Student

BRIGIDINE COLLEGE RANDWICK

PRELIMINARY EXTENSION 1 MATHEMATICS

HALF YEARLY

2009

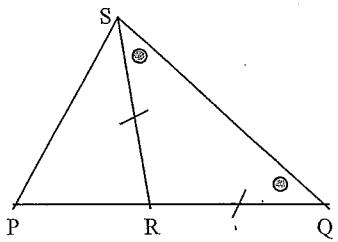
(TIME - 1 HOUR)

Directions to candidates

* *Put your **name at the top** of this paper and on each of the 3 **sections** that are to be collected.*
* *All 3 questions are to be attempted.*
* *All 3 questions are of equal value.*
* *All questions are to be answered on separate pages and will be collected in separate bundles at the end of this exam.*
* *Use PEN to show all necessary working in every question.*
* *Full marks may not be awarded for careless or badly arranged work.*
* *Diagrams are not to scale unless otherwise stated.*

Question 1

a. Which of the following best describes this figure? **1**



A. PRS is a triangle. RP is produced to Q such that QR = RS. QS is joined.

B. PQS is a triangle. Point R is marked on PQ such that ∠RPS = ∠RQS. RS is joined.

C. PRS is a triangle. PR is produced to Q such that ∠RQS = ∠RSQ. QS is joined.

D. PQS is a triangle. R is marked as the midpoint of PQ and RS is joined.

b. Graph the function $2y - 6 = 0$ **1**

c. Consider the function $y = h(x)$. Answer the following question by referring to the incomplete table below.

$y = h(x)$

x	-4	-2	-1	3	4	9
y	5	-3	-6	7	m	0

Find the value of m if $y = h(x)$ is an

i. odd function. **1**

ii. even function. **1**

d. ABC is a triangle in which angle $C = 15^0$, angle $A = 45^0$ and BC = 10 cm. Find side AC in simplest surd form. **2**

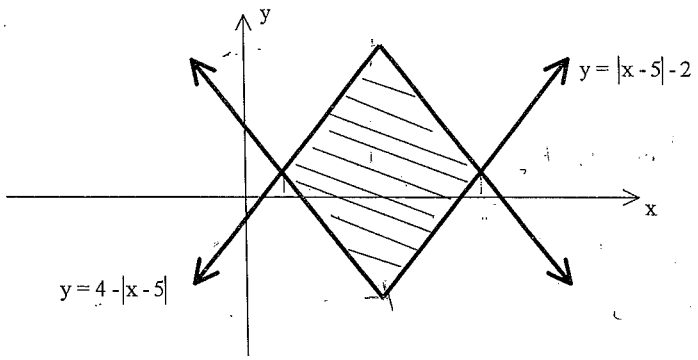
e. If $f(x) = \frac{x^2+3}{2}$ and $g(x) = \sqrt{2x-3}$ find g(a) and hence find f(g(a)). **2**

f. i. Show that $\frac{3x+5}{x+2} = 3 - \frac{1}{x+2}$ **1**

ii. Find the domain and range of $f(x) = \frac{3x+5}{x+2}$ **2**

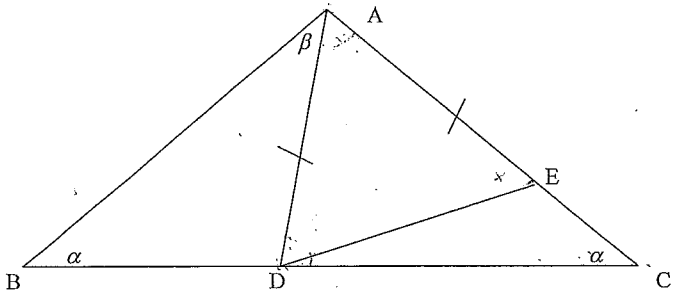
iii. Hence sketch the graph of $f(x) = \frac{3x+5}{x+2}$ **2**

g. Find the area of the shaded region below **3**



Question 2 (Start a new page)

a.



