Student

Teacher

BRIGIDINE COLLEGE RANDWICK

MATHEMATICS

PRELIMINARY YEARLY EXAMINATION

2014

(TIME – 2 HOURS)

Directions to candidates

* *Put your **name at the top** of this paper and on each of the 6 sections that are to be collected.*
* *Answer the multiple choice questions 1-10 on the answer sheet provided.*
* *Free response questions 11- 15 are worth equal marks .*
* *All questions are to be answered IN PEN on separate pages and will be collected in separate bundles at the end of this exam.*
* *All necessary working should be shown in every question.*
* *Full marks may not be awarded for careless or badly arranged work.*

10 marks
Attempt Question 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 - 10.

1. What is the value of $\frac{3.7+6.2}{10.6+4.1}$ correct to 2 decimal places? **1**

   A. 0.67

   B. 4.12

   C. 5.03

   D. 8.38

2. What is $\frac{2}{3-\sqrt{2}}$ as a fraction with a rational denominator? **1**

   A. $6-2\sqrt{2}$

   B. $6+2\sqrt{2}$

   C. $\frac{6-2\sqrt{2}}{7}$

   D. $\frac{6+2\sqrt{2}}{7}$

3. What is the value of $f(-1)$ if $f(x)=x^3-4x$?

   A. $f(-1)=-3$

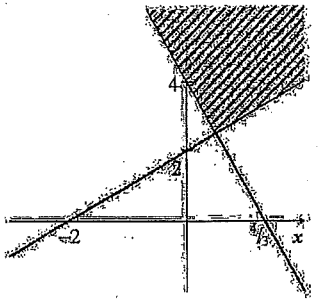
   B. $f(-1)=-5$

   C. $f(-1)=3$

   D. $f(-1)=5$

Marks

4. Which pair of inequalities describe the shaded region? 1



NOT TO SCALE

A. $y \leq 4-3x$
$y \leq x+2$

B. $y \leq 4-3x$
$y \geq x+2$

C. $y \geq 4-3x$
$y \leq x+2$

D. $y \geq 4-3x$
$y \geq x+2$

5. What are the domain and range of the function: 1

$$f(x) = \frac{7}{2x-8}$$

A Domain: {all real $x : x \neq 8$}
Range: {all real y}

B Domain: {all real $x : x \neq 4$}
Range: {all real y}

C Domain: $\{x = 4\}$
Range: {all real $y : y \neq 0$}

D Domain: {all real $x : x \neq 4$}
Range: {all real $y : y \neq 0$}

Marks

6. Which of the following is true for the equation $6x^2 + x - 2 = 0$ 1

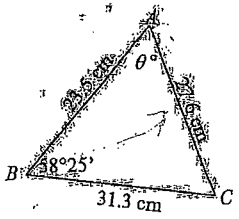
A. no real roots

B. one real root

C. two rational distinct roots

D. two irrational distinct roots.

7. Which of the following is **NOT** a correct expression involving θ in triangle ABC? 1



Not to scale

A. $31.3^2 = 27.6^2 + 23.5^2 - 2 \times 27.6 \times 23.5 \cos\theta$

B. $\cos\theta = \dfrac{23.5^2 + 27.6^2 - 31.3^2}{2 \times 23.5 \times 27.6}$

C. $\dfrac{31.3}{\sin\theta} = \dfrac{27.6}{\sin 58°25'}$

D. $\dfrac{\sin\theta}{31.3} = \dfrac{\sin 58°25'}{23.5}$

8. Write the following expression in simplest form, using no negative indices. 1

$$\frac{(8x)^{-1}}{2^{-6}}$$

A $\dfrac{8}{x}$

B $\dfrac{512}{x}$

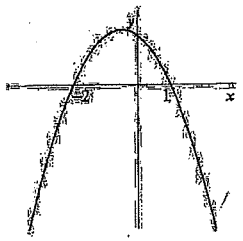
C $\dfrac{1}{8x}$

D $\dfrac{1}{512x}$

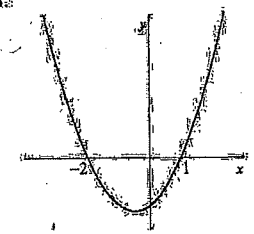
Marks

9. Which graph best represents $y=x^2+x-2$? 1

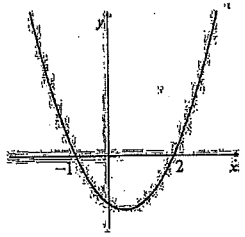
A.



