NAME :

Centre of Excellence in Mathematics

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YEAR 12 – MATHS EXT.2

REVIEW TOPIC (PAPER 1): VOL BY KNOWN X-SECTION

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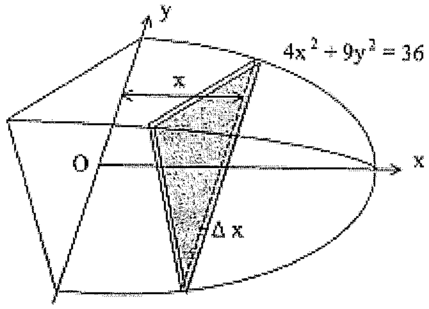
JAMES RUSE 2000 Q2

(a) A symmetrical pier of height 5 metres has an elliptical base with equation $\frac{x^2}{25} + \frac{y^2}{4} = 1$ and slopes to a parallel elliptical top with equation $\frac{x^2}{9} + y^2 = 1$.

If the cross sections of the area parallel to the base are also elliptical find the volume of the pier given that the area of an ellipse with semi-major axis a and semi-minor axis b is πab.

S&G 2001 Q5

c)



The base of the solid K shown in the diagram is the region in the xy plane enclosed between the semi-ellipse $4x^2 + 9y^2 = 36$ and the y axis. Each cross section perpendicular to the x axis is an equilateral triangle.

i) Consider a slice of the solid with thickness Δx and distant x from the y axis. Find the area of this slice in terms of x. 2

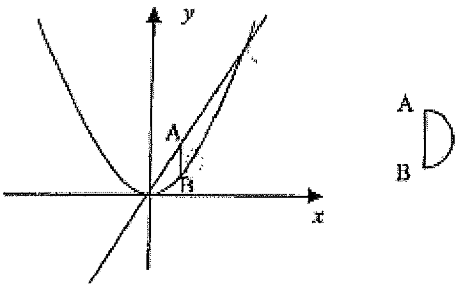
ii) Find the volume of the solid K. 2

iii) Solid J has the same base as solid K but its perpendicular cross sectional slice is an isosceles right angled triangle with its hypotenuse in the xy plane. 2

Find the ratio of volumes of solid K to solid J.

SBHS 2001 Q6

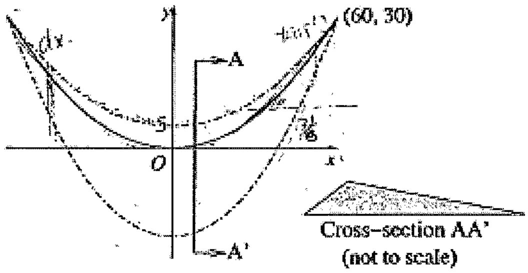
(a) The base of a solid is the region enclosed by $y = 2x$ and $y = x^2$. Cross sections taken perpendicular to the x axis are semicircles with the diameter in the base of the solid (as indicated the diameter AB of the semicircle is perpendicular to the x axis; the semicircle is perpendicular to the xy plane). 5



Find the volume of the solid.

SBHS 2002 Q7

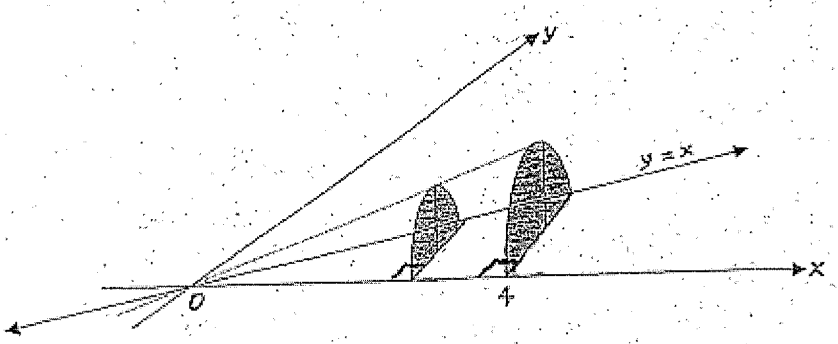
(c) 5



Barcan sand dunes are parabolic in plan view and are triangular in cross section with the inner face having an angle of repose of $\tan^{-1}\frac{3}{4}$ to the horizontal and the outer face at $\tan^{-1}\frac{1}{6}$ to the horizontal. The figure above shows one such dune (dimensions are in metres). Calculate the volume of sand.

