NAME :

Centre of Excellence in Mathematics

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YEAR 12 – MATHS EXT.2

REVIEW TOPIC: GRAPHS I & II

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HSC '96 **Marks**

(4) (b) (i) On the same axes, sketch and label clearly the graphs of the functions $y = x^{\frac{1}{3}}$ and $y = e^x$. **6**

(ii) Hence, on a different set of axes, without using calculus, sketch and label clearly the graph of the function $y = x^{\frac{1}{3}}e^x$.

(iii) Use your sketch to determine for which values of m the equation $x^{\frac{1}{3}}e^x = mx + 1$ has exactly one solution.

$m \leq 0$

HSC '95

Marks

(3) (a) Let $f(x) = -x^2 + 6x - 8$ **10**

On separate diagrams, and without using calculus, sketch the following graphs. Indicate clearly any asymptotes and intercepts with the axes.

(i) $y = f(x)$

(ii) $y = |f(x)|$

(iii) $y^2 = f(x)$

(iv) $y = \dfrac{1}{f(x)}$

(v) $y = e^{f(x)}$

HSC '94

(5) (a) Let $f(x) = \dfrac{(x-2)(x+1)}{5-x}$ for $x \neq 5$.

(i) Show that $f(x) = -x - 4 + \dfrac{18}{5-x}$

(ii) Explain why the graph of $y = f(x)$ approaches that of $y = -x - 4$ as $x \to \infty$ and as $x \to -\infty$.

(iii) Find the values of x for which $f(x)$ is positive, and the values for which $f(x)$ is negative.

(iv) Using part (i), show that the graph of $y = f(x)$ has two stationary points. (There is no need to find the y coordinates of the stationary points).

(v) Sketch the graph $y = f(x)$. Label all asymptotes, and show the x-intercepts.

HSC '93

(4) (a) Let $f(x) = \frac{1-x}{x}$. On separate diagrams sketch the graphs of the following functions. For each graph label any asymptote.

(i) $y = f(x)$

(ii) $y = f(|x|)$

(iii) $y = e^{f(x)}$

(iv) $y^2 = f(x)$. Discuss the behaviour of the curve of (iv) at $x = 1$.

HSC '92

(4) (b) Let $f(x) = \ln(1+x) - \ln(1-x)$ where $-1 < x < 1$.

(i) Show that $f'(x) > 0$ for $-1 < x < 1$.

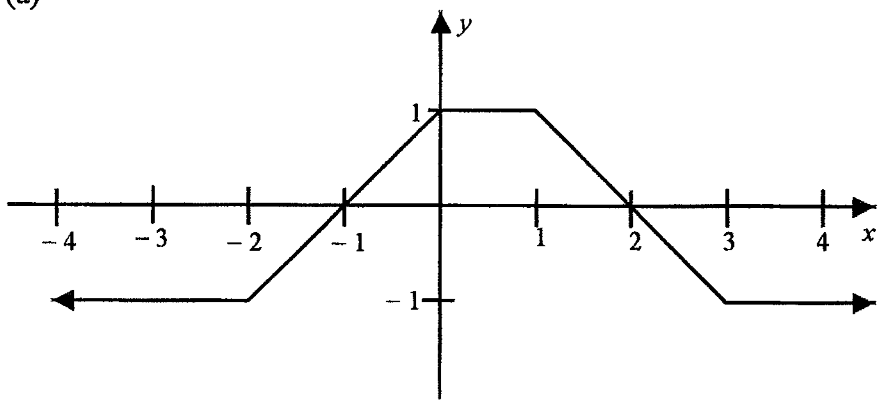
(ii) On the same diagram sketch $y = \ln(1+x)$ for $x > -1$; $y = \ln(1-x)$ for $x < 1$ and $y = f(x)$ for $-1 < x < 1$. Clearly label the three graphs.

