Student _____

BRIGIDINE COLLEGE RANDWICK

PRELIMINARY EXTENSION 1 MATHEMATICS

HALF YEARLY

2010

(TIME - 1 HOUR)
(plus 5 minutes reading)

Directions to candidates

* *Put your **name at the top** of this paper and on each of the 3 **sections** that are to be collected.*
* *All 3 questions are to be attempted.*
* *All 3 questions are of equal value.*
* *All questions are to be answered on separate pages and will be collected in separate bundles at the end of this exam.*
* *All necessary working should be shown in every question.*
* *Full marks may not be awarded for careless or badly arranged work.*

Question 1 ***(Start a new page)***

a. Simplify the expression $|\sqrt{3} - 5|$ 1 m

b. Solve the inequation $2x^2 + 9x \geq 5$ 3 m

c. Solve the following for x

i. $x^3 - 4x^2 - 2x + 8 = 0$ 3 m

ii. $\sqrt{x+13} = x + 1$ 3 m

d. Sketch the graph of the relation $y = |2x - 5| + 1$ 2 m

e. State the natural domain for the following curves

i. $f(x) = x^2 - 4$ 1 m

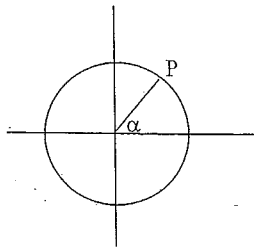
ii. $g(x) = \dfrac{1}{\sqrt{x^2 - 1}}$ 3 m

f. Sketch the graph of $y = (x + 2)|x - 2|$ 4 m

Question 2 *(Start a new page)*

a. Solve for x if the $\sin(180^\circ - x) = \cos 3x$. 2 m

b. Consider the unit circle to the right.



i. Redraw this diagram onto your exam page and write down the equation for this circle. 1 m

ii. Show the point P (x,y) may also be expressed as $(\cos\alpha, \sin\alpha)$. 1 m

iii. Using this information, show that $1 + \tan^2\alpha = \sec^2\alpha$. 1 m

iv. Simplify the expression $(1 - \sin^2\alpha)(1 + \tan^2\alpha)$ 2 m

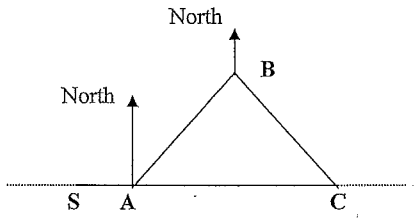
c. Neatly sketch the following curves, showing all features that assisted your sketch.

$y = 2\cos 3x + 1$ for $0^\circ \leq x \leq 150^\circ$ 3 m

d. Solve the following given that 4 m

$\tan^2 x - \sec x = 1$ for $0^\circ \leq x < 360^\circ$.

e. From the start **S**, Sharon rode 3 km due east to **A**. At **A**, she proceeded on a bearing of 055° for 10 km to **B**. At **B**, she changed course to a bearing of 130° and continued in this direction until she reached the finish at **C**. (**C** is due east of the start **S** and **A**)



i. Copy this diagram onto your answer sheet and display this information. 1 m

ii. Show that $\angle ACB = 40^\circ$. 1 m

iii. Use the sine rule to find the distance from **B** to **C** (nearest km). 2 m

iv. It took Sharon 24 minutes to travel from the start to the finish. What was her average speed in km / hr? 2 m

Question 3 *(Start a new page)*

a. Given that $(\sqrt{x} + 2)^2 = a + b\sqrt{2}$, find the values of a, b and x. 3 m

b. Sketch the region defined by $x^2 + y^2 + 2x + 4y + 1 < 0$ 4 m

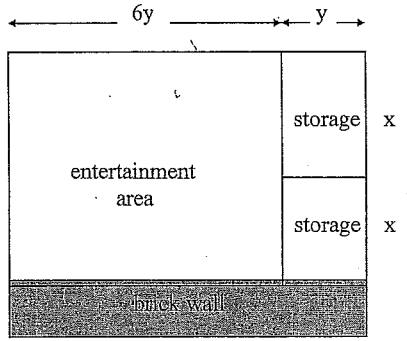
c. i. A point moves in a plane so that is equidistant from the points A (-1,6) and B (3,2). Write this locus as the equation of a line in general form. 3 m

ii. Show that this line represents the perpendicular bisector of these points A and B. 2 m

d. Jessica wishes to construct three rectangular enclosures, as shown to the right, two are for storage and the third as an entertainment area.

The entertainment area is to be six times as long and twice as wide as the storage areas.

One of the storage areas and one side of the entertainment area will be bounded by an existing brick wall.



All other fences are to be constructed from 56 metres of wire mesh.

i. Let x metres be the width of a storage area and y metres be its length. 1 m
Show that $y = 7 - \frac{3}{4}x$.

ii. Hence show that the total area A square metres contained in the three enclosures is given by $A = 14x(7 - \frac{3}{4}x)$. 2 m

iii. Neatly sketch this parabola and show that the area is a maximum when half the wire fencing has been placed parallel to the brick wall. 5 m

