

Mathematics General 2

General Instructions

- Reading time – 5 minutes
- Working time – $2\frac{1}{2}$ hours
- Write using black pen
- Board-approved calculators may be used
- A formulae and data sheet is provided at the back of this paper
- In Questions 26–30, show relevant mathematical reasoning and/or calculations

Total marks – 100

Section I Pages 2–12

25 marks

- Attempt Questions 1–25
- Allow about 35 minutes for this section

Section II Pages 13–36

75 marks

- Attempt Questions 26–30
- Allow about 1 hour and 55 minutes for this section

Section I

25 marks

Attempt Questions 1–25

Allow about 35 minutes for this section

Use the multiple-choice answer sheet for Questions 1–25.

1 What is 1 560 200 km written in scientific notation correct to two significant figures?

- (A) 1.56×10^4 km
- (B) 1.6×10^5 km
- (C) 1.56×10^6 km
- (D) 1.6×10^6 km

2 Which of the following is $4x + 3y - x - 5y$ in its simplest form?

- (A) $3x - 2y$
- (B) $3x + 8y$
- (C) $5x - 2y$
- (D) $5x + 8y$

3 Gayle's gross pay each week is \$952.25.

The following deductions are taken from her gross pay each week:

- tax \$180.93
- superannuation \$85.70
- union membership \$21.40
- health fund \$38.15.

What is Gayle's net pay each week?

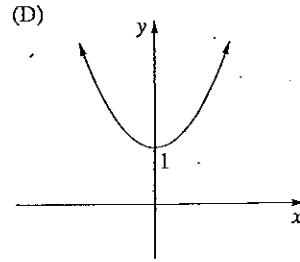
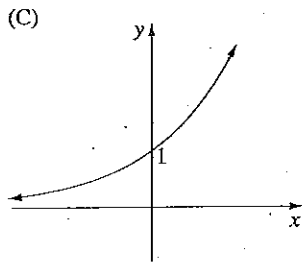
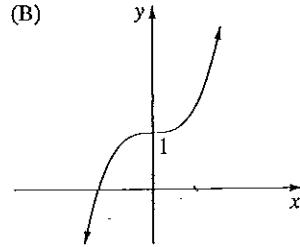
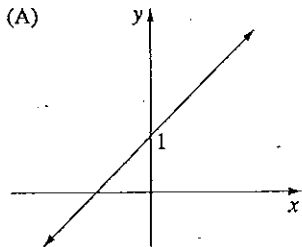
- (A) \$326.18
- (B) \$626.07
- (C) \$771.32
- (D) \$952.25.

- 4 On a school report, a student's record of completing homework is graded using the following codes.

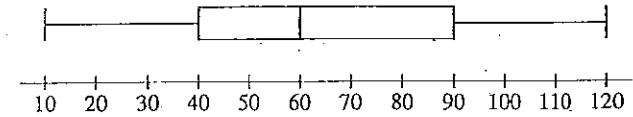
C = consistently
 U = usually
 S = sometimes
 R = rarely
 N = never

What type of data is this?

- (A) Categorical, ordinal
 (B) Categorical, nominal
 (C) Quantitative, continuous
 (D) Quantitative, discrete
- 5 Which of the following graphs best represents the equation $y = x^3 + 1$?



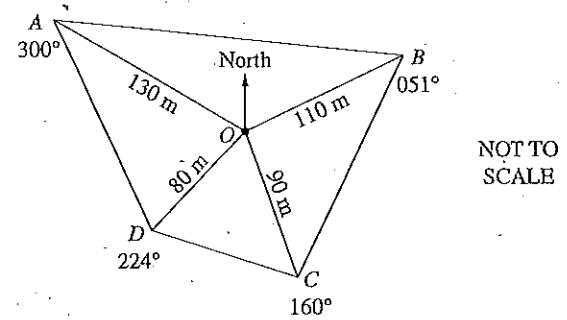
- 6 The times, in minutes, that a large group of students spend on exercise per day are presented in the box-and-whisker plot.



What percentage of these students spend between 40 minutes and 60 minutes per day on exercise?

- (A) 17%
 (B) 20%
 (C) 25%
 (D) 50%

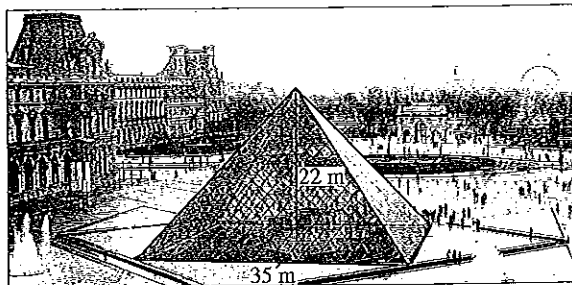
- 7 The diagram shows a radial survey of a field $ABCD$.



In triangle AOB , what is the size of $\angle AOB$?

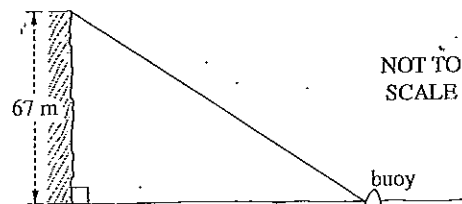
- (A) 51°
 (B) 111°
 (C) 125°
 (D) 249°

- 8 The Louvre Pyramid in Paris has a square base with side length 35 m and a perpendicular height of 22 m.



What is the volume of this pyramid, to the nearest m^3 ?

- (A) 257 m^3
 (B) 1027 m^3
 (C) 8983 m^3
 (D) 26950 m^3
- 9 From the top of a cliff 67 metres above sea level, the angle of depression of a buoy is 42° .

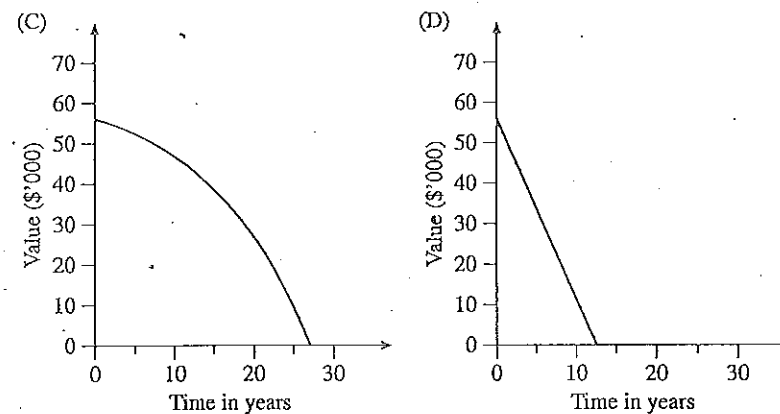
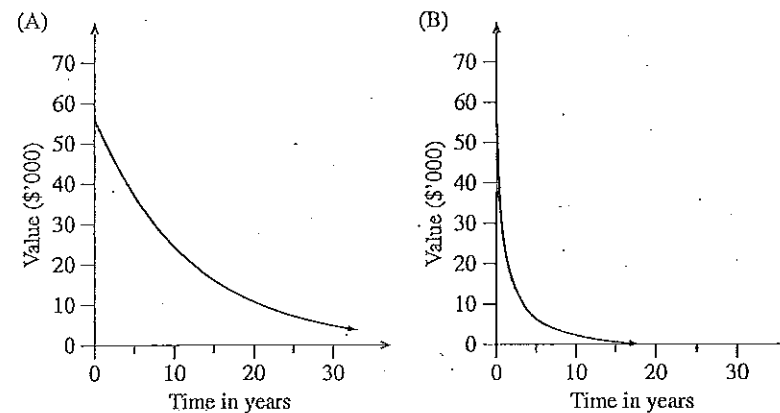


How far is the buoy from the base of the cliff, to the nearest metre?