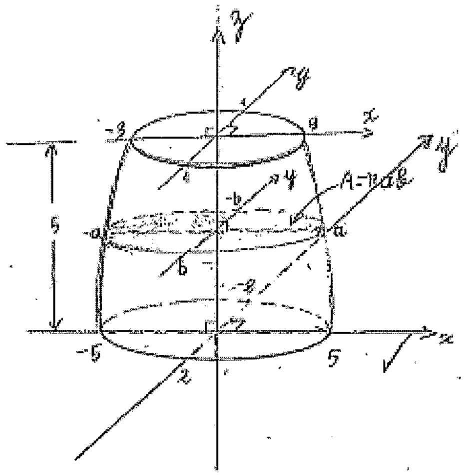
ST IGNATIUS 2002 Q6

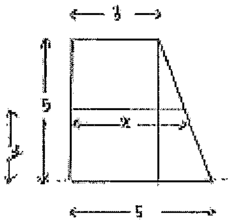
a) The base of a solid is a right-angled triangle on the horizontal x–y plane; bounded by the lines $y = 0$, $x = 4$ and $y = x$. Vertical cross-sections of the solid, parallel to the y –axis, are semicircles with their diameter on the base of the solid as shown in the diagram below.
Find the volume of the solid. 5



SOLUTIONS

JAMES RUSE 2000 Q2





We have: $\frac{z}{5} = \frac{x-3}{2}$

$x = \frac{2z}{5} + 3 \quad \therefore a = \frac{2z}{5} + 3$

Similarly

$\frac{z}{5} = \frac{y-1}{2}$

$y = \frac{2z}{5} + 1 \quad \therefore b = \frac{2z}{5} + 1$

$A = \pi a b = \frac{2z+15}{5} \times \frac{2z+5}{5} = \frac{4z^2+40z+75}{25}$

$\int_0^5 A\,dz = \frac{1}{25}\int_0^5 (4z^2+40z+75)\,dz$

$\frac{1}{25}\left[\frac{4z^3}{3} + \frac{40z^2}{2} + 75z\right]_0^5$

$\frac{1}{5}\left[\frac{4}{3}\times 125 + 20\times 25 + 75\times 5\right] = 41\frac{2}{3}\,u^3$

using similar △'s

$\frac{5-z}{5} = \frac{x-3}{2}$

$\frac{10-2z}{5} = x-3$

$\therefore x = 3 + 2 - \frac{2}{5}z$

$\therefore a = 5 - \frac{2}{5}z$

Similarly $b = 2 - \frac{z}{5}$

$A = \pi a b = \pi\left(5 - \frac{2}{5}z\right)\left(2 - \frac{z}{5}\right)$

$= \pi\left(10 - z - \frac{4z}{5} + \frac{2z^2}{25}\right)$

$= \pi\left(10 - \frac{9z}{5} + \frac{2z^2}{25}\right)$

$V = \pi\int_0^5 10 - \frac{9z}{5} + \frac{2z^2}{25}\,dz$

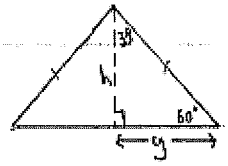
$= \pi\left[10z - \frac{9z^2}{10} + \frac{2z^3}{25}\right]_0^5$

$= \pi\left[50 - \frac{45}{2} + 10\right]$

$= \frac{75\pi}{2}\,u^3$

S&G 2001 Q5

c) ii)



Consider the triangle above, taken as a slice of the solid with thickness Δx.

$\tan 30 = \frac{y}{h}$ $\therefore h = y\sqrt{3}$.

$\therefore$ Area of slice $= y\sqrt{3} \times 2y \times \frac{1}{2} = y^2\sqrt{3}$

As, $4x^2 + 9y^2 = 36$ $\therefore y^2 = \frac{36 - 4x^2}{9}$

$\therefore$ Area of the slice $= \sqrt{3}\left(\frac{36 - 4x^2}{9}\right)$ (2 marks)

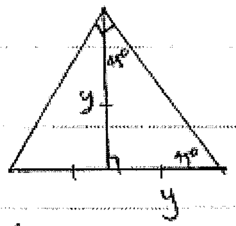
$\therefore$ Volume of $K = \frac{\sqrt{3}}{9}\int_0^3 (36 - 4x^2)\,dx$

$= \frac{\sqrt{3}}{9}\left[36x - \frac{4x^3}{3}\right]_0^3 = \frac{\sqrt{3}}{9}\left[108 - 36\right]$

$= 8\sqrt{3}$ units^3.