In the diagram above, triangle ABC is isosceles where angle ABC = angle ACB = α
The points D and E lie on BC and AC respectively, so that AD = AE , as shown. Let angle BAD = β

i. State why $\angle ADC = \alpha + \beta$. **1**

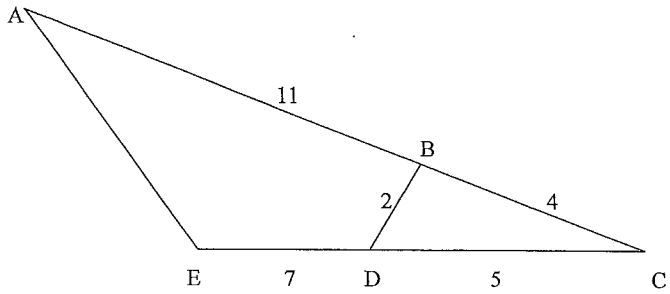
ii. Find $\angle DAC$ in terms of α and β. **1**

iii. Hence or otherwise find $\angle EDC$ in terms of β. **2**

b. i. Prove that $\dfrac{\text{Cos}\,\theta}{1+\text{Sin}\,\theta} + \text{Tan}\,\theta = \text{Sec}\,\theta$ **2**

ii. Hence solve $\dfrac{\text{Cos}\,\theta}{1+\text{Sin}\,\theta} + \text{Tan}\,\theta = 2$ for $-180^0 \le \theta \le 0^0$. **2**

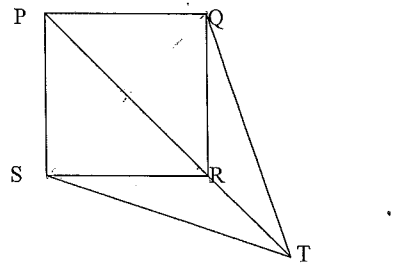
c. The diagram below shows two triangles with sides as marked.



i. Show that triangles ACE and DBC are similar. **3**

ii. Find the length of AE. **1**

d. In the following diagram PQRS is a square with sides x cm and PR is produced to T such that $RT = \frac{1}{2}PR$.



i. Show that PQTS is a kite.(hint : use congruence) **3**

ii. Prove $RT = \dfrac{x\sqrt{2}}{2}$. **1**

Question 3 (Start a new page)

a. For what values of x would it be possible to sketch the curve $y = \sqrt{(3-x)(x+4)}$ **2**

b. The probability that Helen will pass a Maths test is 0.7, an English test is 0.6, and a Science test is 0.8. When she sits for the three tests , find the probability that Helen passes exactly one test of the three tests. **2**

c. In a raffle, 30 tickets are sold and there are two prizes. What is the probability that someone buying 5 tickets wins at least one prise? **2**

d. A chessboard has 32 black squares and 32 white squares. Fredrico chooses three different squares at random.

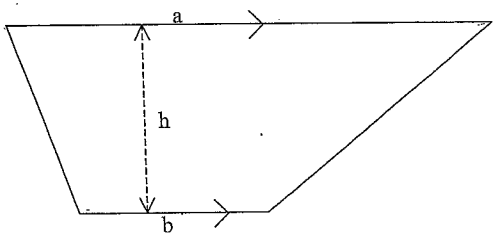
i. What is the probability that Fredrico chooses three white squares? **2**

ii. What is the probability that the three squares Fredrico chooses are the same colour? **1**

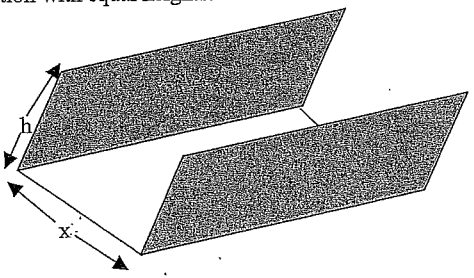
iii. What is the probability that the three squares that Fredrico chooses are not the same colour? **1**

e. Prove that the area of the trapezium below is given by $A = \frac{1}{2} \times h \times (a + b)$. **2**

(hint : join a diagonal)



f. A metal gutter open at the top and ends , is bent up from material 30cm wide to form a rectangular cross section with equal heights.



i. If the base of the gutter is x cm wide, find the height ,h, of the gutter in terms of x. **1**

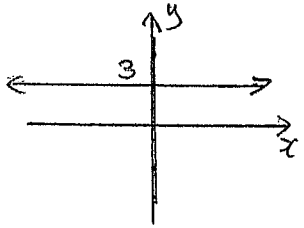
ii. Show that the area of the gutter cross section is given by $A = 15x - \frac{x^2}{2}$ **1**

iii. Find the maximum value of A of the gutter. **2**

Yr 11 Ex 11 2 unit (solutions)

