BRIGIDINE COLLEGE RANDWICK

MATHEMATICS **February 13, 2006**

Extension 2 Common **Time: 50 min**

Write your name at the top of this exam.
Neatness may be taken into consideration in the award of marks.

There are 7 Questions.

1. Completely simplify i^{2006} 2 m

2. If $z_1 = -\sqrt{3} - i$ and $z_2 = 2 + 2i$, Find

   a. $\text{Im}(z_1)$ 1/ 1 m

   b. $|z_1|$ 1/ 1 m

   c. $\text{Arg } z_1$ 2/ 2 m

   d. Express $(z_2)^5$ in the form $x + yi$. 3/ 3 m

   e. Completely simplify $\dfrac{(z_1)^9}{(z_2)^5}$ ie. $\dfrac{(-\sqrt{3} - i)^9}{(2 + 2i)^5}$ 2/3 m

3. Find the fifth roots of i, you may leave answers in cis form 4/4 ~~5~~ m

4. Sketch the region defined by $|z - 4| = |z + i|$, and hence 4 m
   (4,0) (0,-1)
   determine the minimum value of $|z|$.

5. a. Illustrate on the Complex Plane $\arg \dfrac{z - 2}{z + 2} = \dfrac{\pi}{3}$. 2/ 4 m
   (2,0) (-2,0).

   b. Determine the maximum value of $|z|$.

please turn over

xt 2   Feb 03/06

1/ i^{2006}

$= (i^4)^{501}\, i^2$ us cis

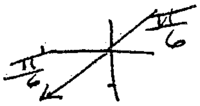
$= -1$ 2

2/ $z_1 = -\sqrt{3} - i$ | $z_2 = 2 + 2i$

a) $Im(z_1) = -1$ 1

b) $|z_1| = \sqrt{(-\sqrt{3})^2 + (-1)^2} = 2$ 1

c) $Arg\, z_1 = \tan^{-1} \frac{-1}{-\sqrt{3}}$



$-\frac{5\pi}{6}$ 2

d) $(z_2)^5 = \left(\sqrt{8}\, \text{cis} \frac{\pi}{4}\right)^5$

$= (2\sqrt{2})^5\, \text{cis} \frac{5\pi}{4}$

$= (2\sqrt{2})^5 \left[\cos \frac{\pi}{4} - \sin \frac{\pi}{4}\right]$

$= -128 - 128i$ 3

e) $\frac{(z_1)^9}{(z_2)^5} = \frac{2^9\, \text{cis}\, 9\left(-\frac{5\pi}{6}\right)}{(\sqrt{8})^5\, \text{cis} \frac{5\pi}{4}}$

$= \frac{2^9}{2^{15/2}}\, \text{cis}\left(-\frac{54}{6}\pi - \frac{5\pi}{4}\right)$

$= 2^{3/2}\, \text{cis}\left(-\frac{41\pi}{4}\right)$

$= 2^{3/2}\, \text{cis}(-1845°)$

$= 2^{3/2}\, \text{cis}(-45)$

us

$= 2^{3/2}\, \text{cis}\left(-\frac{\pi}{4}\right)$ 3

$= 2\sqrt{2}\left(\cos \frac{\pi}{4} - i \sin \frac{\pi}{4}\right)$

$= 2 - 2i$

3/ $z^5 = i$

$z^5 = \text{cis} \frac{\pi}{2}$

$\text{cis}\, 5\theta = \text{cis} \frac{\pi}{2}$ (2)

$5\theta = \frac{\pi}{2} + 2n\pi$

$n = 0, \pm 1, \pm 2$

$\theta = \frac{\pi + 4n\pi}{5 \times 2}$ (2)

$n = 0$ $\text{cis} \frac{\pi}{5 \times 2} = \text{cis} \frac{\pi}{10}$ 4

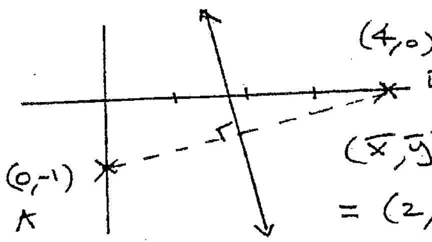
$n = -1$ $\text{cis} \frac{-3\pi}{5 \times 2} = \text{cis} \frac{-3\pi}{10}$