(2 marks)

iii)



Area of slice $= 2y \times y \times \frac{1}{2} = y^2$.

$\therefore$ Volume of J

$= \int_0^3 y^2\,dx$

$= \frac{1}{9}\int_0^3 (36 - 4x^2)\,dx$.

$= \frac{1}{9}\left[36x - \frac{4x^3}{3}\right]_0^3$

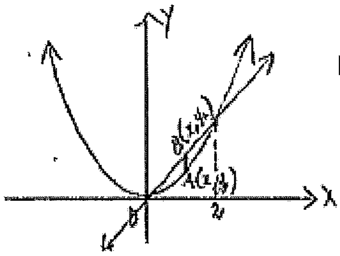
$= \frac{1}{9}\left[108 - 36\right] = 8$ units^3.

$\therefore$ Ratio of volumes of solid K to solid J is $8\sqrt{3} : 8$

$= \sqrt{3} : 1$ (2 marks)

SBHS 2001 Q6

i) (5)



$x^2 = 2x \Rightarrow x = 0$ or 2

Diameter $= y_2 - y_1 = 2x - x^2$

A = Area of semi-circle $= \frac{\pi}{2}\left(x - \frac{x^2}{2}\right)^2$ 1

Thickness of solid δx.

Vol. of element $= \delta V = A\delta x$

$= \frac{\pi}{2}\left(x - \frac{x^2}{2}\right)^2 \delta x$ 1

Total Volume $= \lim_{\delta x \to 0} \sum_{x=0}^{2} \frac{\pi}{2}\left(x - \frac{x^2}{2}\right)^2 \delta x$

$= \frac{\pi}{2}\int_0^2 \left(x - \frac{x^2}{2}\right)^2 dx$

$= \frac{\pi}{2}\int_0^2 \left(x^2 - x^3 + \frac{x^4}{4}\right) dx$ 1

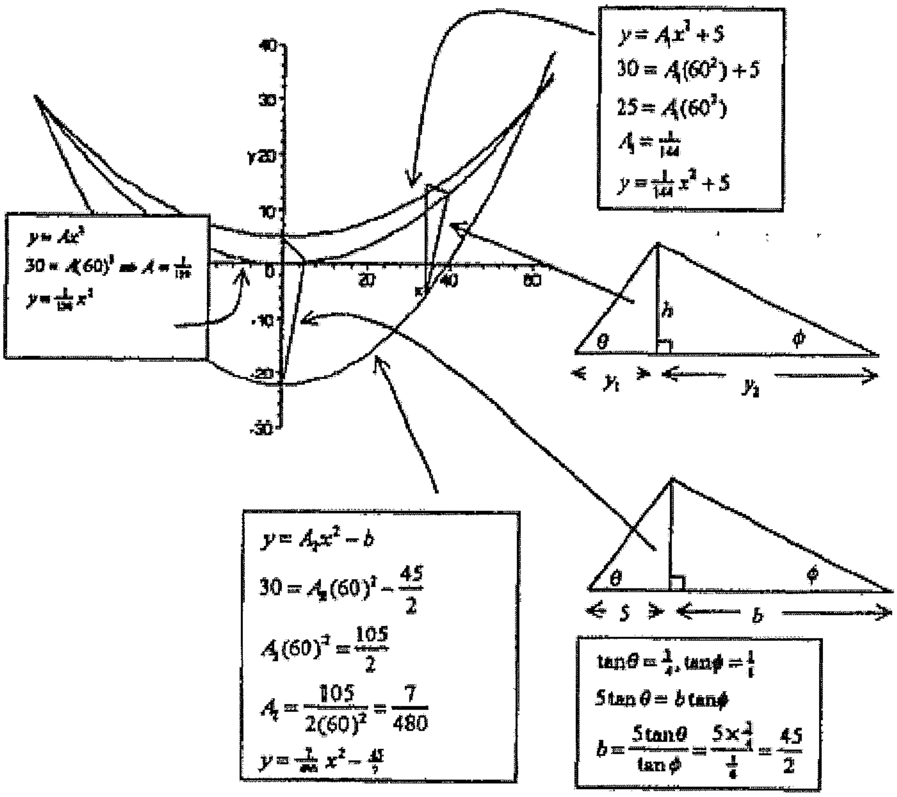
$= \frac{\pi}{2}\left[\frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{20}\right]_0^2$ 1

