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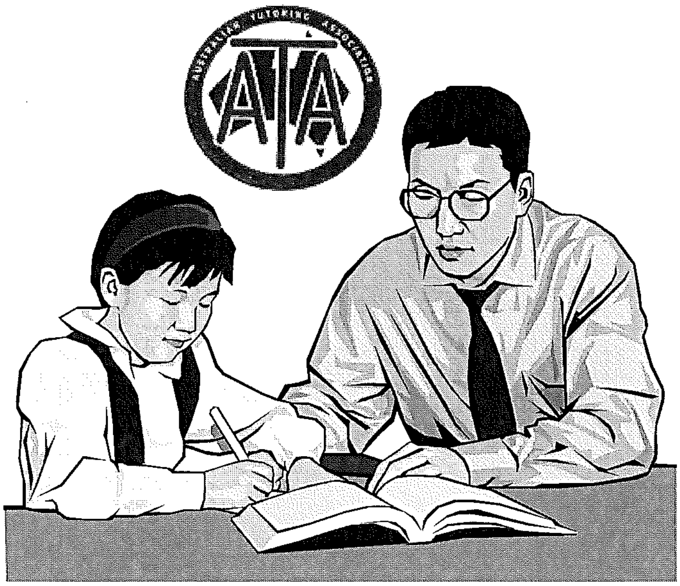

YEAR 12 – MATHS EXT.2

REVIEW TOPIC PAPER 1: INTEGRATION BY PARTS

Received on			**Check corrections on pages:**
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CSSA 2000 Q2

(ii) $\int t\, e^{-t}\, dt$

CSSA 2001 Q5

(a) $I=\int_0^{\pi} x e^{x}\cos x\ dx$ and $J=\int_0^{\pi} e^{x}\cos x\ dx$

(i) Use integration by parts to show that $I-J=-\int_0^{\pi} x e^{x}\sin x\ dx$. 2

(ii) Differentiate xe^{x} and hence find $\int (x+1)e^{x}\ dx$. Hence or otherwise show that 2

$I+J=-\pi\ e^{\pi}+\int_0^{\pi} x e^{x}\sin x\ dx$.

(iii) Evaluate I. 1

HEFFERNAN 2002 Q1

(c) Use integration by parts to evaluate $\int_0^{\frac{\pi}{4}} x\cos 4x\,dx$ **3**

INDEPENDENT 2001 Q2

(d) (i) Use the substitution $x = u^2$, $u > 0$, to show that $\int_4^{16} \frac{\sqrt{x}}{x-1}\,dx = 4 + 2\ln 3 - \ln 5$. 4

(ii) Hence use integration by parts to evaluate $\int_4^{16} \frac{\ln(x-1)}{\sqrt{x}}\,dx$ in simplest exact form. 2

NEAP 2000 Q1

(d) (i) Find $\int x \sin x \, dx$. 3

(ii) Hence find $\int \sin \sqrt{x} \, dx$.

SOLUTIONS

CSSA 2000 Q2

(ii) $$\int t\,e^{-t}\,dt = -t\,e^{-t} + \int e^{-t}\,dt$$
$$= -t\,e^{-t} - e^{-t} + c$$

CSSA 2001 Q5

(i) $I = \int_0^{\pi} x\,e^x \cos x\,dx$, $J = \int_0^{\pi} e^x \cos x\,dx$

$$I - J = \int_0^{\pi} (x-1)\,e^x \cos x\,dx$$
$$= \left[(x-1)\,e^x \sin x\right]_0^{\pi} - \int_0^{\pi} x\,e^x \sin x\,dx$$
$$= -\int_0^{\pi} x\,e^x \sin x\,dx$$

(ii) $\frac{d}{dx} x e^x = e^x + x e^x = (x+1)e^x$

$$\therefore \int (x+1)e^x\,dx = xe^x + c$$

$$I + J = \int_0^{\pi} (x+1)\,e^x \cos x\,dx$$
$$= \left[x\,e^x \cos x\right]_0^{\pi} - \int_0^{\pi} x\,e^x(-\sin x)\,dx$$
$$= -\pi\,e^{\pi} + \int_0^{\pi} x\,e^x \sin x\,dx$$