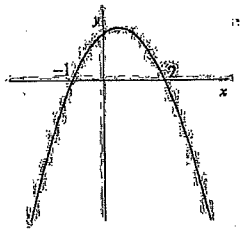
B.



C.



D.



10. What is a solution to the equation $\sin\left(\frac{\theta}{2}+30°\right)=\cos\theta$? 1

A. $\theta=40°$

B. $\theta=60°$

C. $\theta=80°$

D. $\theta=100°$

Question 11

a. If $x=-3$, evaluate $\frac{x^2-x^3}{1+\sqrt[3]{x}}$ to 3 significant figures 2

b. Fully simplify $\sqrt{8}+\sqrt{18}-3\sqrt{32}$. 2

c. Fully simplify $\frac{5x}{6}-\frac{3x-2}{2}$ 2

d. Solve the pair of simultaneous equations 2

$$y=4x$$
$$3x-2y=15$$

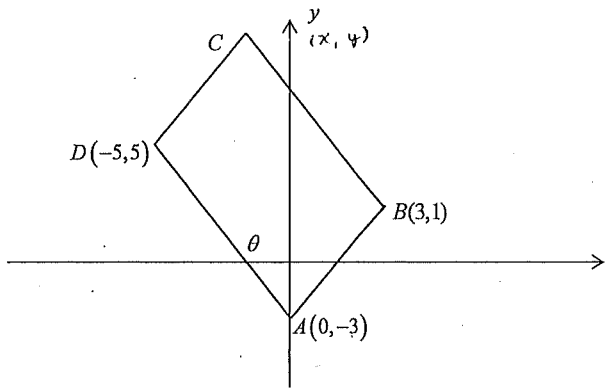
e. Given $f(x)=x^2-x^4$. Is the function, even, odd or neither? Justify your answer. 2

f. Solve $|2x+1|<2$ 2

g. If $f(x)=2x^2-3x-5$, find $\frac{f(a)}{a+1}$ (leaving your answer in fully simplified form). 2

Question 12 (Start a New Page)

a. In the diagram below A, B and D have coordinates (0,-3), (3,1) and (-5,5) respectively.
The angle θ is the angle , the line AD makes with the positive direction of the x axis.



i. Find the gradient of the line AD and hence the value of θ. 2

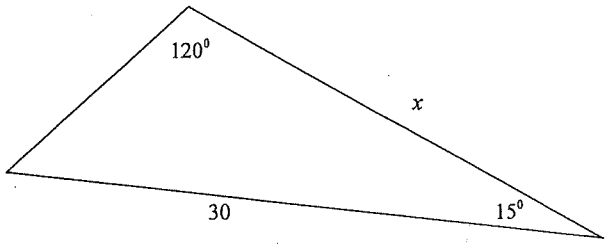
ii. Find the coordinate of C, so that $ABCD$ is a parallelogram. 1

iii. Show that the line AB has equation $4x-3y-9=0$ 2

iv. Find the perpendicular distance between D and AB. 1

v. Find the area of parallelogram $ABCD$. 2

b. Find the exact of x. 3



Question 12 continued .....

c. i. Prove that $\sec^2 x-2\tan x=(\tan x-1)^2$. 2

ii. Hence, or otherwise solve $\sec^2 x-2\tan x=0$ for $0^0 \le x^0 \le 360^0$. 2

Question 13 (Start a New Page)

a. Solve for x: $9^x-28(3)^x+27=0$ 3

b. For the parabola $(y-1)^2=16-8x$

i. State the coordinates of the vertex and focus. 2

ii. Sketch the graph of the parabola showing the above information as well as the directrix.. 2

c. The point $P(x,y)$ moves such that PA is perpendicular to PB. Given that A and B have coordinates (-1,2) and (7,6) respectively,

i. Show that the locus of P can be represented by $x^2-6x+y^2-8y+5=0$ 2

ii. Describe the locus's path geometrically. 2

d. Determine the value(s) of k for which the expression $x^2+(2-k)x+k(2-k)$ is positive definite. 3