$= \frac{\pi}{2}\left[\frac{8}{3} - 4 + \frac{32}{20} - 0\right]$

$= \frac{2\pi}{15}$ units^3. 1

SBHS 2002 Q7

(c)



Area of the cross sectional triangle $\frac{1}{2}h(y_1+y_2)$

$y_1 = \frac{1}{144}x^2 + 5 - \frac{1}{120}x^2 = -\frac{1}{720}x^2 + 5$

$y_2 = \frac{1}{120}x^2 - (\frac{7}{480}x^2 - \frac{45}{2}) = -\frac{1}{160}x^2 + \frac{45}{2}$

$y_1 + y_2 = \frac{1}{144}x^2 + 5 - (\frac{7}{480}x^2 - \frac{45}{2})$

$= -\frac{11}{1440}x^2 + \frac{55}{2}$

$\tan\theta = \frac{h}{y_1} \Rightarrow h = y_1 \tan\theta = y_1 \times \frac{3}{4} = \frac{3y_1}{4}$

$h = \frac{3(-\frac{1}{720}x^2 + 5)}{4} = -\frac{1}{960}x^2 + \frac{15}{4}$

$\text{Area} = \frac{1}{2}\left(-\frac{1}{960}x^2 + \frac{15}{4}\right)\left(-\frac{11}{1440}x^2 + \frac{55}{2}\right) = \frac{11}{2764800}x^4 - \frac{11}{384}x^2 + \frac{825}{16}$

$\Delta V \cong \left(\frac{11}{2764800}x^4 - \frac{11}{384}x^2 + \frac{825}{16}\right)\Delta x$

$V = \int_{-60}^{60}\left(\frac{11}{2764800}x^4 - \frac{11}{384}x^2 + \frac{825}{16}\right)dx$

$= 2\int_0^{60}\left(\frac{11}{2764800}x^4 - \frac{11}{384}x^2 + \frac{825}{16}\right)dx$

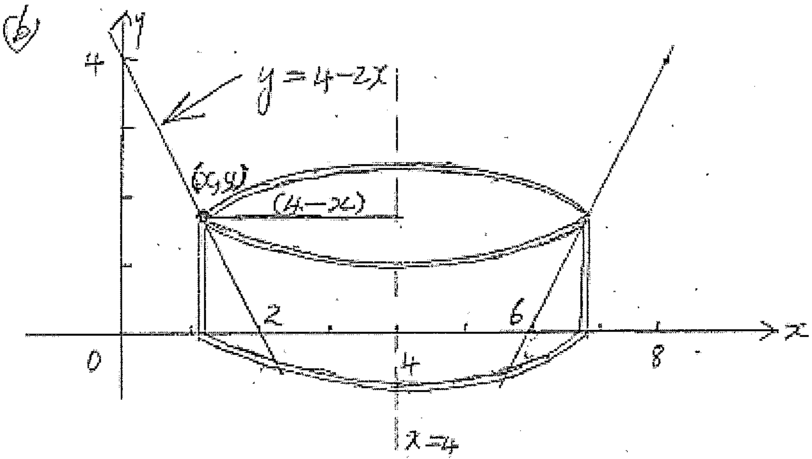
$= 2 \times \left[\frac{11}{13824000}x^5 - \frac{11}{1152}x^3 + \frac{825}{16}x\right]_0^{60}$

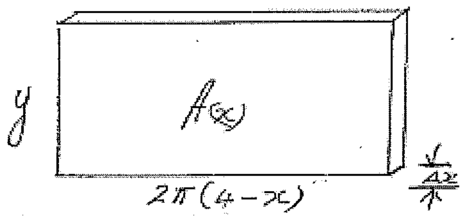
$= 3300$

So the volume of the Barcan dune is 3300 cubic units

ST IGNATIUS 2002 Q6

(b)





$$A(x) = 2\pi(4-x)y$$

$$\Delta V \doteqdot 2\pi(4-x)(4-2x)\Delta x$$

$$V = \lim_{x \to 0} \sum_{x=0}^{2} 2\pi(4-x)(4-2x)\Delta x$$

$$V = 2\pi \int_0^2 (16 - 12x + 2x^2)\,dx$$

$$V = 2\pi \left[16x - 6x^2 + \frac{2x^3}{3}\right]_0^2$$

$$V = 2\pi\left[32 - 24 + \frac{16}{3}\right]$$

$$V = 2\pi\left[\frac{24}{3} + \frac{16}{3}\right]$$

$$V = \frac{8\pi}{3} \text{ units}^3$$