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Q1a) 2·83

2marks: 2·83

1mark: 2·82 or 2·07 or 2·826 or 2·825564723 or write out calculator display then correctly round from it

b) $|2x-6| \leq 4$

$-4 \leq 2x-6 \leq 4$

$2 \leq 2x \leq 10$

$1 \leq x \leq 5$

2marks: $1 \leq x \leq 5$

1mark: $x \geqslant 1$ or a small mistake

c) $2x-4 = x-2(6-3x)+1$

$2x-4 = x-12+6x+1$

$2x-4 = 7x-11$

$7 = 5x$

$x = \frac{7}{5}$

2marks: $\frac{7}{5}$

1mark: correctly expand brackets or one mistake

d) i) $m^2-n^2+5m-5n$

$(m-n)(m+n)+5(m-n)$

$(m-n)(m+n+5)$

i) 3marks: $(m-n)(m+n+5)$

2marks: $(m-n)(m+n)+5(m-n)$

1mark: $(m-n)(m+n)$ or $5(m-n)$

ii) $3x^2+15x-72$

$3(x^2+5x-24)$

$3(x+8)(x-3)$

ii) 3marks: $3(x+8)(x-3)$

2marks: $(3x+24)(x-3)$ or $(x+8)(3x-9)$

1mark: $3(x^2+5x-24)$

Q2a) i) $\frac{2x^2}{x^2-y^2} - \frac{2x}{x^2+xy}$

$\frac{x \times 2x^2}{x \times (x-y)(x+y)} - \frac{2x \times (x-y)}{x(x+y)(x-y)}$

$\frac{2x^3-2x^2+2xy}{x(x-y)(x+y)}$

$\frac{2x^2-2x+2y}{(x-y)(x+y)}$

2ai) 3marks: $\frac{2x^2-2x+2y}{(x-y)(x+y)}$

2marks: $\frac{2x^3-2x^2+2xy}{x(x-y)(x+y)}$

1mark: $\frac{2x^2}{(x-y)(x+y)}$

ii) $\frac{a^2+b^2+2ab}{a+b}$

$\frac{a^2+2ab+b^2}{a+b}$

$\frac{(a+b)(a+b)}{a+b} = a+b$

ii) 2marks: $a+b$

1mark: numerator $= (a+b)(a+b)$

b) $x^2-6x = 5$

$x^2-6x+\left(\frac{6}{2}\right)^2 = 5+\left(\frac{6}{2}\right)^2$

$(x-3)^2 = 14$

$x-3 = \pm\sqrt{14}$

$x = 3 \pm \sqrt{14}$

b) 3marks: $3 \pm \sqrt{14}$

2marks: $(x-3)^2 = 14$

1marks: $x^2-6x+9 = 5+9$

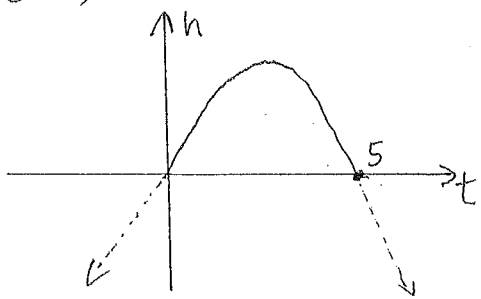
Q2c) i) $h = 10t - 2t^2$

(same as $y = 10x - 2x^2$)

$0 = 10t - 2t^2$

$0 = 2t(5-t)$

$t = 0,\ t = 5$



ii) $t = 5$ seconds

iii) $t = 2.5$ seconds

$h = 10 \times 2.5 - 2 \times 2.5^2$

$= 12.5$m

Q3a) $x^2 + y^2 - 25 = 0$

$x + y = 1 \rightarrow y = 1 - x$

$x^2 + (1-x)^2 - 25 = 0$

$x^2 + 1 - 2x + x^2 - 25 = 0$

$2x^2 - 2x - 24 = 0$

$x^2 - x - 12 = 0$

$(x-4)(x+3) = 0$

$x = 4$ $\qquad$ $x = -3$

$y = -3$ $\qquad$ $y = 4$

i) 1 marks see sketch

ii) 1mark: 5 seconds or correct from students sketch

iii) 2marks: 12.5m

1mark: $t = \frac{1}{2} \times$ part(ii) time

Q3a) 3marks: $x = 4$, $y = -3$ $\qquad$ $x = -3$, $y = 4$

2marks: $x = 4, -3$ or $y = -3, 4$

1mark: sub incorrect x-values to get correct y values from students work. ↑ This MUST be shown

or solving a quadratic correctly.

b) i) $(x-4)^2 = \frac{11-3x}{2}$

$x^2 - 8x + 16 = \frac{11-3x}{2}$

$2x^2 - 16x + 32 = 11 - 3x$

$2x^2 - 13x + 21 = 0$

$(2x-7)(x-3) = 0$

$x = \frac{7}{2}, 3$

ii) $(5-4x)^3 = 64$

$5 - 4x = \sqrt[3]{64} = 4$

$4x = 1$

$x = \frac{1}{4}$

c) i) $\sqrt{32} + 2\sqrt{18} - 5\sqrt{28}$

$4\sqrt{2} + 2 \times 3\sqrt{2} - 5 \times 2\sqrt{7}$

$4\sqrt{2} + 6\sqrt{2} - 10\sqrt{7}$

$10\sqrt{2} - 10\sqrt{7}$

ii) $(2\sqrt{3} - \sqrt{5})^2$

$(2\sqrt{3})^2 - 2 \times 2\sqrt{3} \times \sqrt{5} + (\sqrt{5})^2$

$12 - 4\sqrt{15} + 5$

$17 - 4\sqrt{15}$

bi) 3marks: $x = \frac{7}{2}, 3$

2marks: $2x^2 - 13x + 21 = 0$

1mark: expanding $(x-4)^2$ correctly or correctly solving students quadratic.

ii) 2marks: $x = \frac{1}{4}$

1mark: $\sqrt[3]{64}$

ci) 2marks: $10\sqrt{2} - 10\sqrt{7}$

1mark: 2 correct surd simplification ie $4\sqrt{2}$ or $6\sqrt{2}$ or $-10\sqrt{7}$

ii) 2marks: $17 - 4\sqrt{15}$

1mark: correct expansion or correct simplification.