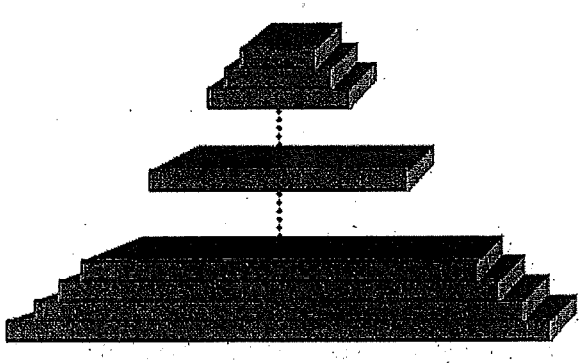
Question 14 (Start a New Page)

a. The third term of a geometric series is 54 and the sixth term is 2. Find:

i. the common ratio and the first term. 2

ii. the sum of the first 6 terms. 2

b. A pyramid is built using 1536 blocks on the base level. The next layer contains 1472 blocks and the next 1408, and so on.



i. How many blocks are used for the sixteenth layer? 2

ii. Before it is capped with a single pyramid block, the top layer has 64 blocks. How many layers are there before the cap is put on? 1

iii. How many blocks are used in the construction of the pyramid? 1

c. Consider the geometric series

$$3-6x+12x^2-24x^3+.....$$

i. For what values of x does the series have a limiting sum? 2

ii. If the limiting sum of the series is 2.5, find the value of x. 2

d. If $2=x^{0.6}$ and $3=x^{0.8}$, find $\log_x 6$ 2

Question 15 (Start a New Page)

a. Evaluate $\lim_{x\to\infty}\dfrac{x-x^2}{1+3x^2}$ 2

b. Differentiate the following with respect to x.

i. $y=x(2x+3)^4$ leaving your answer in fully factorised form. 3

ii. $y=\dfrac{1}{\sqrt{2-x}}$ leaving your answer in surd form. 2

iii. $y=\dfrac{x-1}{x-3}$ 2

c. The curve $y=4x^2-bx+9$ has a gradient of -2 when $x=1$. 2
Find the value of b.

d. Find the equation of the tangent to the curve $y=2x^3-3x+1$ at the point where the curve cuts the y axis. 3

Multiple Choice Answer Sheet

Name :

Select Your Answers

Teacher :

1.	A	B	C	D
2.	A	B	C	D
3.	A	B	C	D
4.	A	B	C	D
5.	A	B	C	D
6.	A	B	C	D
7.	A	B	C	D
8.	A	B	C	D
9.	A	B	C	D
10.	A	B	C	D

10

Question 11

14

a) $$\frac{(-3)^2 - (-3)^3}{1 + \sqrt[3]{-3}}$$

$$= \frac{9 + 27}{1 + \sqrt[3]{-3}}$$

$$= \frac{36}{1 + \sqrt[3]{-3}}$$

$$= -81.402\ldots$$

$$= -81.4 \text{ (3sf)}$$

b) $$\sqrt{8} + \sqrt{18} - 3\sqrt{32}$$

$$= 2\sqrt{2} + 3\sqrt{2} - 12\sqrt{2}$$

$$= -7\sqrt{2}$$

c) $$\frac{5x}{6} - \frac{3x-2}{2}$$

$$= \frac{5x}{6} - \frac{3(3x-2)}{6}$$

$$= \frac{5x - 3(3x-2)}{6}$$

$$= \frac{5x - 9x + 6}{6}$$

$$= \frac{-4x + 6}{6}$$

$$= \frac{6 - 4x}{6}$$

$$= \frac{3 - 2x}{3}$$

d) $y = 4x$

$3x - 2y = 15$

$3x - 2(4x) = 15$

$3x - 8x = 15$

$-5x = 15$

$y = 4(-3)$

$= -12$

$\therefore x = -3,\ y = -12$

e) $f(x) = x^2 - x^4$

$f(-x) = (-x)^2 - (-x)^4$

$= x^2 - x^4$

The function is even as

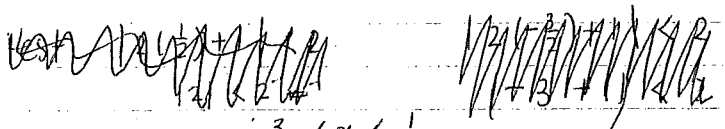
$f(-x) = f(x)$ ✓✓

f) $|2x+1| < 2$

$2x + 1 < 2$ or $2x + 1 > -2$

$2x < 1$ or $2x > -3$

$x < \frac{1}{2}$ $x > -\frac{3}{2}$ ✓



$\therefore \quad -\frac{3}{2} < x < \frac{1}{2}$ ✓

g) $f(x) = 2x^2 - 3x - 5$

$f(a) = 2a^2 - 3a - 5$

$$\frac{f(a)}{a+1} = \frac{2a^2 - 3a - 5}{a+1}$$

$$= \frac{2a^2 - 5a + 2a - 5}{a+1}$$ ✓

$$= \frac{a(2a-5) + (2a-5)}{a+1}$$ ✓

$$= \frac{(2a-5)(a+1)}{a+1}$$

$$= 2a - 5$$

Question 12 (15)

