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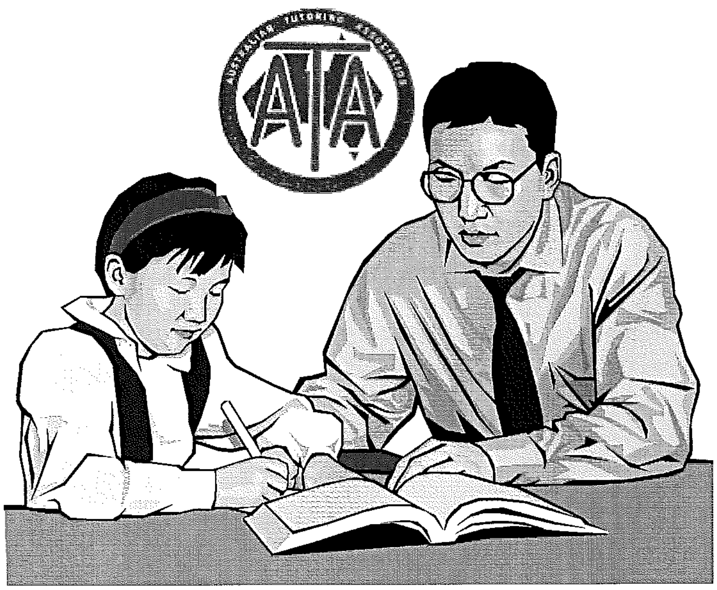

YEAR 12 – MATHS EXT.2

REVIEW TOPIC:

INTEGRATION 1-PAPER2

INTEGRATION I

REVIEW EXERCISES

(1) Show by integration that $\int \frac{1}{x^2 - a^2}\,dx = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + c$

(1) (a) Find the values of the pronumerals given that :

$$\frac{x^5 - x^4 + 4x^3 - 4x^2 + 8x - 4}{\left(x^2+2\right)^3} \equiv \frac{Ax+B}{x^2+2} + \frac{Cx+D}{\left(x^2+2\right)^2} + \frac{Ex+F}{\left(x^2+2\right)^3}$$

$\boxed{A = 1, B = -1, C = D = F = 0, E = 4}$

(b) Hence, find $\displaystyle\int \frac{x^5 - x^4 + 4x^3 - 4x^2 + 8x - 4}{\left(x^2+2\right)^3}\,dx$

$$\frac{1}{2}\ln\left(x^2+2\right) - \frac{\sqrt{2}}{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right) - \frac{1}{\left(x^2+2\right)^2} + c$$

(3) $\displaystyle\int \frac{6x^5 - 3x^4 - 2x^3 + 2x^2 + 6x - 4}{(2x-1)(3x^2-1)}\,dx$

(Hint: You may assume the result $\displaystyle\int \frac{1}{x^2 - a^2}\,dx = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + c$).

$$\boxed{\frac{1}{3}x^3+\frac{3}{2}\ln|2x-1|-\frac{2}{3}\ln\left|3x^2-1\right|+\frac{\sqrt{3}}{6}\ln\left|\frac{\sqrt{3}x-1}{\sqrt{3}x+1}+c\right|}$$

HSC '88

(1) (b) Use the substitution $t = \tan\frac{\theta}{2}$ to find the exact value of

$$\int_0^{\frac{\pi}{2}} \frac{d\theta}{\sin\theta + 2}$$

$$\boxed{\frac{\pi}{3\sqrt{3}}}$$

HSC '89

(2) (a) (iii) Evaluate : $\int_{\sqrt{3}}^{2} \sqrt{4 - x^2}\ dx$

$$\boxed{\frac{\pi}{3} - \frac{\sqrt{3}}{2}}$$

HSC '90

(1) (c) Let $I_n = \int_0^x (1+t^2)^n dt,\ n = 1, 2, 3, \ldots$

Use integration by parts to show that

$I_n = \frac{1}{2n+1}(1+x^2)^n x + \frac{2n}{2n+1} I_{n-1}$. (Hint: Use $(1+t^2)^{n-1} + t^2(1+t^2)^{n-1} = (1+t^2)^n$)

HSC '90

(2) (a) Find the exact value of

(i) $\displaystyle\int_2^3 \frac{x+1}{\sqrt{x^2+2x+5}}\,dx$

$\boxed{\sqrt{20}-\sqrt{13}}$

HSC '90

(2) (a) (ii) Evaluate $\displaystyle\int_0^{\sqrt{2}} \sqrt{4-x^2}\,dx$

$\boxed{\frac{\pi}{2}+1}$

(Qu2)

(b) Find (i) $\int \frac{dx}{(x+1)(x^2+2)}$

$$\boxed{\frac{1}{3}\ln|x+1| - \frac{1}{6}\ln(x^2+2) + \frac{1}{3\sqrt{2}}\tan^{-1}\frac{x}{\sqrt{2}} + c}$$

(Qu2)

(b) (ii) Find $\int \cos^3 x\, dx$ by writing $\cos^3 x = \left(1 - \sin^2 x\right)\cos x$, or otherwise.

$\boxed{\sin x - \frac{1}{3}\sin^3 x + c}$

HSC '91

(1) (a) Find :

(i) $\int \frac{t-1}{t^3}\, dt$

$\boxed{-\frac{1}{t} + \frac{1}{2t^2} + c}$

(ii) $\int \frac{e^x}{e^{2x}+9}\,dx$, using the substitution $u = e^x$

$\boxed{\frac{1}{3}\tan^{-1}\left(\frac{e^x}{3}\right)+c}$

(b) (i) Evaluate $\int_0^1 \frac{5}{(2t+1)(2-t)}\,dt$

$\boxed{\ln 6}$

HSC '91

(1) (b) (ii) By using the substitution $t = \tan\frac{\theta}{2}$, evaluate $\int_0^{\frac{\pi}{2}} \frac{d\theta}{3\sin\theta + 4\cos\theta}$.