(iii) Find an expression for the inverse function $y = f^{-1}(x)$.

$$\boxed{f^{-1}(x) = \frac{e^x - 1}{e^x + 1}}$$

HSC '91

(4) (a)



The diagram is a sketch of the function $y = f(x)$. On separate diagrams sketch :

(i) $y = -f(x)$

(ii) $y = |f(x)|$

(iii) $y = f\left(|x|\right)$

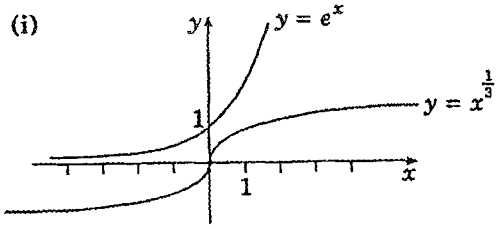
(iv) $y = \sin^{-1}\left(f\left(x\right)\right)$

SOLUTIONS TO HSC QUESTIONS:

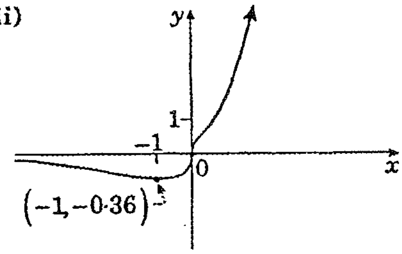
HSC '96

(4)

(b) **(i)**



(ii)

