NAME :

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YEAR 12 – MATHS EXT.2

REVIEW TOPIC (PAPER 1): RECTANGULAR HYPERBOLA

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RC 2002 Q4

b) $P\left(4p, \frac{4}{p}\right)$ and $Q\left(4q, \frac{4}{q}\right)$ are points on the rectangular hyperbola $xy = 16$.

i) Derive the equations of the tangents at P and Q.. 2

ii) The tangents at P and Q intersect at the point R.
Derive the coordinates of the point R 2

iii) If the chord PQ passes through the point $(4,0)$, derive the locus of R. 4

ST IGNATIUS 2002 Q4

d) A and B are variable points on the rectangular hyperbola $xy = c^2$

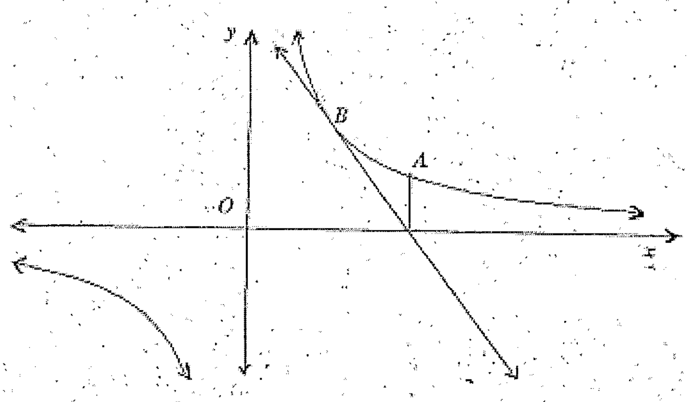


Diagram not to scale.

(i) The tangent at B passes through the foot of the ordinate of A. If A and B have parameters t_1 and t_2, show that $t_1 = 2t_2$ 4

(ii) Hence prove that the locus of the midpoint of AB is a rectangular hyperbola. 2

CSSA 2004 Q4

(b) (i) Sketch the graph of the rectangular hyperbola $xy = 1$, showing clearly the coordinates of the foci and the equations of the directrices. **2**

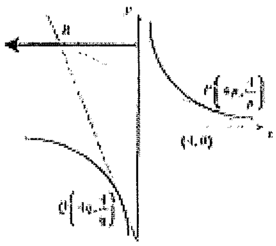
(ii) $P\left(p, \frac{1}{p}\right)$ and $Q\left(q, \frac{1}{q}\right)$ are two points on the rectangular hyperbola $xy = 1$. Show that the chord PQ has equation $x + pqy - (p + q) = 0$. **2**

(iii) If O is the origin, show that ΔOPQ has area $\dfrac{|p^2 - q^2|}{2|pq|}$ square units. **3**

SOLUTIONS

RC 2002 Q4

b)



i)

$y=\frac{16}{x}$

$\frac{dy}{dx}=-\frac{16}{x^2}$ 1

At P $\frac{dy}{dx}=-\frac{16}{16p^2}=-\frac{1}{p^2}$

Tangent at P is $y-\frac{4}{p}=-\frac{1}{p^2}(x-4p)\Rightarrow x+p^2y=8p$ 1

Tangent at P is $x+q^2y=8q$

ii) Solving simultaneously,

$(p^2-q^2)y=8(p-q)$

$y=\frac{8}{p+q}$ 1

$x+\frac{8p^2}{p+q}=8p$

$x=\frac{8pq}{p+q} \quad \Rightarrow \quad R\left(\frac{8pq}{p+q},\frac{8}{p+q}\right)$ 1

iii) Chord PQ:

$m_{PQ}=\frac{\frac{4}{p}-\frac{4}{q}}{4p-4q}=-\frac{1}{pq}$ 1

PQ: $y-\frac{4}{p}=-\frac{1}{pq}(x-4p)$

$(4,0)\Rightarrow -\frac{4}{p}=-\frac{1}{pq}(4-4p)$

$4q=4-4p$ 1

$p+q=1$

R $x=8pq$ $y=8$ (since $p+q=1$) 1

Hence the locus of R is the ray $y=8,\ x<0$ 1

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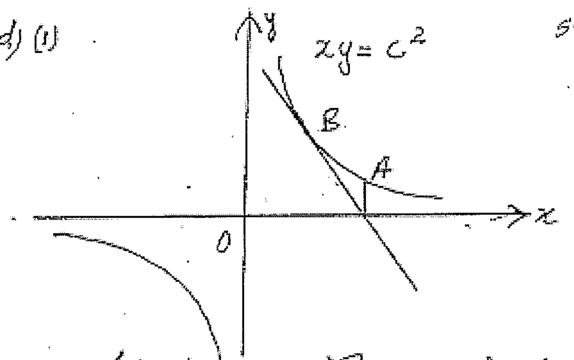
(d) (i)



Let N = Foot of the ordinate of A.

$B\left(ct_2, \frac{c}{t_2}\right)$; $A\left(ct_1, \frac{c}{t_1}\right)$

tangent at B: $y' = -\frac{c^2}{x^2}$

gradient m at B is $m = -\frac{c^2}{c^2 t_2^2} = -\frac{1}{t_2^2}$

Eqn: $y - \frac{c}{t_2} = -\frac{1}{t_2^2}(x - ct_2)$

$t_2^2 y - ct_2 = -x + ct_2$

For Co-ordinates of N let $y = 0$

$\therefore x = 2ct_2$

But this is equal to the x co-ordinate of A.

$c.t_1 = 2ct_2$

$\therefore t_1 = 2t_2$ --- (1)

(ii) Co-ordinates of the mid point of AB

$x = \frac{c}{2}(t_1 + t_2)$ --- (2)

$y = \frac{c}{2}\left(\frac{1}{t_1} + \frac{1}{t_2}\right)$ --- (3)

Eliminate parameters.

sub (1) in both (2) and (3)

$x = \frac{c}{2}(3t_2)$ --- (2a)

$y = \left(\frac{1}{2t_2} + \frac{1}{t_2}\right)\frac{c}{2}$ ---

$y = \left(\frac{1+2}{2t_2}\right)\frac{c}{2}$

$y = \frac{c}{2}\left(\frac{3}{2t_2}\right)$ --- (3a)

(2a) × (3a)

$xy = \frac{c}{2}(3t_2) \times \frac{c}{2}\left(\frac{3}{2t_2}\right)$

$xy = \frac{c^2}{8} \times 9$

$8xy = 9c^2$.

which is a rectangular hyperbola.

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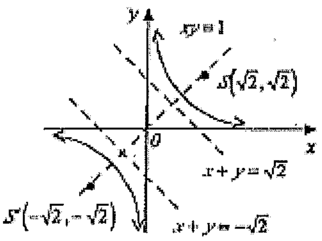
i.



ii. $P\left(p,\frac{1}{p}\right)$, $Q\left(q,\frac{1}{q}\right)$

$gradient\ PQ = \dfrac{\frac{1}{p}-\frac{1}{q}}{p-q} = \dfrac{q-p}{pq(p-q)} = -\dfrac{1}{pq}$

Equation is

$y-\frac{1}{p} = -\frac{1}{pq}(x-p)$

$pq\,y - q = -x + p$

$x + pq\,y - (p+q) = 0$

iii. ⊥distance from O to the chord PQ is

$d = \dfrac{|p+q|}{\sqrt{1+(pq)^2}}$

$PQ^2 = (p-q)^2 + \left(\frac{1}{p}-\frac{1}{q}\right)^2$

$= (p-q)^2 + \dfrac{(p-q)^2}{(pq)^2}$

$= \dfrac{(p-q)^2}{(pq)^2}\left\{(pq)^2+1\right\}$

Area of ΔOPQ is given by $A = \frac{1}{2} PQ \times d$

$\therefore A = \frac{1}{2}\left|\dfrac{p-q}{pq}\right|\sqrt{(pq)^2+1} \times \dfrac{|p+q|}{\sqrt{1+(pq)^2}}$

$= \dfrac{|p^2-q^2|}{2|pq|}$