Q1 a) C

b)

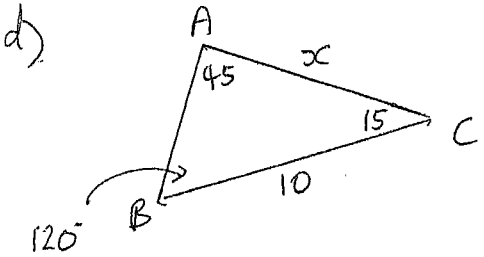


$2y-6=0$
$2y=6$
$y=3$

c) i) -5

ii) 5

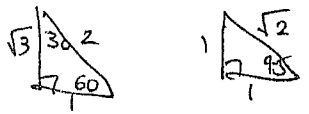
d)



$$\frac{x}{\sin 120} = \frac{10}{\sin 45}$$

$\sin 120 = \sin 60$

$$\frac{x}{\sin 60} = \frac{10}{\sin 45}$$



$$\frac{x}{\frac{\sqrt{3}}{2}} = \frac{10}{\frac{1}{\sqrt{2}}}$$

$$\frac{2x}{\sqrt{3}} = \frac{\sqrt{2}\,10}{1}$$

$$2x = 10\sqrt{6}$$

$$x = 5\sqrt{6}$$

e) $g(a) = \sqrt{2a-3}$

$$f(g(a)) = \frac{(\sqrt{2a-3})^2 + 3}{2}$$

$$= \frac{2a-3+3}{2}$$

$$= \frac{2a}{2} = a$$

f i) $$\frac{3x+5}{x+2} = \frac{3x+6-1}{x+2}$$

$$= \frac{3(x+2)-1}{x+2}$$

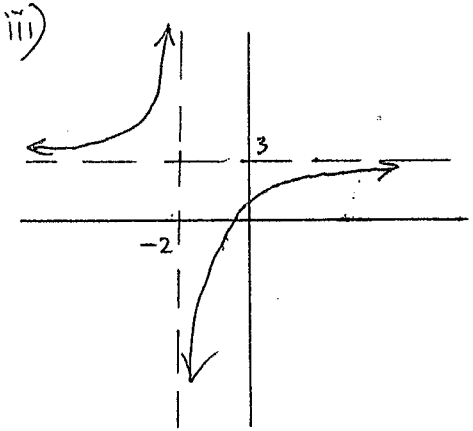
$$= \frac{3(x+2)}{x+2} - \frac{1}{x+2}$$

$$= 3 - \frac{1}{x+2}$$

ii) $D: x \neq -2$

$R: y \neq 3$

iii)



g) intersection:

$|x-5| - 2 = 4 - |x-5|$

$2|x-5| = 6$

$|x-5| = 3$

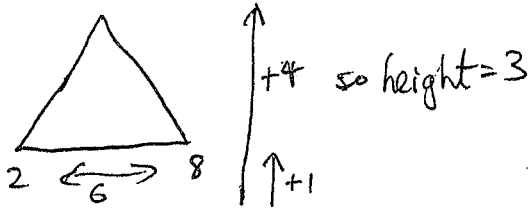
$x-5 = 3$ or $x-5 = -3$

$x = 8$ or $x = 2$

so $y = 1$ $y = 1$

$(8,1)$ $(2,1)$

∴ Area of top Δ



$A = \frac{1}{2} \times 6 \times 3 = 9$

total area $= 9 \times 2 = 18$ units^2

Q2a) i) ext ∠ Δ = sum int opp ∠'s

ii) $180 = (\alpha+\beta) + (\alpha) + \angle DAC$

$\angle DAC = 180 - 2\alpha - \beta$

iii) $\angle ADE = \angle AED$ (isos Δ)

$\therefore \angle DAC + 2 \times \angle ADE = 180$ (∠ sum Δ)