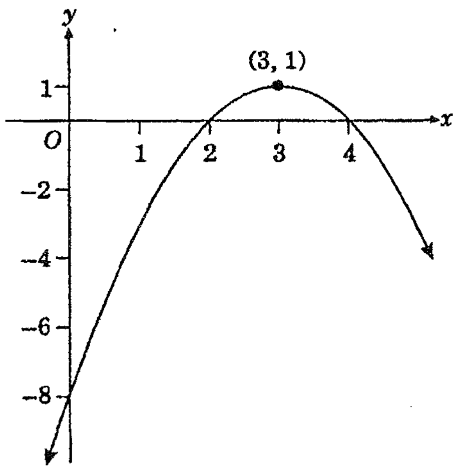


HSC '95

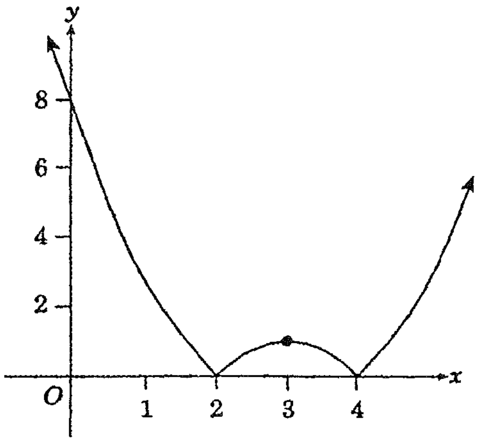
QUESTION THREE

(a) (i) $y = f(x)$

$$(-x^2 + 6x - 8) = -(x^2 - 6x + 8)$$
$$= -(x-2)(x-4)$$



(ii) $y = |f(x)|$



(iii) $y^2 = f(x)$

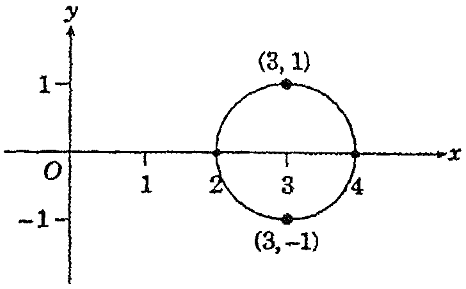
$\therefore \quad y^2 = -x^2 + 6x - 8$

$\therefore \quad x^2 + y^2 - 6x = -8$

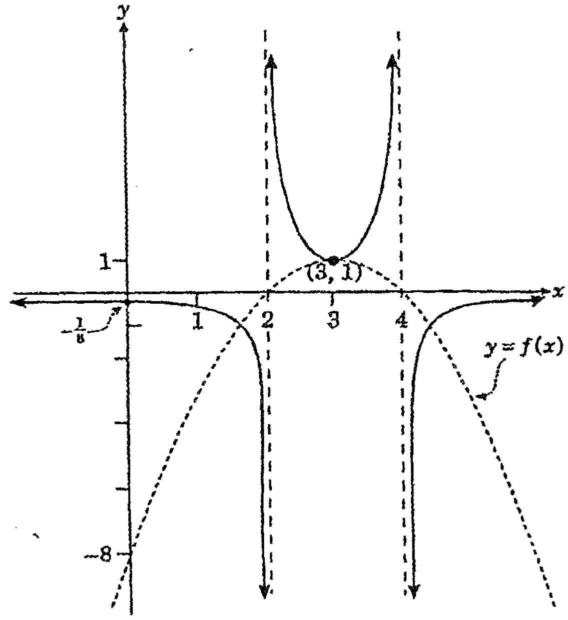
$x^2 - 6x + y^2 = -8 + 9$

$(x-3)^2 + y^2 = 1.$

That is, circle, centre $(3, 0)$ and radius 1 unit.