(iii) $I = \frac{1}{2}\{(I+J)+(I-J)\} = -\frac{1}{2}\pi\,e^{\pi}$

HEFFERNAN 2002 Q1

(c) The parts formula states that

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx.$$

For $\int_0^{\frac{\pi}{4}} x\cos(4x)dx$, let $u = x$ and $\frac{dv}{dx} = \cos(4x)$

So, $\frac{du}{dx} = 1$ and $v = \frac{1}{4}\sin(4x)$

(1 mark)

So $\int_0^{\frac{\pi}{4}} x\cos(4x)dx = \left[\frac{x}{4}\sin(4x)\right]_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} \frac{1}{4}\sin(4x)dx$ **(1 mark)**

$$= \left\{\frac{\pi}{16}\sin\pi - 0\right\} - \frac{1}{4}\left[-\frac{1}{4}\cos(4x)\right]_0^{\frac{\pi}{4}}$$
$$= \frac{\pi}{16}\times 0 + \frac{1}{16}\{\cos\pi - \cos 0\}$$
$$= \frac{1}{16}(-1-1)$$
$$= -\frac{1}{8}$$

(1 mark)

INDEPENDENT 2001 Q2

(i)

$x = u^2,\quad u > 0$

$dx = 2u\,du$

$\dfrac{\sqrt{x}}{x-1} = \dfrac{u}{u^2-1}$

$x = 4 \Rightarrow u = 2$

$x = 16 \Rightarrow u = 4$

$$I = \int_4^{16} \frac{\sqrt{x}}{x-1}\,dx = \int_2^4 \frac{2u^2}{u^2-1}\,du = \int_2^4 \frac{2(u^2-1)+2}{u^2-1}\,du$$

$$I = \int_2^4 \left(2 + \frac{2}{u^2-1}\right) du = \int_2^4 \left(2 + \frac{1}{u-1} - \frac{1}{u+1}\right) du$$

$$\therefore I = \left[2u + \ln\left|\frac{u-1}{u+1}\right|\right]_2^4 = 2(4-2) + \ln\tfrac{3}{5} - \ln\tfrac{1}{3}$$

$$\therefore I = 4 + \ln 3 - \ln 5 + \ln 3 = 4 + 2\ln 3 - \ln 5$$

(ii)

$$\int_4^{16} \frac{\ln(x-1)}{\sqrt{x}}\,dx = \left[2\sqrt{x}\ln(x-1)\right]_4^{16} - 2\int_4^{16} \frac{\sqrt{x}}{x-1}\,dx$$

$$= 2(4\ln 15 - 2\ln 3) - 2(4 + 2\ln 3 - \ln 5)$$

$$= 2(4\ln 5 + 2\ln 3) - 2(4 + 2\ln 3 - \ln 5)$$

$$= 10\ln 5 - 8$$

3

NEAP 2000 Q1

(d) (i) $\int x\sin x\,dx$

Let $u = x \quad v' = \sin x$

$\therefore u' = 1 \quad v = -\cos x$

$\left[\int u'v\,dx = uv - \int uv'\,dx\right]$

$\therefore \int x\sin x\,dx = -x\cos x - \int 1 \times (-\cos x)\,dx$

$= -x\cos x + \int \cos x\,dx$

$= -x\cos x + \sin x + C$ ✓

(ii) $\int \sin(\sqrt{x})dx$

Let $u = \sqrt{x}$. Then $u^2 = x$, so $2u\,du = dx$

Hence $\int \sin\sqrt{x}\,dx = \int 2u\sin u\,du$ ✓

$= 2(-u\cos u + \sin u) + C$ from (i)

$= -2\sqrt{x}\cos\sqrt{x} + 2\sin\sqrt{x} + C$ ✓