$180 - 2\alpha - \beta + 2 \times \angle ADE = 180$

$$\angle ADE = \frac{2\alpha+\beta}{2} = \alpha + \frac{\beta}{2}$$

$\angle ADC = \angle ADE + \angle EDC$

$\alpha + \beta = \alpha + \frac{\beta}{2} + \angle EDC$

$$\angle EDC = \frac{\beta}{2}$$

b) i) $$\frac{\cos\theta}{1+\sin\theta} + \tan\theta = \sec\theta$$

$$\frac{\cos\theta}{1+\sin\theta} + \frac{\sin\theta}{\cos\theta}$$

$$\frac{\cos^2\theta + \sin\theta(1+\sin\theta)}{\cos\theta(1+\sin\theta)}$$

$$\frac{\cos^2\theta + \sin\theta + \sin^2\theta}{\cos\theta(1+\sin\theta)}$$

$$\frac{1+\sin\theta}{\cos\theta(1+\sin\theta)}$$

$$= \frac{1}{\cos\theta}$$

$= \sec\theta = RHS$

ii) $\sec\theta = 2$

$$\frac{1}{\cos\theta} = 2$$

$$\frac{1}{2} = \cos\theta$$

$\theta = 60°$

$\sin\theta$ $-180 \leq \theta \leq 0$
(3rd ; 4th quad)

$\theta = -60°$

c) i) $\angle C$ is common

$\frac{BC}{EC} = \frac{DC}{AC}$

$\frac{4}{12} = \frac{5}{15}$

$\frac{1}{3} = \frac{1}{3}$

$\therefore$ 2 sides in ratio and included $\angle$ = $\equiv$

$\therefore \triangle ACE \,|||\, \triangle DBC$

ii) $\frac{AE}{2} = \frac{15}{5}$

$AE = 6$

d) i) (S) $PS = PQ$ (sides of a square)

(A) $\angle SPR = 45° = \angle QPR$ (diags of a square bisect $\angle$'s)

(S) PT common

$\therefore$ by SAS $\triangle SPT \equiv \triangle QPT$

$\therefore PS = PQ$ and $ST = QT$ (matching sides of congruent $\triangle$'s)

$\therefore$ PQTS = kite (a pair of adjacent sides =)

ii) $PR^2 = PS^2 + SR^2$

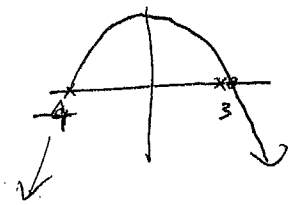
$PR^2 = x^2 + x^2$

$PR^2 = 2x^2$

$RT = \frac{1}{2} PR$

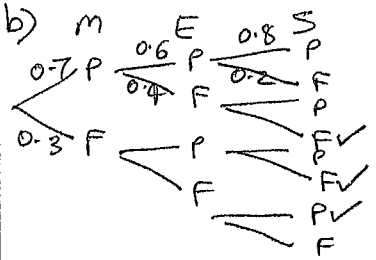
so $RT = \frac{1}{2} \times \sqrt{2}x = \frac{\sqrt{2}x}{2}$

a) $(3-x)(x+4) \geq 0$



$-4 \leq x \leq 3$

b)



$(0.7 \times 0.4 \times 0.2) + (0.3 \times 0.6 \times 0.2)$

$+ (0.3 \times 0.4 \times 0.8) = 0.188 = \frac{47}{250}$

c) $P(\text{at least 1}) = 1 - P(\text{none})$

$= 1 - \frac{25}{30} \times \frac{24}{29}$

$= \frac{9}{29}$

d) i) $\frac{32}{64} \times \frac{31}{63} \times \frac{30}{62} = \frac{5}{42}$

ii) $P(3 \text{ white}) = \frac{5}{42}$ (from i)

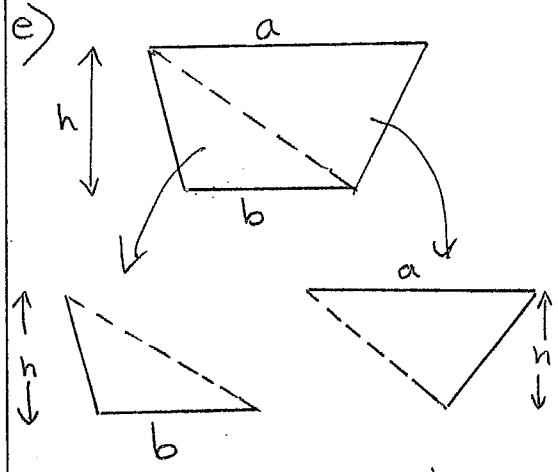
or $P(3 \text{ black}) = \frac{5}{42}$ (same as i)