- (A) 60 m
 (B) 74 m
 (C) 90 m
 (D) 100 m

- 10 A piece of machinery, initially worth \$56 000, depreciates at 8% per annum.

Which graph best shows the salvage value of this piece of machinery over time?



- 11 Which of the following is $3x^0 + 5x$ in its simplest form?

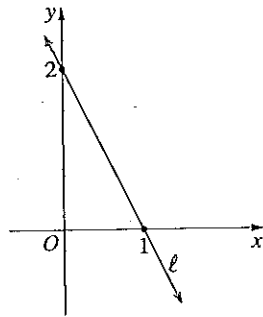
- (A) $6x$
 (B) $8x$
 (C) $1 + 5x$
 (D) $3 + 5x$

- 12 The length of a fish was measured to be 49 cm, correct to the nearest cm.

What is the percentage error in this measurement, correct to one significant figure?

- (A) $\pm 0.01\%$
- (B) $\pm 0.5\%$
- (C) $\pm 1\%$
- (D) $\pm 2\%$

- 13 What is the equation of the line ℓ ?



- (A) $y = -2x + 2$
- (B) $y = 2x + 2$
- (C) $y = -\frac{x}{2} + 2$
- (D) $y = \frac{x}{2} + 2$

- 14 Stockholm is located at $59^\circ\text{N } 18^\circ\text{E}$ and Darwin is located at $13^\circ\text{S } 131^\circ\text{E}$.

What is the time difference between Stockholm and Darwin? (Ignore time zones and daylight saving.)

- (A) 184 minutes
- (B) 288 minutes
- (C) 452 minutes
- (D) 596 minutes

- 15 A camera costs \$449, including 12% GST.

What is the price of the camera without GST, correct to the nearest dollar?

- (A) \$395
- (B) \$401
- (C) \$437
- (D) \$503

- 16 The probability of winning a game is $\frac{7}{10}$.

Which expression represents the probability of winning two consecutive games?

- (A) $\frac{7}{10} \times \frac{6}{9}$
- (B) $\frac{7}{10} \times \frac{6}{10}$
- (C) $\frac{7}{10} \times \frac{7}{9}$
- (D) $\frac{7}{10} \times \frac{7}{10}$

- 17 What amount must be invested now at 4% per annum, compounded quarterly, so that in five years it will have grown to \$60 000?

- (A) \$8919
- (B) \$11 156
- (C) \$49 173
- (D) \$49 316

- 18 A Student Representative Council (SRC) consists of five members. Three of the members are being selected to attend a conference.

In how many ways can the three members be selected?

- (A) 10
(B) 20
(C) 30
(D) 60

- 19 The table shows the life expectancy (expected remaining years of life) for females at selected ages in the given periods of time.

Life expectancy for females

Period of time	Remaining years of life			
	at age 0	at age 25	at age 45	at age 65
1965–1967	74.2	51.2	32.3	15.7
1975–1977	76.6	53.1	34.0	17.1
1985–1987	79.2	55.4	36.1	18.6
1995–1997	81.3	57.1	37.7	19.8

In 1975, a 45-year-old female used the information in the table to calculate the age to which she was expected to live. Twenty years later she recalculated the age to which she was expected to live.

What is the difference between the two ages she calculated?

- (A) 2.7 years
(B) 3.1 years
(C) 3.7 years
(D) 5.8 years

- 20 A machine produces cylindrical pipes. The mean of the diameters of the pipes is 8 cm and the standard deviation is 0.04 cm.

Assuming a normal distribution, what percentage of cylindrical pipes produced will have a diameter less than 7.96 cm?

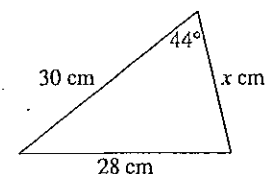
- (A) 16%
(B) 32%
(C) 34%
(D) 68%

- 21 Four cards, labelled 2, 3, 5 and 7, are placed on a table to form a four-digit number.

How many different numbers greater than 3000 can be formed?

- (A) 6
(B) 12
(C) 18
(D) 24

- 22 The area of the triangle shown is 250 cm^2 .



NOT TO SCALE

What is the value of x , correct to the nearest whole number?

- (A) 11
(B) 18
(C) 22
(D) 24

- 23 The number of 'standard drinks' in various glasses of wine is shown.

Number of standard drinks			
White Wine		Red Wine	
small glass	large glass	small glass	large glass
0.9	1.4	1.0	1.5

A woman weighing 62 kg drinks three small glasses of white wine and two large glasses of red wine between 8 pm and 1 am.

What would be her blood alcohol content (BAC) estimate at 1 am, correct to three decimal places?

- (A) 0.030
 (B) 0.037
 (C) 0.046
 (D) 0.057
- 24 Consider the equation $\frac{2x}{3} - 4 = \frac{5x}{2} + 1$.

Which of the following would be a correct step in solving this equation?

- (A) $\frac{2x}{3} - 3 = \frac{5x}{2}$
 (B) $\frac{2x}{3} = \frac{5x}{2} + 5$
 (C) $2x - 4 = \frac{15x}{2} + 3$
 (D) $\frac{4x}{6} - 8 = 5x + 2$

- 25 An insurance company offers customers the following discounts on the basic annual premium for car insurance.

Type of discount	Discount
Multi-policy discount (Owner has more than 2 insurance policies with the company)	15%
No-claim bonus (Owner has had at least 5 years without an insurance claim)	20%
Combined CTP and comprehensive insurance (Owner has both insurances with the company)	\$50

If a customer is eligible for more than one discount, subsequent discounts are applied to the already discounted premium. The combined compulsory third party (CTP) and comprehensive insurance discount is always applied last.

Jamie has three insurance policies, including combined CTP and comprehensive insurance, with this company. He has used this company for 8 years and he has never made a claim.

The basic annual premium for his car insurance is \$870.

How much will Jamie need to pay after the discounts are applied?

- (A) \$482.44
 (B) \$515.50
 (C) \$541.60
 (D) \$557.60

Centre Number

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Student Number

Section II

75 marks

Attempt Questions 26–30

Allow about 1 hour and 55 minutes
 for this section

Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.

Your responses should include relevant mathematical reasoning and/or calculations.

Extra writing space is provided on pages 35 and 36. If you use this space, clearly indicate which question you are answering.

Write your Student Number at the top of this page.

Please turn over

Question 26 (15 marks)

- (a) A farmer used the 'capture-recapture' technique to estimate the number of chickens he had on his farm. He captured, tagged and released 18 of the chickens. Later, he caught 26 chickens at random and found that 4 had been tagged. 2

What is the estimate for the total number of chickens on this farm?

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- (b) Clark's formula is used to determine the dosage of medicine for children. 2

$$\text{Dosage} = \frac{\text{weight in kg} \times \text{adult dosage}}{70}$$

The adult daily dosage of a medicine contains 3150 mg of a particular drug.

A child who weighs 35 kg is to be given tablets each containing 525 mg of this drug.

How many tablets should this child be given daily?