a) i. $m_{AD} = \frac{5-(-3)}{-5-0}$

$= \frac{5+3}{-5}$

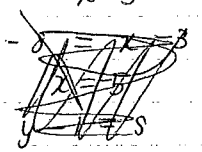
$= -\frac{8}{5}$ ✓

$\tan\theta = -\frac{8}{5}$

$\theta = 122°$ (to nearest degree) ✓

ii. $m_{AD} = m_{CB}$

$-\frac{8}{5} = \frac{y-1}{x-3}$



x - negative

y = positive

$y - 1 = 8$

$y = 9$

$-5 = x - 3$

$x = -2$ ✓

$\therefore$ coordinates of C = (-2, 9)

iii. $m_{AB} = \frac{1-(-3)}{3-0}$

$= \frac{4}{3}$ ✓

$y - (-3) = \frac{4}{3}(x - 0)$

$y + 3 = \frac{4}{3}x$

$3y + 9 = 4x$ ✓

$4x - 3y - 9 = 0$

iv. USE FORMULA

perpendicular distance $= \frac{|ax+by+c|}{\sqrt{a^2+b^2}}$

$= \frac{|4(-5) - 3(5) - 9|}{\sqrt{4^2+(-3)^2}}$

$= \frac{|-20-15-9|}{\sqrt{16+9}}$

$= \frac{|-44|}{\sqrt{25}}$

$= \frac{44}{5}$

$\therefore$ perpendicular distance $= 8\frac{4}{5}$

$= 8.8$.

v. Area of parallelogram $= b \times h$

$AB^2 = (3-0)^2 + (1-(-3))^2$

$= 3^2 + 4^2$

$= 9 + 16$

$= 25$

$AB = \sqrt{25}$

$= 5$.

$\therefore$ Area of parallelogram $= 5 \times \frac{44}{5}$

$= 44$ units^2

b) $\frac{30}{\sin 120} = \frac{x}{\sin 45}$

$\sin 120 = \frac{\sqrt{3}}{2}$

$180 - 120 = 60$

$x = \frac{30 \sin 45}{\sin 120}$

$x = \frac{30 \times \frac{1}{\sqrt{2}}}{\frac{\sqrt{3}}{2}} = \frac{30}{\sqrt{2}} \times \frac{2}{\sqrt{3}}$

$= \frac{60 \times \sqrt{6}}{6} = \frac{60\sqrt{6}}{6} = 10\sqrt{6}$

c. i. $\sec^2 x - 2\tan x = (\tan x - 1)^2$

LHS : $\sec^2 x - 2\tan x$

$= 1 + \tan^2 x - 2\tan x$

$= \tan^2 x - 2\tan x + 1$

$= (\tan x - 1)^2$

$=$ RHS

ii. $\tan x = 1$

$x = 45°, 225°$

S | A

T | C

$(180 + 45)$

$= 225$.