$n = 1$ $\text{cis} \frac{\pi}{2} = \text{cis} \frac{\pi}{2}$

$n = -2$ $\text{cis} \frac{-7\pi}{5 \times 2} = \text{cis} \frac{-7\pi}{10}$

$n = 2$ $\text{cis} \frac{9\pi}{5 \times 2} = \text{cis} \frac{9\pi}{10}$

4/



$= (2, -\frac{1}{2})$ ✓

$m_{AB} = \frac{1}{4} \therefore \perp -4$

$y - -\frac{1}{2} = -4(x - 2)$

$2y + 1 = -8x + 16$ ✓

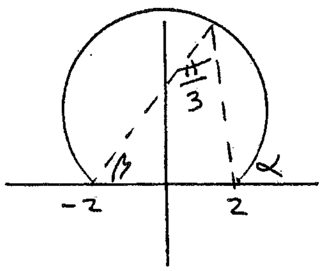
$8x + 2y - 15 = 0$

$d = \frac{|(8)(0) + (2)(0) - 15|}{\sqrt{(8)^2 + (2)^2}}$ ✓✓

$= \frac{15}{\sqrt{68}} \sim \frac{15\sqrt{17}}{34}$

Ext 2 Feb 03/06

5)



$$\arg \frac{z-2}{z+2} = \frac{\pi}{3}$$

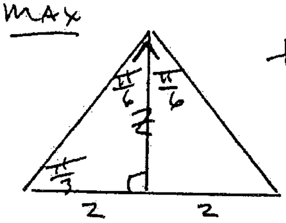
$$\arg(z-2) - \arg(z+2) = \frac{\pi}{3}$$

$$\arg(z-2) = \arg(z+2) + \frac{\pi}{3}$$

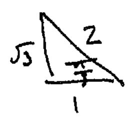
Let $\alpha = \beta + \frac{\pi}{3}$

$\therefore$ True ②

MAX



$$\tan \frac{\pi}{3} = \frac{z}{2}$$



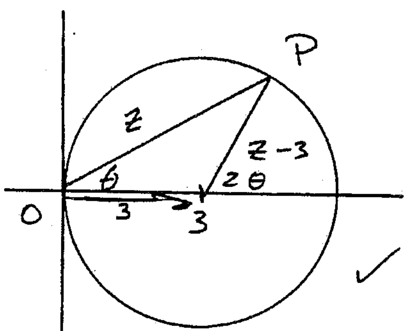
②

$$\sqrt{3} = \frac{z}{2}$$

$$2\sqrt{3}\ u = z$$

4

6)



from diagram Let

$\arg z = \theta$

$\therefore \arg(z-3) = 2\theta$ | central twice ins ∠

$\therefore \arg(z-3) = 2 \arg z$ ✓

$\arg(z-3) = \arg z^2$

3

7) $x^2 + y^2 = 8$

a) $x = 2,\ y = \pm 2$

$$2x + 2y \frac{dy}{dx} = 0$$ ✓

$$\therefore \frac{dy}{dx} = \frac{-2x}{2y} = \frac{-x}{y}$$

$$m = -1$$

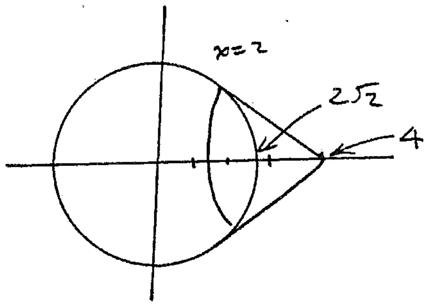
$$y - 2 = -(x-2)$$

$$y - 2 = -x + 2$$

$$y = -x + 4$$ ✓

2

b)



$\therefore V_{cone} - V_{by\ cylinder}$ ✓

$$V_c = \frac{\pi}{3}(2)^2(2) = \frac{8\pi}{3}$$

$$V_{cyl} = \pi \int_2^{2\sqrt{2}} y^2\, dx$$

$$= \pi \int (8 - x^2)\, dx$$

$$= \pi \left[8x - \frac{x^3}{3} \right]_2^{2\sqrt{2}}$$

$$= \pi \left[\left(16\sqrt{2} - \frac{16\sqrt{2}}{3}\right) - \left(16 - \frac{8}{3}\right) \right]$$

$$= \pi \left[\frac{48\sqrt{2} - 16\sqrt{2}}{3} - \frac{40}{3} \right]$$

$$= \pi \left[\frac{32\sqrt{2} - 40}{3} \right]$$ ✓

$$V = \frac{\pi}{3}\left[8 + 40 - 32\sqrt{2}\right] u^3$$ ✓

$$\Rightarrow \frac{\pi}{3}\left[48 - 32\sqrt{2}\right] u^3$$

NB REWORD $\therefore V_{cone} - V_{hemisphere}$

$$\frac{1}{3}\pi(4)^2(4) - \frac{1}{2} \cdot \frac{4}{3}\pi(2\sqrt{2})^3$$

$$\frac{128\pi - 64\sqrt{2}\pi}{3}$$