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Question 26 continues on page 15

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Question 26 (continued)

- (c) Two cities lie on the same meridian of longitude. One is 40° north of the other. 2

What is the distance between the two cities, correct to the nearest kilometre?

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- (d) A family currently pays \$320 for some groceries. 2

Assuming a constant annual inflation rate of 2.9%, calculate how much would be paid for the same groceries in 5 years' time.

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Question 26 continues on page 16

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Question 26 (continued)

- (e) The table shows the relative frequency of selecting each of the different coloured jelly beans from packets containing green, yellow, black, red and white jelly beans.

Colour	Relative frequency
Green	0.32
Yellow	0.13
Black	0.14
Red	
White	0.24

- (i) What is the relative frequency of selecting a red jelly bean? 1

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- (ii) Based on this table of relative frequencies, what is the probability of NOT selecting a black jelly bean? 1

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- (f) Approximately 71% of Earth's surface is covered by water. Assume Earth is a sphere. 2

Calculate the number of square kilometres covered by water.

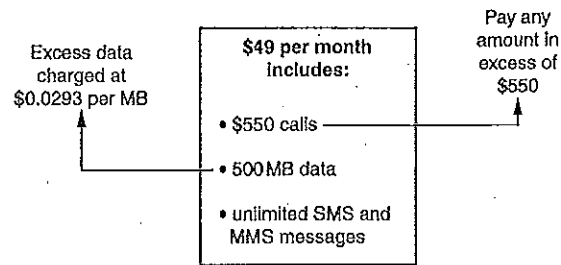
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Question 26 continues on page 17

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Question 26 (continued)

(g) Pat's mobile phone plan is shown.



Last month Pat:

- made calls to the value of \$561
- sent 152 SMS messages
- sent 37 MMS messages
- used 1.7 GB of data.

What was the total of Pat's phone bill for last month?

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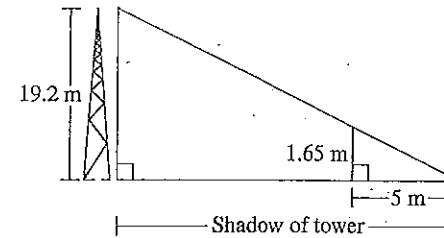
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End of Question 26

Question 27 (15 marks)

(a) At a particular time during the day, a tower of height 19.2 metres casts a shadow. At the same time, a person who is 1.65 metres tall casts a shadow 5 metres long.



NOT TO SCALE

What is the length of the shadow cast by the tower at that time?

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(b) A patient requires 2400 mL of fluid to be delivered at a constant rate by means of a drip over 12 hours. Each mL of fluid is equivalent to 15 drops.

How many drops per minute need to be delivered?

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Question 27 continues on page 19

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Question 27 (continued)

- (c) Ariana's parents have given her an interest-free loan of \$4800 to buy a car. She will pay them back by paying \$x immediately and \$y every month until she has repaid the loan in full.

After 18 months Ariana has paid back \$1510, and after 36 months she has paid back \$2770.

This information can be represented by the following equations.

$$x + 18y = 1510$$

$$x + 36y = 2770$$

- (i) Solve these equations simultaneously to find the values of x and y. 2

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- (ii) How many months will it take Ariana to repay the loan in full? 2

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Question 27 continues on page 20

Question 27 (continued)

- (d) In a small business, the seven employees earn the following wages per week:

\$300, \$490, \$520, \$590, \$660, \$680, \$970.

- (i) Is the wage of \$970 an outlier for this set of data? Justify your answer with calculations. 3

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- (ii) Each employee receives a \$20 pay increase. 1

What effect will this have on the standard deviation?

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Question 27 continues on page 21

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Question 27 (continued)

- (e) A 42 megabyte (MB) file is to be downloaded at a rate of 500 kilobits per second (kbps), where 1 kilobit = 1000 bits.

How long would it take to download this file? Give your answer in minutes and seconds, correct to the nearest second.

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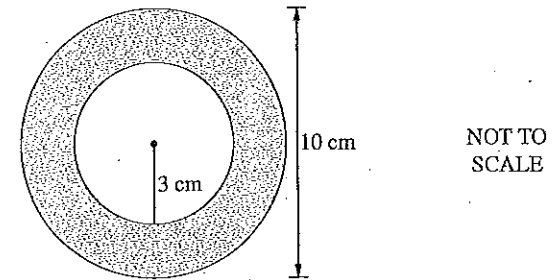
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End of Question 27

Please turn over

Question 28 (15 marks)

- (a) The diagram shows an annulus.



Calculate the area of the annulus.

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Question 28 continues on page 23

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Question 28 (continued)

- (b) The results of two tests are normally distributed. The mean and standard deviation for each test are displayed in the table.

2

	Mathematics	English
\bar{x}	70	75
s	6.5	8

Kristoff scored 74 in Mathematics and 80 in English. He claims that he has performed better in English.

Is Kristoff correct? Justify your answer using appropriate calculations.

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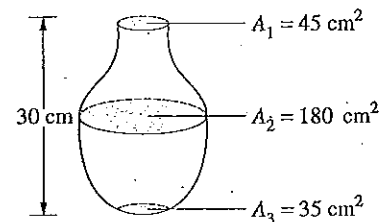
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Question 28 continues on page 24

Question 28 (continued)

- (c) Three equally spaced cross-sectional areas of a vase are shown.

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Use Simpson's rule to find the approximate capacity of the vase in litres.

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- (d) The formula $C = \frac{5}{9}(F - 32)$ is used to convert temperatures between degrees Fahrenheit (F) and degrees Celsius (C).

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Convert 3°C to the equivalent temperature in Fahrenheit.

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Question 28 continues on page 25

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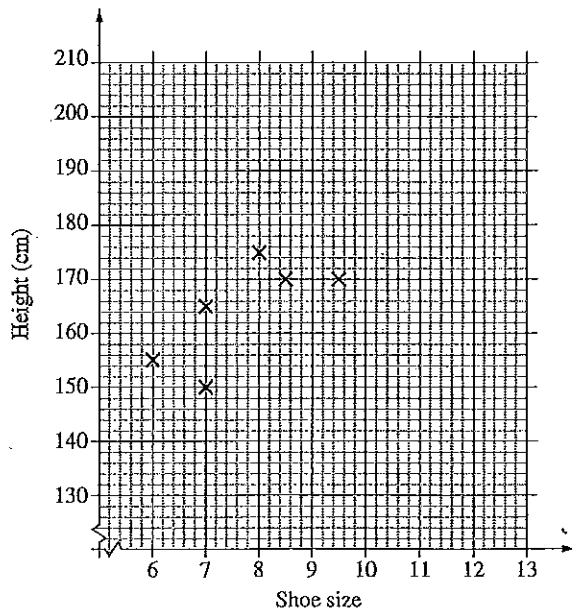
Question 28 (continued)

(e) The shoe size and height of ten students were recorded.

Shoe size	6	7	7	8	8.5	9.5	10	11	12	12
Height	155	150	165	175	170	170	190	185	200	195

(i) Complete the scatter plot AND draw a line of fit by eye.

2



(ii) Use the line of fit to estimate the height difference between a student who wears a size 7.5 shoe and one who wears a size 9 shoe.

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(iii) A student calculated the correlation coefficient to be 1 for this set of data. Explain why this cannot be correct.