Question 13

a) $9^x - 28(3)^x + 27 = 0$

let $3^x = u$

$u^2 - 28u + 27 = 0$

$(u - 27)(u - 1) = 0$

$\therefore u = 27, \; u = 1$

$3^x = 27 \qquad 3^x = 1$

$x = 3 \qquad x = 0$ ✓✓

b) i. $(y-1)^2 = 16 - 8x$

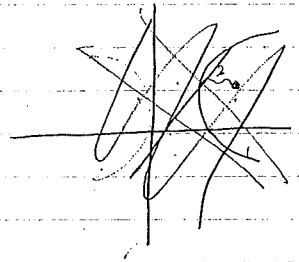
$(y-1)^2 = -8(x - 2)$

$-8 = -4a$

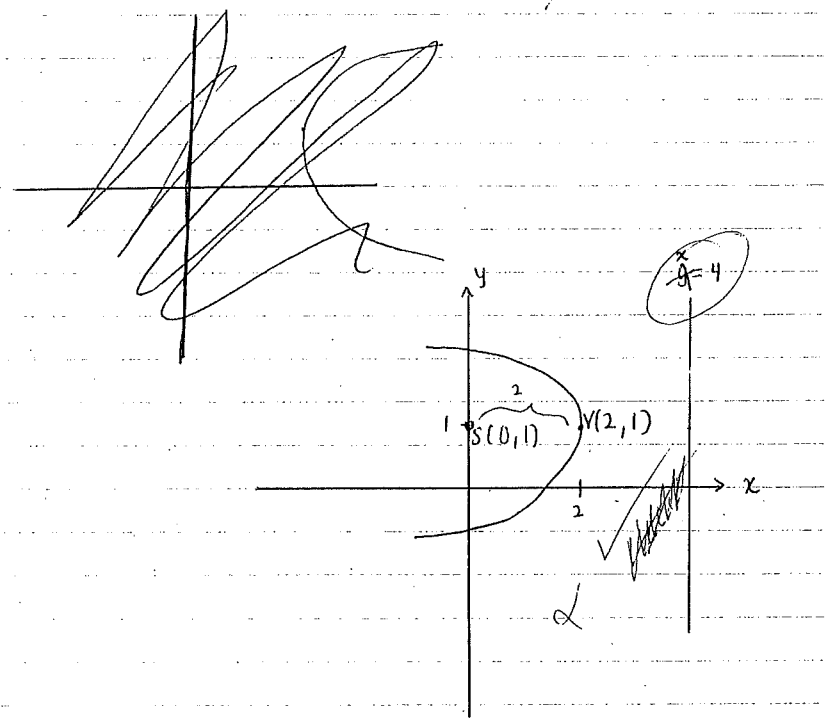
$\therefore a = 2$

vertex = $(2, 1)$ ✓

focus = $(0, 1)$ ✓

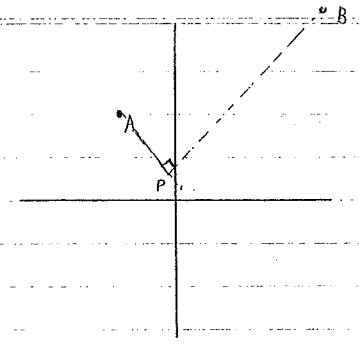


ii.



✓ ✓

c.



i. $m_{PA} = \dfrac{2-y}{-1-x}$ $\qquad m_1 = -\dfrac{1}{m_2}$

$m_{PB} = \dfrac{6-y}{7-x}$ $\qquad m_1 \times m_2 = -1$

$\dfrac{2-y}{-1-x} \times \dfrac{6-y}{7-x} = -1$

$\dfrac{(2-y)(6-y)}{(-1-x)(7-x)} = -1$

$(2-y)(6-y) = -(-1-x)(7-x)$

$(2-y)(6-y) = (1+x)(7-x)$

$12 - 2y - 6y + y^2 = 7 - x + 7x - x^2$

$12 - 8y + y^2 = 7 + 6x - x^2$

$x^2 - 6x + y^2 - 8y + 5 = 0$ ✓

ii. the locus is a circle $\qquad (x-3)^2 + (y-4)^2 = 2$

with centre (3, 4)

and radius $= \sqrt{20}$

$= 2\sqrt{5}$ ✓

d) $\Delta > 0$

$\Delta = b^2 - 4ac$

$= (2-k)^2 - 4(1)(k+(2-k))$

$= (2-k)^2 - 4(2k - k^2)$

$= (2-k)^2 - 8k + 4k^2 \qquad = 5k^2 - 12k + 4$

$5k^2 - 12k + 4$

$5k^2 - 10k - 2k + 4$

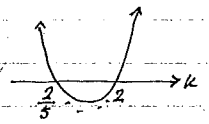
$5k(k-2) - 2(k-2)$

$(k-2)(5k-2)$

$\therefore k = 2, \frac{2}{5}$

$\Delta > 0$

$\therefore k < \frac{2}{5}$ or $k > 2$



$\frac{2}{5} < k < 2$

Question 14

a) i. $ar^2 = 54 \quad \ldots (1)$

$ar^5 = 2 \quad \ldots (2)$

Devide (2) by (1)

$r^3 = \frac{2}{54}$

$= \frac{1}{27}$

$r = \frac{1}{3}$

Sub $r = \frac{1}{3}$ into (1)

