Student ______________________________

Teacher ______________________________

BRIGIDINE COLLEGE RANDWICK

Preliminary Mathematics

- 2008 -

HALF-YEARLY

Time 1.5 Hour

DIRECTIONS TO CANDIDATES

- * *Put your **name** at the top of this paper and on **each of the 6 sections** to be collected.*
- * *All **6 questions** may be attempted.*
- * *All **6 questions** are to be answered on **separate pages** and will be collected in **separate bundles** at the end of this exam.*
- * *All questions are of equal value.*
- * *All necessary working should be shown in every question.*
- * *Full marks may not be awarded for careless or badly arranged work.*

QUESTION 1 ***(start a new page)***

a. Melissa paid \$ 39.95 for her Maths text book. She later sold this for \$ 32.50. Determine her Percentage Loss for this book to 1 decimal place. **2 m**

b. Show that $\frac{5}{\sqrt{2}} + \sqrt{2}$ may be written in the form $A\sqrt{2}$ and state the value of A. **3 m**

c. Simplify the following, leaving denominators rational when necessary

i. $2\sqrt{7} - 3\sqrt{28} + \sqrt{63}$ **2 m**

ii. $\sqrt{\frac{12}{8}}$ **2 m**

d. Completely simplify the following expressions

i. $\frac{5x + 15}{x+1} \div \frac{x + 3}{2x^2 + x - 1}$ **3 m**

ii. $\frac{3x+1}{3x-1} - \frac{3x-1}{3x+1}$ **3 m**

QUESTION 2 *(start a new page)*

a. Express $2.\overline{21}$ (ie 2.212121212 ....) as a fraction. 3 m

b. Solve the following equations

i. $4\left(\frac{1}{x} + 2\right) = 5 - \frac{2}{x}$ 3 m

ii. $5 - \frac{2x + 1}{3} = 15$ 3 m

iii. $(2x+3)^2 = (2x - 1)(2x+5)$ 3 m

c. Solve the simultaneous equations given by 3 m

$2x - y = 1$ and $4x + 2y = 5$.

QUESTION 3 *(start a new page)*

a. Solve for x in the following $|4 + 5x| \leq 20$ 3 m

b. Determine the natural domain of these curves :

i. $y = \frac{2x}{x^2 - 4}$ 2 m

ii. $y = \sqrt{1 - 3x}$ 2 m

c. Shade in the region on the number plane where the following inequalities 3 m

hold simultaneously $x + y \geq 1$ and $x^2 + y^2 \leq 1$

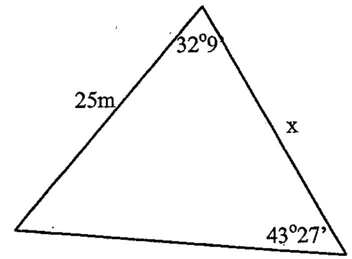
d. Sketch the following curves (showing all important features that assisted your sketch)

i. $y = \frac{1}{x - 2} + 1$ 3 m

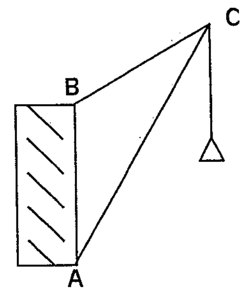
ii. $y = |2x - 1| + 2$ 2 m

QUESTION 4 *(start a new page)*

a. For this figure below, evaluate x to 1 decimal place 3 m



b. The diagram below is of a crane where AB = 7.5m, AC =12.6m and BC = 6m.



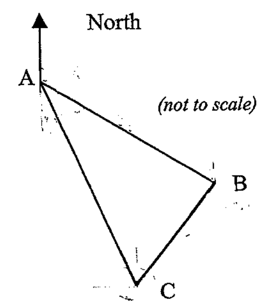
Redraw this crane onto to your exam page and find:

i. < ABC (nearest degree) 2 m

ii. The height of point C above the ground. (nearest metre) 3 m

c. A plane leaves airport A and flies on a bearing of 133° a distance of 350 km to point B.

It then flies on a bearing of 234° for 190 km to point C, as shown in this diagram to the right.



i. Redraw this diagram onto you exam page, marking in all the above information and show that < ABC = 79°. 2 m

ii. Show that the distance from C to A is 365 km (nearest km). 3 m

iii. Find the bearing of A from C (to the nearest degree). 2 m

QUESTION 5 *(start a new page)*

a. Solve the inequation $x^2 + 2 \geq 3x$ 3 m

b. Without the use of a calculator, find the exact value of:

i. $\cos 150°$ 3 m

ii. $\text{cosec } 225°$ 3 m

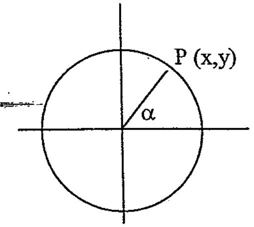
c. Solve for α where $0 \leq \alpha \leq 360°$, if $\sin \alpha = \frac{-1}{2}$. 3 m

d. Sketch one cycle of the curve $y = 2 \sin 3x$. 3 m

- question 6 last page -

QUESTION 6 *(start a new page)*

a. Consider the unit circle and the Point P given to the right.