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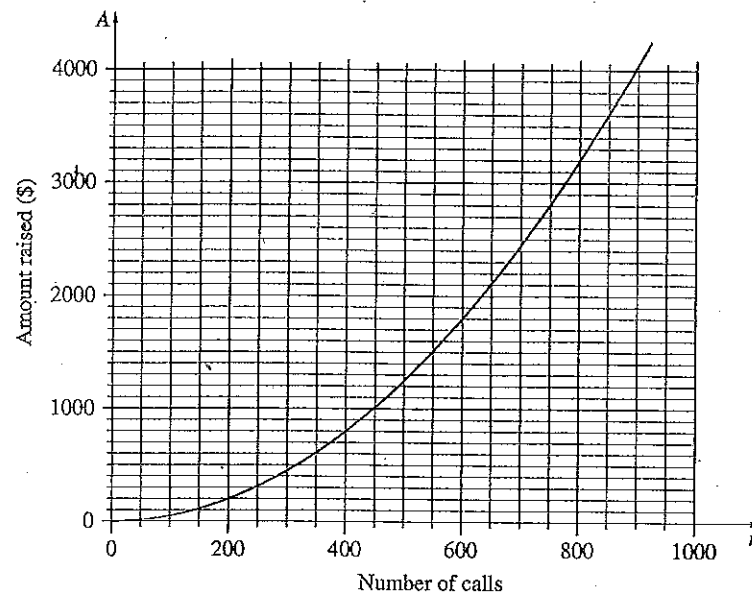
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Question 28 continues on page 26

Question 28 (continued)

(f) A charity seeks to raise money by telephoning people at random from a call centre and asking them to donate.

Over the years, this charity has found that the amount of money raised (\$A) is related to the number of telephone calls made (n). A graph of this relationship is shown.



It costs the charity \$2100 per week to run the call centre. It also costs an average of 50 cents per telephone call.

(i) Write an equation to represent the total cost, C , of running the call centre for a week in which n phone calls are made.

1

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(ii) By graphing this equation on the axes above, determine the number of phone calls the charity needs to make in order to break even.

2

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End of Question 28

Question 29 (15 marks)

- (a) On 20 August, tickets were purchased for \$425 using a credit card. No other purchases were made using this card in August. Simple interest was charged at a rate of 18.4% per annum. There was no interest-free period. The period for which interest was charged included the date of purchase and the date of payment.

What amount was paid when the account was paid in full on 31 August?

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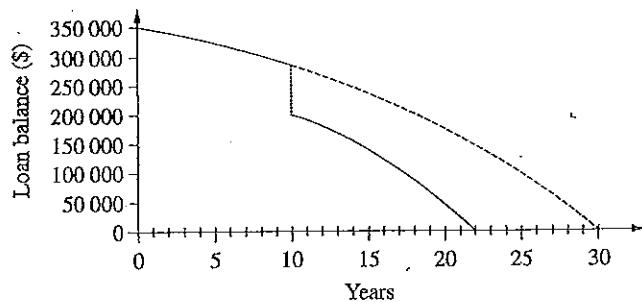
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- (b) Jamal borrowed \$350 000 to be repaid over 30 years, with monthly repayments of \$1880. However, after 10 years he made a lump sum payment of \$80 000. The monthly repayment remained unchanged. The graph shows the balances owing over the period of the loan.



Over the period of the loan, how much less did Jamal pay by making the lump sum payment?

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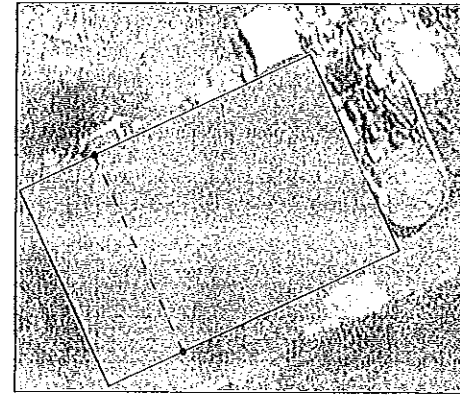
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Question 29 continues on page 28

Question 29 (continued)

- (c) The image shows a rectangular farm shed with a flat roof.



The width of the shed indicated by the dotted line was measured using an online ruler tool, and found to be approximately 12 metres.

- (i) By measurement and calculation, show that the area of the roof of the shed is approximately 216 m².

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- (ii) All the rain that falls onto this roof is diverted into a cylindrical water tank which has a diameter of 3.6 m. During a storm, 5 mm of rain falls onto the roof.

Calculate the increase in the depth of water in the tank due to the rain that falls onto the roof during the storm.

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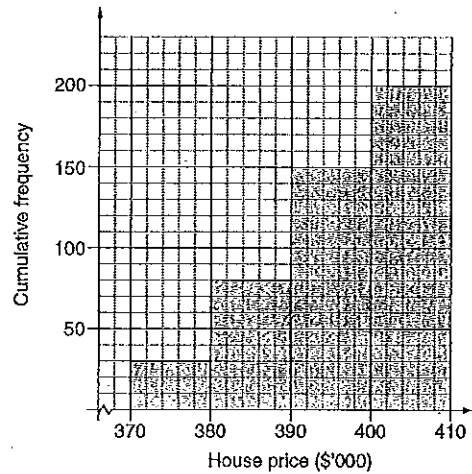
Question 29 continues on page 29

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Question 29 (continued)

- (d) Data from 200 recent house sales are grouped into class intervals and a cumulative frequency histogram is drawn.



- (i) Use the graph to estimate the median house price. 1

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- (ii) By completing the table, calculate the mean house price. 3

Class Centre (\$'000)	Frequency

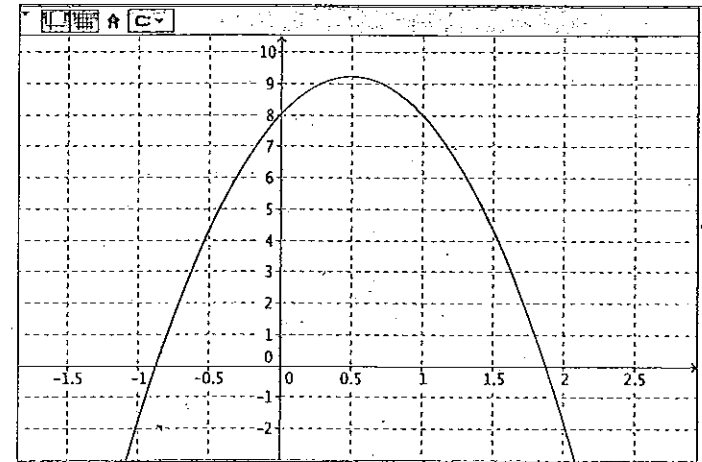
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Question 29 continues on page 30

Question 29 (continued)

- (e) A diver springs upwards from a diving board, then plunges into the water. The diver's height above the water as it varies with time is modelled by a quadratic function. Graphing software is used to produce the graph of this function.

2



Explain how the graph could be used to determine how high above the height of the diving board the diver was when he reached the maximum height.

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End of Question 29

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Question 30 (15 marks)

- (a) The energy consumption of a computer in standby mode is 21 watts. The cost of electricity is 31 cents per kWh.

A school computer room has 20 computers.

How much will the school save by switching off all 20 computers during 11 weeks of school holidays?