$a\left(\frac{1}{3}\right)^2 = 54$

$\frac{a}{9} = 54$

$a = 486$

$\therefore$ first term = 486

common ratio = $\frac{1}{3}$

ii. $S_n = \frac{a(1-r^n)}{1-r}$

$S_6 = \frac{486\left(1-\left(\frac{1}{3}\right)^6\right)}{1-\frac{1}{6}}$

$= \frac{486 - \frac{486}{729}}{\frac{5}{6}}$

$= 582.4$

b. 1536, 1472, 1408 . . . .

i. $1536 - 1472 = 64$ $\quad 1472 - 1536 = -64$

$1472 - 1408 = 64$ $\quad 1408 - 1472 = -64$

$\therefore$ AP

$T_{16} = a + (n-1)d$

$= 1536 + (16-1) \times -64$

$= 1536 - (15 \times 64)$

$= 576$

ii. $64 = 1536 + (n-1) \times -64$

$= 1536 - 64n + 64$

$= 1600 - 64n$

$64n = 1536$

$n = 24$

$\therefore$ there are 24 layers

iii. $\frac{n}{2}(a+l)d$ =

$\frac{64}{2}(1536+64) \times -64 =$

$32 \times (1600) \times -64 =$ $\frac{n}{2}(a+l)d$

$\frac{n}{2}(2a+(n-1)d$

$\frac{64}{2}(2(1536)+(64-1)d$

iii. $S_{24} = \frac{24}{2}(1536+64) \times -64$?

$= 1228800$

$\therefore$ Amount of blocks $= 1228800 + 1$

$= 1228801$

c) i. $-1 < x < 1$

ii. $2.5 = \frac{3}{1-r}$ $\frac{-6x}{3} = 2x$ $\frac{12x^2}{-6x} = -2x$

$r = -2x$

$2.5 = \frac{3}{1+2x}$

$2.5(1+2x) = 3$

$2.5 + 5x = 3$

$5x = 0.5$

$x = 0.1$

$x = \frac{1}{10}$

d) $2 = x^{0.6}$ $\log_x 2 = 0.6$

$3 = x^{0.8}$ $\log_x 3 = 0.8$

$\log_x 6 = \log_x 2 + \log_x 3$

$= 0.6 + 0.8$

$= 1.4$

Question 15

a) $\lim_{x \to \infty} \frac{x - x^2}{1+3x^2}$

$= \frac{\frac{x}{x^2} - \frac{x^2}{x^2}}{\frac{1}{x^2} + \frac{3x^2}{x^2}}$

$= \frac{0-1}{0+3}$

$= -\frac{1}{3}$

13

b) i. $y = x(2x+3)^4$

let $u = x$

$u' = 1$

let $v = (2x+3)^4$

$v' = 4(2x+3)^3 \times 2$

$v' = 8(2x+3)^3$

$y' = 8x(2x+3)^3 + (2x+3)^4$

$y' = (2x+3)^3 (8x + (2x+3))$

$y' = (2x+3)^3 (8x+2x+3)$

$y' = (2x+3)^3 (10x+3)$

ii. $y = \frac{1}{\sqrt{2-x}}$

$y = (2-x)^{-\frac{1}{2}}$

$y' = -\frac{1}{2}(2-x)^{-\frac{3}{2}} \times -1$

$= -\frac{1}{2} \times \frac{1}{(\sqrt{2-x})^3}$

$= \frac{1}{2(\sqrt{2-x})^3}$

$= \frac{1}{2(2-x)\sqrt{2-x}}$

$= \frac{1}{4-2x}$

$= -\frac{1}{2(\sqrt{2-x})^3}$

$= -\frac{1}{2(2-x)^2\sqrt{2-x}}$

iii. $y = \frac{x-1}{x-3}$

$y' = \frac{vu' - uv'}{v^2}$

let $u = x - 1$

$u' = 1$

let $v = x - 3$

$v' = 1$

$y' = \frac{(x-3) - (x-1)}{(x-3)^2}$

$= \frac{x - 3 - x + 1}{(x-3)^2}$

$= -\frac{2}{(x-3)^2}$ ✓✓

c) $y = 4x^2 - bx + 9$

$y' = 8x - b$

$-2 = 8(1) - b$

$-2 = 8 - b$

$-b = -10$ ✓✓

$b = 10$

d) curve cuts the y-axis $(x = 0, y = 1)$

$y = 2x^3 - 3x + 1$

$y' = 6x^2 - 3$

$y' = 6(0)^2 - 3$

$= -3$

$y - y_1 = m(x - x_1)$

$y - 1 = -3(x - 0)$ ✓✓✓

$y - 1 = -3x$

equation : $3x + y - 1 = 0$