[Hint: partial fractions may be needed]

$\frac{1}{5}\log_e 6$

HSC '91

(1) (c) Let $I_n = \int_0^{\frac{\pi}{2}} \sin^n x \, dx$, where n is a non-negative integer.

(i) Show that $I_n = (n-1)\int_0^{\frac{\pi}{2}} \sin^{n-2} x \cos^2 x \, dx$, when $n \geq 2$.

(ii) Deduce that $I_n = \frac{n-1}{n} I_{n-2}$ when $n \geq 2$.

(iii) Evaluate I_4.

$\frac{3\pi}{16}$

HSC '92

(1) (a) (ii) $\int \frac{2x+6}{x^2+6x+1}\,dx$

$\boxed{\ln\left|x^2+6x+1\right|+c}$

(b) $\int_{\frac{3}{2}}^{\frac{5}{2}} \frac{dx}{\sqrt{(x-1)(3-x)}}$ by using the substitution $u = x - 2$.

$\boxed{\frac{\pi}{3}}$

(c) Evaluate $\int_0^1 \frac{5(1-t)}{(t+1)(3-2t)}\,dt$

$\ln\left(\frac{4\sqrt{3}}{3}\right)$

HSC '92

(4) (a) Each of the following statements is either true or false. Write TRUE or FALSE for each of statement and give brief reasons for your answers. (You are not asked to find the primitive functions.)

(i) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^7\theta \, d\theta = 0$

TRUE

(ii) $\int_{0}^{\pi} \sin^7\theta \, d\theta = 0$

FALSE

(iii) $\int_{-1}^{1} e^{-x^2} \, dx = 0$

FALSE

(iv)

$$\int_0^{\frac{\pi}{2}} \left(\sin^8\theta - \cos^8\theta\right)\, d\theta = 0$$

TRUE

(v) For $n = 1, 2, 3, \ldots,$ $\displaystyle\int_0^1 \frac{dt}{1+t^n} \le \int_0^1 \frac{dt}{1+t^{n+1}}$

TRUE

HSC 93

(1) (a) Evaluate $\int_3^8 \frac{x}{(x+1)-\sqrt{x+1}}\,dx$ by using the substitution $x+1=u^2$.

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HSC '93

(1) (b) Evaluate $\int_0^{\frac{\pi}{2}} \frac{d\theta}{2+\cos\theta}$ by using the substitution $t = \tan\frac{\theta}{2}$.

$\boxed{\frac{\pi}{3\sqrt{3}}}$

(d) (i) Find the real numbers a, b and c such that $\frac{4x+3}{(x^2+1)(x+2)} = \frac{ax+b}{x^2+1} + \frac{c}{x+2}$

$\boxed{a = 1, b = 2, c = -1}$

(ii) Hence find $\int \frac{4x+3}{(x^2+1)(x+2)}\,dx$

$\boxed{\tfrac{1}{2}\ln|x^2+1| + 2\tan^{-1}x - \ln|x+2| + c}$

HSC '94

(1) (a) Find :

(i) $\int \frac{4x-12}{x^2-6x+13}\,dx$

$\boxed{2\ln|x^2-6x+13| + c}$

(ii) $\int \frac{1}{x^2-6x+13}\,dx$

$\boxed{\frac{1}{2}\tan^{-1}\frac{x-3}{2} + c}$

HSC 94

(1) (b) Evaluate $\int_{\frac{3}{2}}^{3} \sqrt{9-u^2}\, du$

$$\boxed{\frac{9}{2}\left(\frac{\pi}{3}-\frac{\sqrt{3}}{4}\right)}$$

(c) Evaluate $\int_1^2 \frac{11-2t}{(2t-1)(3-t)}\,dt$

$\boxed{\ln 18}$

HSC 95

(1) (a) Find $\int \frac{1}{x(\ln x)^2}\,dx$ **2**

$\boxed{-\frac{1}{\ln x}+c}$

(c) Show that $\int_{1}^{4} \frac{6t+23}{(2t-1)(t+6)}\,dt = \ln 70$ **4**

HSC '95

(1) (e) Use the substitution $t = \tan\frac{x}{2}$, or otherwise, calculate

$$\int_0^{\frac{\pi}{2}} \frac{dx}{5 + 3\sin x + 4\cos x}$$

$\boxed{\frac{1}{6}}$

(d) Find $\frac{d}{dx}\left(x\sin^{-1}x\right)$, and hence find $\int \sin^{-1}x\,dx$ **3**

$$\boxed{x\sin^{-1}x+\sqrt{1-x^2}+c}$$

<u>HSC '94</u>

(6) (a) (i) Given that $\sin x > \frac{2x}{\pi}$ for $0 < x < \frac{\pi}{2}$, explain why

$$\int_0^{\frac{\pi}{2}} e^{-\sin x}\,dx < \int_0^{\frac{\pi}{2}} e^{-\frac{2x}{\pi}}\,dx.$$

(ii) Show that $\int_{\frac{\pi}{2}}^{\pi} e^{-\sin x}\, dx = \int_{0}^{\frac{\pi}{2}} e^{-\sin x}\, dx$

(iii) Hence show that $\int_{0}^{\pi} e^{-\sin x}\, dx < \frac{\pi}{e}(e-1)$.