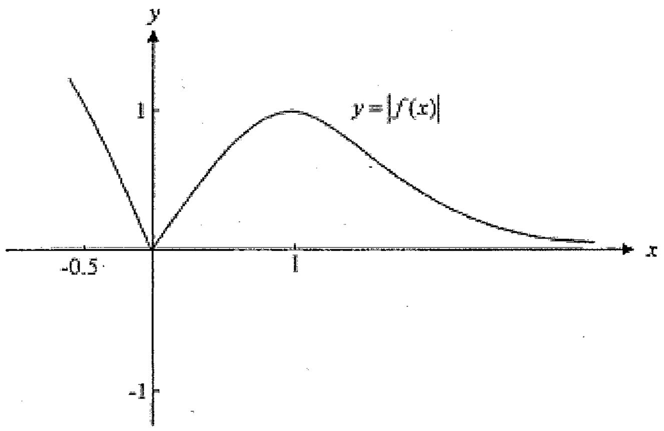
(3)

(a) (i)



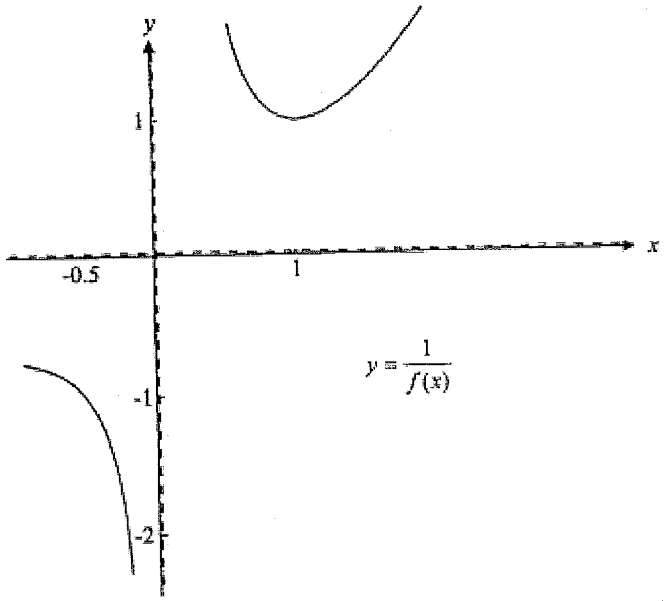
(1 mark)

(ii)



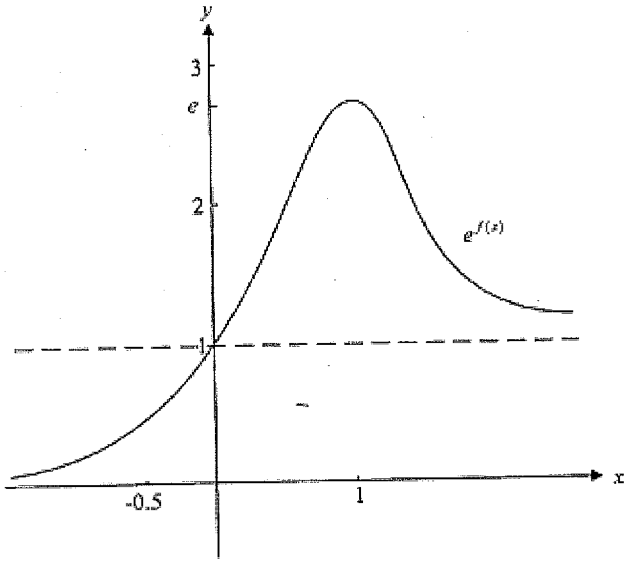
(1 mark)

(iii)



(1 mark) for asymptotes **(1 mark)** for graph including the points (-0.5, -1) and (1,1)

(iv)



(1 mark) for points (0,1) and (1,e) **(1 mark)** for graph

Note that whilst it is probably outside the domain of what can reasonably be shown in the sketch here of $y = e^{f(x)}$, it is interesting to note that as $x \to \infty, f(x) \to 0$ from above, so $e^{f(x)} \to 1$ from above. So the graph of $y = e^{f(x)}$ has a horizontal asymptote at $y = 1$.