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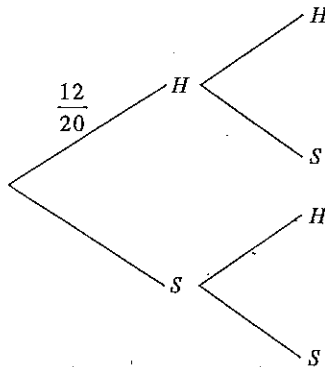
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- (b) On a tray there are 12 hard-centred chocolates (H) and 8 soft-centred chocolates (S). Two chocolates are selected at random. A partially completed probability tree is shown.



What is the probability of selecting one of each type of chocolate?

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Question 30 continues on page 32

Question 30 (continued)

- (c) The table gives the present value interest factors for an annuity of \$1 per period, for various interest rates (r) and numbers of periods (N).

Table of present value interest factors					
$r \backslash N$	Interest rate per period (as a decimal)				
	0.0075	0.0080	0.0085	0.0090	0.0095
70	54.30462	53.43960	52.59397	51.76724	50.95891
71	54.89293	54.00754	53.14226	52.29657	51.46995
72	55.47685	54.57097	53.68593	52.82118	51.97618
73	56.05643	55.12993	54.22502	53.34111	52.47764
74	56.63169	55.68446	54.75957	53.85641	52.97438

- (i) Oscar plans to invest \$200 each month for 74 months. His investment will earn interest at the rate of 0.0080 (as a decimal) per month.

Use the information in the table to calculate the present value of this annuity.

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- (ii) Lucy is using the same table to calculate the loan repayment for her car loan. Her loan is \$21 500 and will be repaid in equal monthly repayments over 6 years. The interest rate on her loan is 10.8% per annum.

Calculate the amount of each monthly repayment, correct to the nearest dollar.

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Question 30 continues on page 33

Question 30 (continued)

- (d) Claire is driving on a motorway at a speed of 110 kilometres per hour and has to brake suddenly. She has a reaction time of 2 seconds and a braking distance of 59.2 metres.

Calculate her stopping distance.

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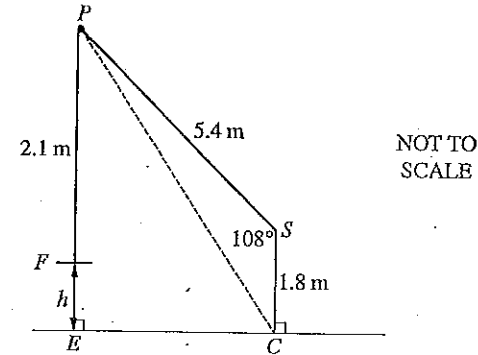
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Question 30 continues on page 34

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Question 30 (continued)

- (e) From point S , which is 1.8 m above the ground, a pulley at P is used to lift a flat object F . The lengths SP and PF are 5.4 m and 2.1 m respectively. The angle PSC is 108° .



- (i) Show that the length PC is 6.197 m, correct to 3 decimal places.

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- (ii) Calculate h , the height of the object above the ground.

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End of paper

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Section II extra writing space

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FORMULAE AND DATA SHEET

Financial Mathematics

Simple Interest

$$I = Prn$$

- P is initial amount
- r is interest rate per period, expressed as a decimal
- n is number of periods

Compound interest

$$A = P(1 + r)^n$$

- A is final amount
- P is initial amount
- r is interest rate per period, expressed as a decimal
- n is number of compounding periods

Present value and future value

$$PV = \frac{FV}{(1 + r)^n}, \quad FV = PV(1 + r)^n$$

- r is interest rate per period, expressed as a decimal
- n is number of compounding periods

Straight-line method of depreciation

$$S = V_0 - Dn$$

- S is salvage value of asset after n periods
- V_0 is initial value of asset
- D is amount of depreciation per period
- n is number of periods

Declining-balance method of depreciation

$$S = V_0(1 - r)^n$$

- S is salvage value of asset after n periods
- V_0 is initial value of asset
- r is depreciation rate per period, expressed as a decimal
- n is number of periods

Data Analysis

Mean of a sample

$$\bar{x} = \frac{\text{sum of scores}}{\text{number of scores}}$$

z-score

For any score x ,

$$z = \frac{x - \bar{x}}{s}$$

- \bar{x} is mean
- s is standard deviation

Outlier(s)

score(s) less than $Q_L - 1.5 \times IQR$

or

score(s) more than $Q_U + 1.5 \times IQR$

- Q_L is lower quartile
- Q_U is upper quartile
- IQR is interquartile range

Least-squares line of best fit

$$y = \text{gradient} \times x + \text{y-intercept}$$

$$\text{gradient} = r \times \frac{\text{standard deviation of } y \text{ scores}}{\text{standard deviation of } x \text{ scores}}$$

$$\text{y-intercept} = \bar{y} - (\text{gradient} \times \bar{x})$$

- r is correlation coefficient
- \bar{x} is mean of x scores
- \bar{y} is mean of y scores

Normal distribution

- approximately 68% of scores have z-scores between -1 and 1
- approximately 95% of scores have z-scores between -2 and 2
- approximately 99.7% of scores have z-scores between -3 and 3

Spherical Geometry

Circumference of a circle

$$C = 2\pi r \text{ or } C = \pi D$$

r is radius

D is diameter

Arc length of a circle

$$l = \frac{\theta}{360} 2\pi r$$

r is radius

θ is number of degrees in central angle

Radius of Earth

(taken as) 6400 km

Time differences

For calculation of time differences using longitude:
15° = 1 hour time difference

Area

Circle

$$A = \pi r^2$$

r is radius

Sector

$$A = \frac{\theta}{360} \pi r^2$$

r is radius

θ is number of degrees in central angle

Annulus

$$A = \pi(R^2 - r^2)$$

R is radius of outer circle

r is radius of inner circle

Trapezium

$$A = \frac{h}{2}(a + b)$$

h is perpendicular height

a and b are the lengths of the parallel sides

Area of land and catchment areas

unit conversion: 1 ha = 10 000 m²

Surface Area

Sphere

$$A = 4\pi r^2$$

r is radius

Closed cylinder

$$A = 2\pi r^2 + 2\pi rh$$

r is radius

h is perpendicular height

Volume

Prism or cylinder

$$V = Ah$$

A is area of base

h is perpendicular height

Pyramid or cone

$$V = \frac{1}{3}Ah$$

A is area of base

h is perpendicular height

Volume and capacity

unit conversion: 1 m³ = 1000 L

Approximation Using Simpson's Rule

Area

$$A \approx \frac{h}{3}(d_f + 4d_m + d_l)$$

h is distance between successive measurements

d_f is first measurement

d_m is middle measurement

d_l is last measurement

Volume

$$V \approx \frac{h}{3}(A_L + 4A_M + A_R)$$

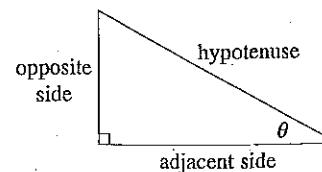
h is distance between successive measurements

A_L is area of left end

A_M is area of middle

A_R is area of right end

Trigonometric Ratios



$$\sin \theta = \frac{\text{opposite side}}{\text{hypotenuse}}$$

$$\cos \theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

$$\tan \theta = \frac{\text{opposite side}}{\text{adjacent side}}$$

Sine rule

In $\triangle ABC$,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Area of a triangle

In $\triangle ABC$,

$$A = \frac{1}{2}ab \sin C$$

Cosine rule

In $\triangle ABC$,

$$c^2 = a^2 + b^2 - 2ab \cos C$$

or

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Units of Memory and File Size