(iv) $y = \dfrac{1}{f(x)}$

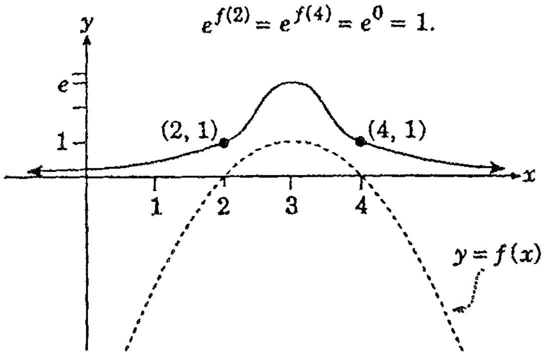


(v) $y = e^{f(x)}$

$f(3) = 1$

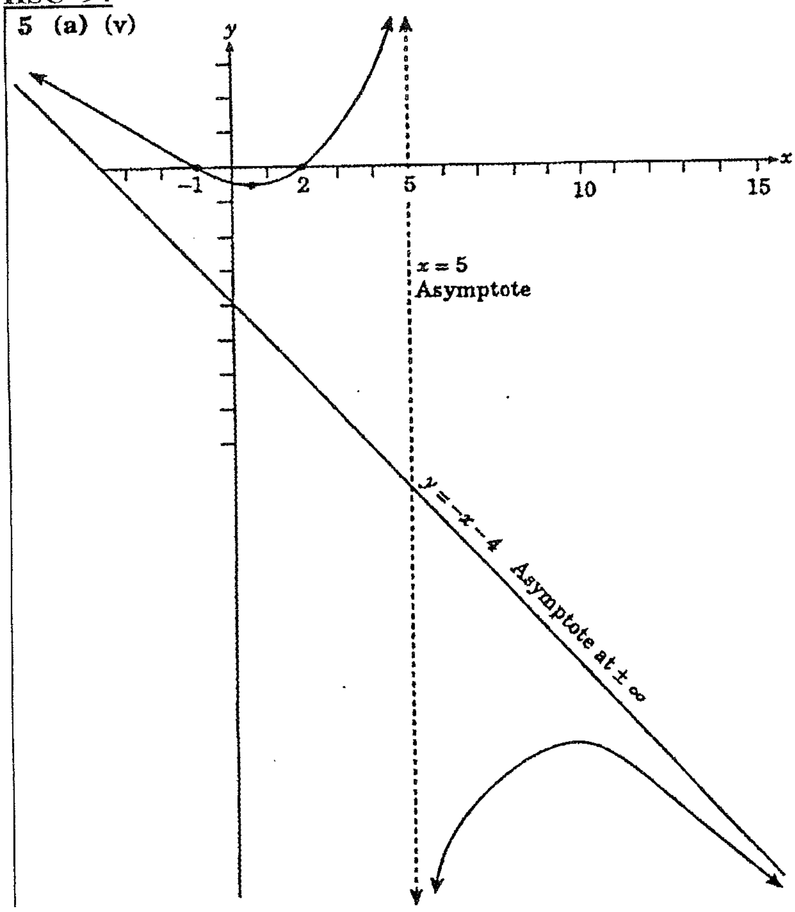
$\therefore \quad e^{f(3)} = e$

$e^{f(2)} = e^{f(4)} = e^0 = 1.$



HSC '94

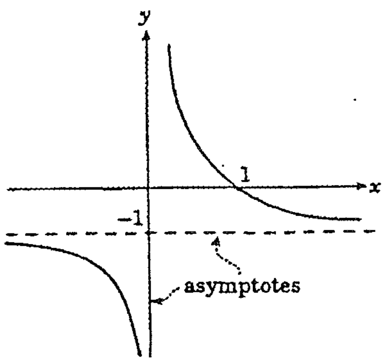
5 (a) (v)



HSC '93

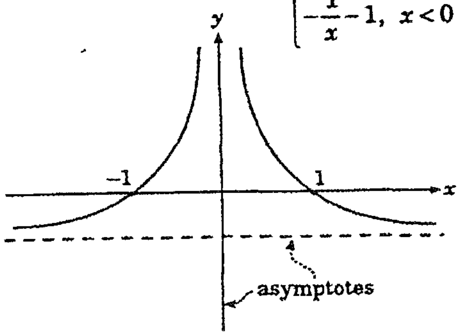
QUESTION FOUR

(a) (i) $f(x)=\dfrac{1-x}{x}=\dfrac{1}{x}-1.$



(ii) $$y=\frac{1-|x|}{|x|}=\begin{cases}\frac{1-x}{x}, & x>0\\ \frac{1+x}{-x}, & x<0\end{cases}$$

$$=\begin{cases}\frac{1}{x}-1, & x>0\\ -\frac{1}{x}-1, & x<0\end{cases}$$



(iii) **From the graph in (i).**

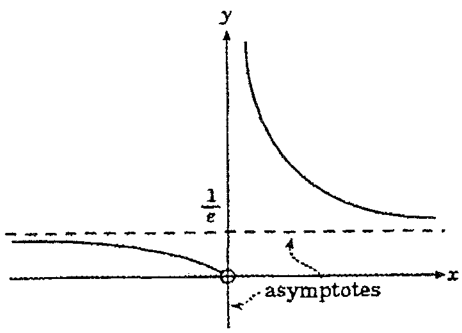
As $x\to0^+$, $y\to e^{\infty}=\infty$

$x\to0^-$, $y\to e^{-\infty}=0$

$x\to\infty$, $y\to e^{-1}=\frac{1}{e}$

$x\to-\infty$, $y\to e^{-1}=\frac{1}{e}$

[OR $y=e^{\frac{1}{x}-1}=\frac{1}{e}e^{\frac{1}{x}}$ from which limits can be determined.]



(iv) $y^2 = f(x)$

$y = \pm\sqrt{f(x)}$. The function is symmetrical about the x axis.
The function is defined only for $f(x) \geq 0$, i.e. $0 < x \leq 1$.

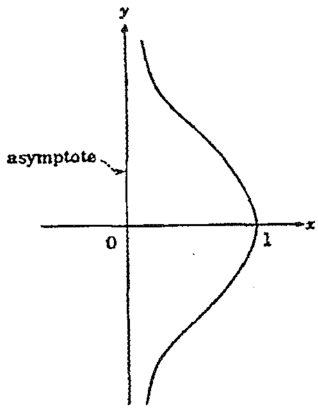
Now $y^2 = \frac{1}{x} - 1$.

Differentiating: $2y\frac{dy}{dx} = -\frac{1}{x^2}$

$$\frac{dy}{dx} = -\frac{1}{2x^2 y}$$

At (1, 0), $\frac{dy}{dx}$ is undefined.

There is a vertical tangent at (1, 0).