$\therefore \frac{5}{42} + \frac{5}{42} = \frac{5}{21}$

iii) $P(3 \text{ not same}) = 1 - P(3 \text{ same})$

$= 1 - \frac{5}{21} = \frac{16}{21}$

e)



$A = \frac{1}{2} \times b \times h$

$= \frac{1}{2} bh$

$A = \frac{1}{2} \times a \times h$

$= \frac{1}{2} ah$

$\therefore$ Total $= \frac{1}{2} bh + \frac{1}{2} ah$

$= \frac{1}{2} h(b+a)$

f) i) $h + x + h = 30$

$2h = 30 - x$

$h = 15 - \frac{x}{2}$

ii) $A = x \times h$

$= x(15 - \frac{x}{2})$

$= 15x - \frac{x^2}{2}$

iii) $A = 15x - \frac{x^2}{2}$

vertex $= x = \frac{-b}{2a}$

$x = \frac{-15}{2 \times -\frac{1}{2}} = 15$

$\therefore$ max A when ...

$\therefore A = 15 \times 15 - \frac{15^2}{2}$

$A = 112.5 \text{ cm}^2$

EXII yrII ½ yrIV

(marking scheme)

a) (C)

b) see diagram (1mark)

c) i) (1mark) -5

ii) (1mark) 5

d) (2marks) $x = 5\sqrt{6}$

(1mark) $\frac{x}{\sin 60} = \frac{10}{\sin 45}$

or $\frac{10}{\frac{1}{\sqrt{2}}}$ on one side.

e) (2marks) $g(a) = \sqrt{2a-3}$ and $f(g(a)) = a$

(1mark) either of the above.

f) i) (1mark) showing one side = other

ii) (2marks) D: $x \neq -2$
R: $y \neq 3$

(1mark) either one of the above

iii) (2marks) See sketch with labelling

(1mark) a hyperbolic sketch with an assymptote at $x = -2$

g) (3marks) 18 units^2

(2marks) (8,1) and (2,1)

(1mark) $|x-5| = 3$
or (8,1) or (2,1)
or $x = 8$ or $x = 2$

Q2 i) ext $\angle \Delta$ = sum int opp $\angle$'s
(1mark)

ii) (1mark) $180 - 2\alpha - \beta$

iii) (2marks) $\frac{\beta}{2}$

(1mark) $\angle ADE = \alpha + \frac{\beta}{2}$
or something on the right track

b) i) (2marks) proof

(1mark) 1 mistake

ii) (2marks) $\theta = -60$

(1mark) $\theta = 60$ or $\cos\theta = \frac{1}{2}$

c) i) (3marks) see proof
(2marks) showing 2 sides ratio

(1mark) $\angle C$ is common

ii) (1mark) = 6

d) i) (3marks) see proof
(2marks) proving SAS

(1mark) 2 steps of congruence proof correct.

ii) (1mark) see proof.

Q3 a) (2marks) $-4 \leq x \leq 3$

(1mark) $x \leq -4, x \geq 3$
or $x < -4$ or $x > 3$
or $-4 < x < 3$

b) (2marks) $\frac{47}{250}$ or 0.188

(1mark) $0.7 \times 0.4 \times 0.2$
or $0.3 \times 0.6 \times 0.2$
or $0.3 \times 0.4 \times 0.8$

c) (2marks) $\frac{9}{29} = 0.31\ldots$

(1mark) $\frac{25}{30} \times \frac{24}{29}$

d) i) (2marks) $= \frac{5}{42}$

(1mark) $\frac{32}{64} \times \frac{31}{64} \times \frac{30}{64}$

~~or $\frac{32}{64} \times \frac{31}{63}$~~

ii) (1marks) $\frac{5}{21}$ or 2x part i answer (MUST BE SHOWN)

iii) (1mark) $\frac{16}{21}$ or 1 − part(ii) answer (MUST BE SHOWN)

~~Q3d) iii)~~

Q3 e) (2marks) see proof.

(1mark) showing $A = \frac{1}{2}bh$ or $A = \frac{1}{2}ah$

f) i) $h = 15 - \frac{x}{2}$ (1mark)

ii) (1mark) showing A

iii) (2marks) $112.5 cm^2$

(1mark) $x = 15$