1 byte = 8 bits

1 kilobyte = 2¹⁰ bytes = 1024 bytes

1 megabyte = 2²⁰ bytes = 1024 kilobytes

1 gigabyte = 2³⁰ bytes = 1024 megabytes

1 terabyte = 2⁴⁰ bytes = 1024 gigabytes

Blood Alcohol Content Estimates

$$BAC_{\text{male}} = \frac{10N - 7.5H}{6.8M}$$

or

$$BAC_{\text{female}} = \frac{10N - 7.5H}{5.5M}$$

N is number of standard drinks consumed

H is number of hours of drinking

M is person's mass in kilograms

Distance, Speed and Time

$$D = ST, \quad S = \frac{D}{T}, \quad T = \frac{D}{S}$$

$$\text{average speed} = \frac{\text{total distance travelled}}{\text{total time taken}}$$

$$\text{stopping distance} = \left\{ \begin{array}{l} \text{reaction-time} \\ \text{distance} \end{array} \right\} + \left\{ \begin{array}{l} \text{braking} \\ \text{distance} \end{array} \right\}$$

Probability of an Event

The probability of an event where outcomes are equally likely is given by:

$$P(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

Straight Lines

Gradient

$$m = \frac{\text{vertical change in position}}{\text{horizontal change in position}}$$

Gradient-intercept form

$$y = mx + b$$

m is gradient

b is y-intercept

2015 Higher School Certificate Solutions Mathematics General 2

SECTION I

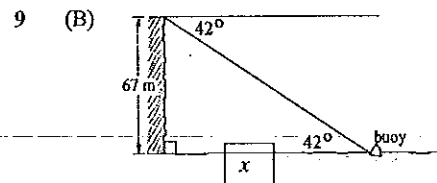
Summary

1 D	6 C	11 D	16 D	21 C
2 A	7 B	12 C	17 C	22 D
3 B	8 C	13 A	18 A	23 D
4 A	9 B	14 C	19 D	24 B
5 B	10 A	15 B	20 A	25 C

SECTION I

- 1 (D) $1\ 560\ 200 = 1\ 600\ 000$ (2 s.f.)
 $= 1.6 \times 10^6$ (2 s.f.)
- 2 (A) $4x + 3y - x - 5y = 4x - x + 3y - 5y$
 $= 3x - 2y$
- 3 (B) Deductions $= \$180.93 + \85.70
 $+ \$21.40 + \38.15
 $= \$326.18$
 \therefore Net Pay $= \$952.25 - \326.18
 $= \$626.07$
- 4 (A) The data is categorical and also the values can be compared on a scale therefore it is also ordinal
- 5 (B) $y = x^3$ is a cubic curve. $y = x^3 + 1$ is the same shape translated vertically 1 unit. \therefore B.
- 6 (C) 40 is the end of the first quartile (25%). 60 is the median (50%). The gap is 25%.
- 7 (B) $\angle AOB = (360^\circ - 300^\circ) + 51^\circ$
 $= 111^\circ$

8 (C) $V = \frac{1}{3} Ah$
 $= \frac{1}{3} \times 35^2 \times 22$
 $= 8983 \text{ m}^3$ (nearest m^3).



$$\tan 42^\circ = \frac{67}{x}$$

$$x \tan 42^\circ = 67$$

$$x = \frac{67}{\tan 42^\circ}$$

$$= 74 \text{ m (nearest m)}$$

- 10 (A) Only A and B are show declining balance depreciation. Testing a value say $n = 10$ in the depreciation formula.
 $S = V_0(1-r)^n$
 $= 56000(1-0.08)^{10}$
 $= \$24326$
 Only A could pass through (10, 24 326).

11 (D) $3x^0 + 5x = 3(1) + 5x$
 $= 3 + 5x$

12 (C) % Error $= \pm \frac{\text{Error}}{\text{Value}} \times 100$
 $= \pm \frac{0.5}{49} \times 100$
 $= \pm 1\%$ (1 s.f.)

13 (A) Gradient $= -\frac{2}{1} = -2$
 y-intercept $= 2$
 $y = -2x + 2$

14 (C) Difference in longitude $= 131^\circ - 18^\circ$
 $= 113^\circ$
 $360^\circ = 24 \text{ hours}$
 $1^\circ = \frac{24 \times 60}{360} \text{ minutes}$
 $= 4 \text{ minutes}$
 $113^\circ = 4 \times 113$
 $= 452 \text{ minutes}$

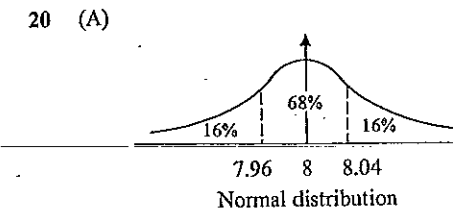
15 (B) $112\% = 449$
 $1\% = \frac{449}{112}$
 $100\% = \frac{449}{112} \times 100$
 $= 400.8928\dots$
 $= \$401$ (nearest \$).

16 (D) From the question the probability is fixed at $\frac{7}{10}$ to win.
 $P(\text{WW}) = \frac{7}{10} \times \frac{7}{10}$

17 (C) $r = 4\%$ p.a.
 $= 1\%$ per quarter
 $n = 5 \text{ years}$
 $= 5 \times 4$
 $= 20 \text{ quarters}$
 $A = P(1+r)^n$
 $60\ 000 = P(1+0.01)^{20}$
 $P = \frac{60\ 000}{1.01^{20}}$
 $= \$49\ 173$

18 (A) This combination is unordered therefore from a group of 5, choose 3:
 ${}^5C_3 = \frac{5 \times 4 \times 3}{3 \times 2 \times 1}$
 $= 10$

19 (D) At 45:
 Life expectancy $= 45 + 34$
 $= 79$
 At 65:
 Life expectancy $= 65 + 19.8$
 $= 84.8$
 Difference $= 84.8 - 79.0$
 $= 5.8 \text{ years}$



Thus 68% of the diameters lie between 7.96 and 8.04. For the remainder:
 $\frac{100 - 68}{2} = 16\%$

This is the percentage of diameters that are less than 7.96.

- 21 (C) There are 3 choices for the first digit. That is 3, 5 or 7.
 Once selected, there are 3 choices left for the next digit. Finally 1 choice for the last digit.

Number of selections is:
 $3 \times 3 \times 2 \times 1 = 18$

22 (D) Area $= \frac{1}{2} ab \sin C$
 $250 = \frac{1}{2} \times 30 \times x \times \sin 44^\circ$
 $x = \frac{250}{15 \sin 44}$
 $= 24 \text{ cm}$ (nearest cm).

23 (D) Number of standard drinks:
 $3 \times 0.9 + 2 \times 1.5 = 5.7$
 $BAC_{\text{female}} = \frac{10N - 7.5H}{5.5M}$
 $= \frac{10(5.7) - 7.5 \times 5}{5.5 \times 62}$
 $= 0.057$ (3 s.f.).

24 (B) $\frac{2x}{3} - 4 = \frac{5x}{2} + 1$
 Add 4 to both sides:
 $\frac{2x}{3} = \frac{5x}{2} + 5$

25 (C) Price after Multi-policy discount:
 $870 \times (1 - 0.15) = \739.50
 Price after No-claim bonus:
 $739.50 \times (1 - 0.2) = \591.60
 Price after Combined Policy discount:
 $591.60 - 50 = \$541.60$

SECTION II

Question 26

(a) $\frac{18}{x} = \frac{4}{26}$
 $26 \times 18 = 4x$
 $x = \frac{26 \times 18}{4}$
 $x = 117$ chickens.