HSC '92

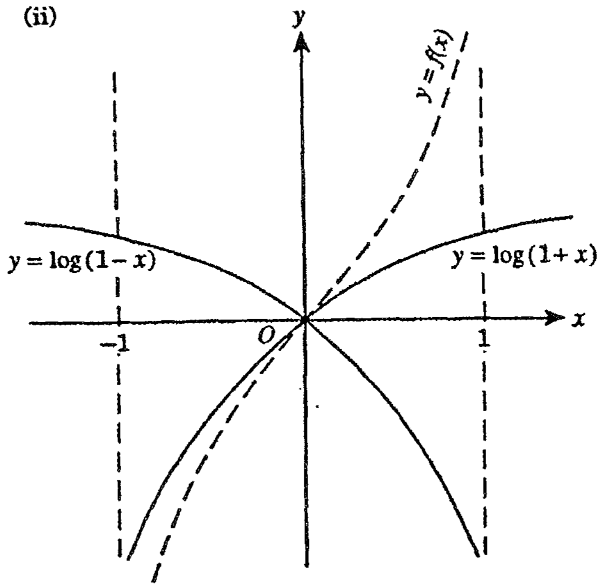
(4)

(b) (i) $f(x) = \log(1+x) - \log(1-x), \quad -1 < x < 1$

$$f'(x) = \frac{1}{1+x} + \frac{1}{1-x}$$
$$= \frac{2}{1-x^2}$$
$$> 0$$

since $1 - x^2 > 0$ for $-1 < x < 1$.

(ii)

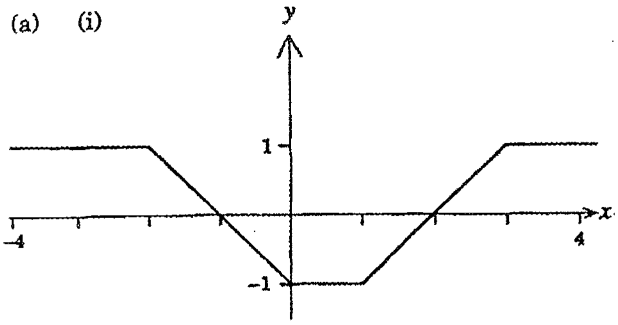


N.B. $f'(x) > 0$ for $-1 < x < 1$
$f'(0) = 2$

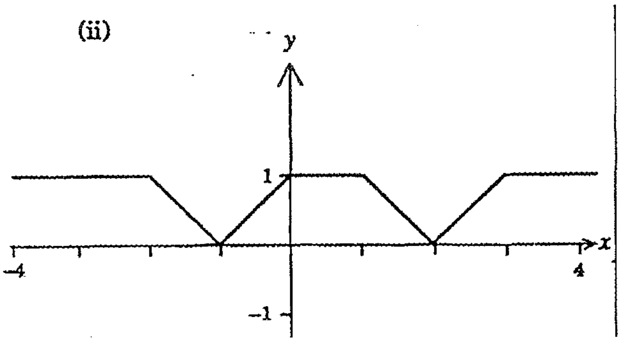
HSC '91

QUESTION FOUR

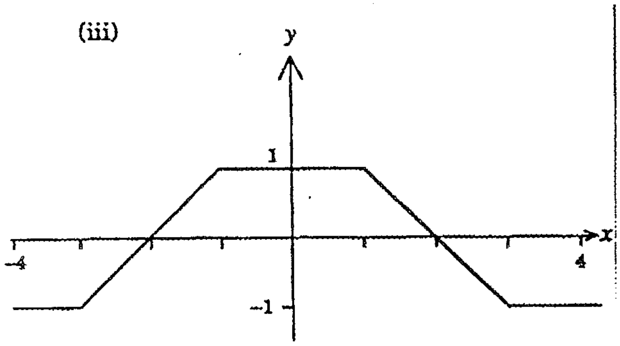
(a) (i)



(ii)



(iii)



(iv)