(b) Dosage = $\frac{35 \times 3150}{70}$
 $= 1575$ mg
 Number of tablets = $\frac{1575}{525} = 3$.

(c) The radius of the earth is 6400km.
 Distance = $\frac{\theta}{360} \times 2\pi r$
 $= \frac{40}{360} \times 2\pi \times 6400$
 $= 4468$ km.

(d) $A = P(1+r)^n$
 $= 320(1+0.029)^5$
 $= \$369.17$.

(e) (i) Relative frequency of a red:
 $= 1 - (0.32 + 0.13 + 0.14 + 0.24)$
 $= 1 - 0.83$
 $= 0.17$.

(ii) $P(\text{not black}) = 1 - P(\text{black})$
 $= 1 - 0.14$
 $= 0.86$.

(f) The radius of the earth is 6400km.
 The surface area is $4\pi r^2$.
 $A = 71\% \text{ of } 4\pi r^2$
 $= \frac{71}{100} \times 4 \times \pi \times 6400^2$
 $= 365450163.7 \text{ km}^2$
 $= 3.65 \times 10^8 \text{ km}^2$.

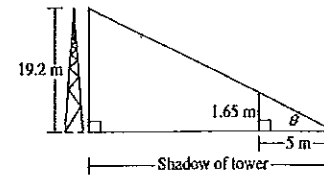
(g) Excess Call Charges = $561 - 550$
 $= \$11$
 1 GB = 1024 MB
 Excess Data Charges
 $= 1.7 \times (1024 - 500) \times 0.0293$
 $= \$36.36$
 Bill = $\$11 + \$36.36 + \$49$
 $= \$96.36$ (nearest cent).

Question 27

(a) *Method 1:*
 Using similar triangles:
 $\frac{1.65}{5} = \frac{19.2}{x}$
 $1.65 \times x = 19.2 \times 5$
 $x = \frac{19.2 \times 5}{1.65}$
 $= 58.18$
 $= 58.2\text{m}$ (1 d.p.).

OR

Method 2:



$\tan \theta = \frac{1.65}{5}$
 $\theta = 18.26 \dots^\circ$
 $\tan \theta = \frac{19.2}{\text{shadow}}$
 $\text{shadow} = \frac{19.2}{\tan \theta}$
 $= 58.18$
 $= 58.2\text{m}$ (1 d.p.).

(b) 12 hours = $12 \times 60 = 720$ mins
 $2400\text{ml} \times 15 = 36000$ drops

Rate = $\frac{\text{number of drops}}{\text{time}}$
 $= \frac{36000}{720}$
 $= 50$ drops per minute.

(c) (i) Solving simultaneously:
 $x + 18y = 1510$ ①
 $x + 36y = 2770$ ②
 ② - ①
 $18y = 1260$
 $y = \frac{1260}{18}$
 $y = 70$
 Substitute in ①
 $x + 18(70) = 1510$
 $x = 250$
 $\therefore x = 250, y = 70$

(c) (ii) Let n be the number of months to pay off the loan.
 $x + ny = 4800$
 Using the values from part (i)

$x + ny = 4800$
 $250 + 70n = 4800$
 $70n = 4550$
 $n = 65$
 \therefore it will take 65 months.

(d) (i) $IQR = Q_U - Q_L$
 $= 680 - 490$
 $= 190$
 For an Upper Outlier:
 Outlier $> Q_U + 1.5 \times IQR$
 $> 680 + 1.5 \times 190$
 > 965
 Since $970 > 965$ then 970 is an outlier.

(ii) No effect.

An increase of \$20 does not alter the dispersion of the data in any way. Standard deviation is a measure of dispersion. Hence there is no effect on the standard deviation.

(e) 1 MB = 1024 kB
 1 kB = 1024 bytes
 1 byte = 8 bits
 $42 \text{ MB} = 42 \times 1024 \text{ kB}$
 $= 42 \times 1024 \times 1024 \text{ bytes}$
 $= 42 \times 1024^2 \times 8 \text{ bits}$
 $= 352\,321\,536 \text{ bits}$
 $= 352\,321.536 \text{ kilobits}$
 Time = $\frac{\text{data (kb)}}{\text{download speed (kbps)}}$
 $= \frac{352\,321.536}{500}$
 $= 704.643 \dots \text{ seconds}$
 $= 11 \text{ min } 45 \text{ secs}$ (nearest sec).

Question 28

(a) $A = \pi(R^2 - r^2)$
 $= \pi(5^2 - 3^2)$
 $= 16\pi$
 $= 50.27\text{cm}^2$ (2 d.p.).

- (b) Since the results have different means and standard deviations, consider the z scores of each.

$$\text{Using the formula } z = \frac{x - \bar{x}}{s}$$

$$z_{\text{Maths}} = \frac{74 - 70}{6.5} = 0.6153\dots$$

$$z_{\text{English}} = \frac{80 - 75}{8} = 0.625$$

The z score for English is greater than the z score for Maths.

∴ Kristoff performed slightly better in English.

(c) $V = \frac{h}{3}(A_L + 4A_M + A_R)$

where $h = 30 \div 2 = 15$

$$V = \frac{15}{3}(45 + 4 \times 180 + 35) = 4000 \text{ cm}^3 = 4000 \text{ mL} = 4 \text{ L}$$

(d) $C = \frac{5}{9}(F - 32)$

$$3 = \frac{5}{9}(F - 32)$$

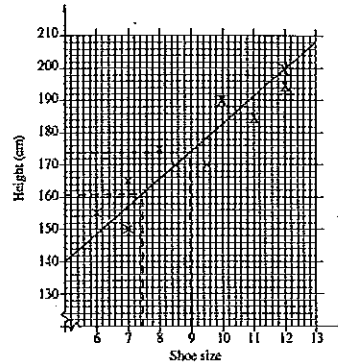
$$27 = 5(F - 32)$$

$$27 = 5F - 160$$

$$5F = 187$$

$$F = 37.4^\circ$$

- (e) (i) When doing a line of best fit by eye, there should be an equal number of points above and below the line. [See graph in next column.]



- (ii) From the graph:

Shoe 7.5 gives a height of 162 cm.

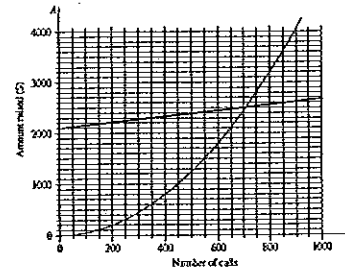
Shoe 9 gives a height of 174 cm.

Height difference = $174 - 162 = 12$ cm.

- (iii) The plotted points do not form an exact straight line. So the correlation coefficient cannot be 1.

(f) (i) $C = 0.5n + 2100$

n	0	200	400
C	2100	2200	2300



From graph we need the point of intersection. This gives $n = 700$ calls.

Question 29

(a) $I = PRN$
 Interest = $425 \times \frac{18.4}{100} \times \frac{12}{365} = \2.57 (nearest cent)

Amount Paid = $\$425 + \$2.57 = \$427.57$

- (b) Without the \$80 000 payment, it takes 30 years to pay the loan:
 $30 \times 12 \times \$1880 = \$676\,800$

With the \$80 000 payment, it takes 22 years to pay the loan:
 $22 \times 12 \times \$1880 + \$80\,000 = \$576\,320$

Amount saved by Jamal by making the lump sum payment:
 $\$676\,800 - \$576\,320 = \$100\,480$

- (c) (i) 4 cm on map represents 12 m. So 1 cm on map represents 3 m. Length of the shed on map = 6 cm. Area = $4 \text{ cm} \times 6 \text{ cm}$ (on map) = $12 \text{ m} \times 18 \text{ m}$ (using scale) = 216 m^2 .

- (ii) Volume of water on the roof:
 $V = A \times h = 216 \text{ m}^2 \times 5 \text{ mm} \div 1000 = 1.08 \text{ m}^3$

That same volume enters the cylindrical tank:

$$V = \pi r^2 h$$

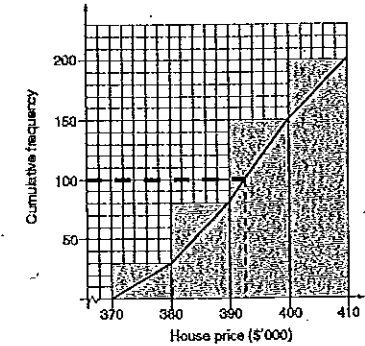
$$1.08 = \pi \times 1.8^2 \times h$$

$$h = \frac{1.08}{1.8^2 \pi}$$

$$= 0.106103\dots \text{ m}$$

$$= 106 \text{ mm (nearest mm)}$$

- (d) (i)



From the graph, the median is \$393 000.

- (ii) From the graph, each increase is the frequency for that class centre.

Class Centre (x)	Frequency (f)	fx
375	30	11 250
385	50	19 250
395	70	27 650
405	50	20 250
200		78 400

$$\text{Mean} = \frac{78\,400}{200}$$

$$= 392$$

∴ Mean house price = \$392 000.

- (e) For the starting height: For $t = 0$, $h = 8 \text{ m}$ (from graph)

For the highest point: For $t = 0.5$, $h = 9.2 \text{ m}$ (from graph)

∴ Height above board = $9.2 - 8 = 1.2 \text{ m}$.

Question 30

- (a) 20 computers for 11 weeks at 21 watts per hour:
 $20 \times 11 \times 7 \times 24 \times 21 = 776\,160$ watts
 $= 776.160 \text{ kW}$
 $776.160 \times 0.31 = \$240.61$ (nearest cent)

- (b) The probability of one of each type of chocolate is:

$$P(\text{HS or SH}) = P(\text{HS}) + P(\text{SH})$$

$$= \frac{12}{20} \times \frac{8}{19} + \frac{8}{20} \times \frac{12}{19}$$

$$= \frac{48}{95}$$

(c) (i) $PV = 200 \times 55.68446$
 $= \$11136.89$

(ii) $10.8\% \text{ p.a.} = \frac{10.8\%}{12}$
 $= 0.9\% \text{ per month}$
 $= 0.009 \text{ per month}$
 $6 \text{ years} = 6 \times 12$
 $= 72 \text{ months}$

Let M be the monthly payment.
 Using the table with $r = 0.0090$
 and $N = 72$, the present value
 of \$1 is 52.82118.

$$52.82118 \times M = 21500$$

$$M = \frac{21500}{52.82118}$$

$$= \$407 \text{ (nearest \$)}$$

(d) $110 \text{ km per hour} = 110\,000 \text{ m per hour}$
 $= \frac{110\,000}{60} \text{ m per minute}$
 $= \frac{110\,000}{60 \times 60} \text{ m per second}$
 $= 30.56 \text{ m/s}$

$$D = \left[\frac{\text{reaction time}}{\text{distance}} \right] + \left[\frac{\text{braking}}{\text{distance}} \right]$$

$$= 30.56 \times 2 + 59.2$$

$$= 120.32 \text{ m}$$

(e) (i) Using the cosine rule:
 $PC^2 = 5.4^2 + 1.8^2 - 2 \times 5.4 \times 1.8 \times \cos 108^\circ$
 $= 38.407\dots$
 $PC = 6.197 \text{ m (3 d.p.)}$

(ii) *Method 1:*
 $\frac{\sin \theta}{5.4} = \frac{\sin 108^\circ}{6.197}$
 $\sin \theta = \frac{5.4 \times \sin 108^\circ}{6.197}$
 $\theta = 55.96957\dots^\circ$
 $\approx 56^\circ$

$$\angle PCE = 90^\circ - 56^\circ$$

$$= 34^\circ$$

$$\sin 34^\circ = \frac{PE}{6.197}$$

$$PE = 6.197 \times \sin 34^\circ$$

$$= 3.4653\dots$$

$$\approx 3.5$$

$$h = PE - PF$$

$$= 3.5 - 2.1$$

$$= 1.4 \text{ m (1 d.p.)}$$

OR

Method 2:
 $\cos \angle PCS = \frac{6.197^2 + 1.8^2 - 5.4^2}{2 \times 6.197 \times 1.8}$
 $= 0.559536\dots$

$$\angle PCS = 55.976258\dots^\circ$$

$$\angle PCE = 90^\circ - 55.976258\dots^\circ$$

$$= 34.023741\dots^\circ$$

$$\frac{PE}{PC} = \sin \angle PCE$$

$$\frac{PE}{6.197} = \sin 34.0237$$

$$PE = 6.197 \sin 34.0237$$

$$= 3.467$$

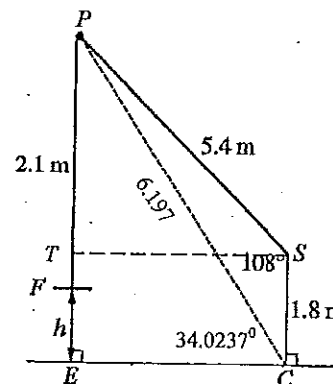
$$h = PE - PF$$

$$= 3.467\dots - 2.1$$

$$= 1.367\dots$$

$$= 1.4 \text{ m (1 d.p.)}$$

OR



Method 3:
 Construct TS parallel to EC as shown.

$$\frac{TS}{PS} = \cos \angle PST$$

$$\frac{TS}{5.4} = \cos \angle (108^\circ - 90^\circ)$$

$$TS = 5.4 \times \cos 18^\circ$$

$$= 5.1357\dots$$

$$EC = 5.1357\dots$$

$$PE^2 = PC^2 - EC^2$$

$$= 6.197^2 - 5.1357^2 \text{ using (i)}$$

$$= 3.468\dots$$

$$h = PE - 2.1$$

$$= 3.468 - 2.1$$

$$= 1.368\dots$$

$$= 1.4 \text{ m (1 d.p.)}$$

OR

Method 4:
 Construct TS parallel to EC as shown.

$$\frac{PT}{PS} = \sin \angle PST$$

$$\frac{PT}{5.4} = \sin \angle (108^\circ - 90^\circ)$$

$$PT = 5.4 \sin 18^\circ$$

$$= 1.66869\dots$$

$$PE = PT + SC$$

$$= 1.66869\dots + 1.8$$

$$= 3.46869\dots$$

$$h = PE - PF$$

$$= 3.46869\dots - 2.1$$

$$= 1.3686\dots$$

$$= 1.4 \text{ m (1 d.p.)}$$

End of Mathematics solutions