i. Redraw this figure onto you exam page and state the equation of this circle. 1 m

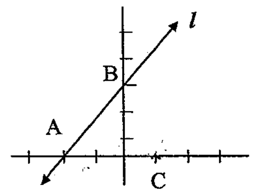
ii. Show that the $\cos \alpha = x$. 1 m

iii. Show that $\sin^2 \alpha + \cos^2 \alpha = 1$. 1 m

iv. Prove that $3 \cos^2 \alpha - 2 = 1 - 3 \sin^2 \alpha$. 2 m

v. By considering statement iii., show that $\sec^2 x - 1 = \tan^2 x$. 2 m

b.



To the left is a diagram of line ***l*** which passes through the points A (-2,0) and B (0,3). *(not to scale)*

The point C is given by the point (1,0).

i. Show that the equation of line ***l*** may be given by $3x - 2y + 6 = 0$. 2 m

ii. Show that the perpendicular distance of the point C to the line ***l*** is $\frac{9}{\sqrt{13}}$ units. 2 m

iii. Determine the area of triangle ABC. 2 m

iv. Determine the coordinates of a point D, such that ABCD would form a parallelogram. 2 m

- end of exam -

BRIGIDINE COLLEGE RANDWICK

PRELIMINARY MATHEMATICS
HALF YEARLY – June, 2007

FORTITER ET SUAVITER

SOLUTIONS AND MARKING SCHEME: Q1 & Q3

QUESTION 1

(a) Loss $= \$39{\cdot}95 - \$32{\cdot}50$

$= \$7{\cdot}45$

Loss as a percentage of cost

$= \dfrac{7{\cdot}45}{399{\cdot}95} \times 100\%$

$= 18{\cdot}6\%$ (1*dp*)

(b) $\dfrac{5}{\sqrt{2}} + \sqrt{2} = \dfrac{5}{\sqrt{2}} \times \dfrac{\sqrt{2}}{\sqrt{2}} + \sqrt{2}$

$= \dfrac{5\sqrt{2}}{2} + \sqrt{2}$

$= \left(\dfrac{5}{2} + 1\right)\sqrt{2}$

$= \dfrac{7}{2}\sqrt{2}$

$\therefore$ A $= \dfrac{7}{2}$

(c)

(i) $2\sqrt{7} - 3\sqrt{28} + \sqrt{63}$

$= 2\sqrt{7} - 3\sqrt{4 \times 7} + \sqrt{9 \times 7}$

$= 2\sqrt{7} - 6\sqrt{7} + 3\sqrt{7}$

$= -\sqrt{7}$

(ii) $\sqrt{\dfrac{12}{8}} = \sqrt{\dfrac{3}{2}}$

$= \dfrac{\sqrt{3}}{\sqrt{2}}$

$= \dfrac{\sqrt{3}}{\sqrt{2}} \times \dfrac{\sqrt{2}}{\sqrt{2}}$

$= \dfrac{\sqrt{6}}{2}$

(d)

(i) $\dfrac{5x+15}{x+1} \div \dfrac{x+3}{2x^2+x-1}$

$= \dfrac{5x+15}{x+1} \times \dfrac{2x^2+x-1}{x+3}$

$= \dfrac{5(x+3)}{(x+1)} \times \dfrac{(2x-1)(x+1)}{(x+3)}$

$= \dfrac{5}{1} \times \dfrac{(2x-1)}{1}$

$= 5(2x-1)$

(ii) $\dfrac{(3x+1)}{(3x-1)} - \dfrac{(3x-1)}{(3x+1)}$

$= \dfrac{(3x+1)}{(3x-1)} \times \dfrac{(3x+1)}{(3x+1)} - \dfrac{(3x-1)}{(3x+1)} \times \dfrac{(3x-1)}{(3x-1)}$

$= \dfrac{(3x+1)^2 - (3x-1)^2}{(3x-1)(3x+1)}$

$= \dfrac{9x^2+6x+1-(9x^2-6x+1)}{(3x-1)(3x+1)}$

$= \dfrac{9x^2+6x+1-9x^2+6x-1}{(3x-1)(3x+1)}$

$= \dfrac{12x}{(3x-1)(3x+1)}$

QUESTION 3

(a) $|4+5x| \le 20$

$4 + 5x \le 20$ and $4 + 5x \ge -20$

$5x \le 16$ and $5x \ge -24$

$x \le \dfrac{16}{4}$ and x>=-24/5

i.e. $-\dfrac{24}{5} \le x \le \dfrac{16}{5}$

(b)

(i) $y = \dfrac{2x}{x^2-4}$

$\dfrac{2x}{x^2-4}$ is not defined for $x^2 - 4 = 0$

i.e. $x^2 = 4$

$x = \pm 2$

Domain: All real x, $x \ne \pm 2$

(ii) $y = \sqrt{1-3x}$

$\sqrt{1-3x}$ is defined for $1 - 3x \ge 0$

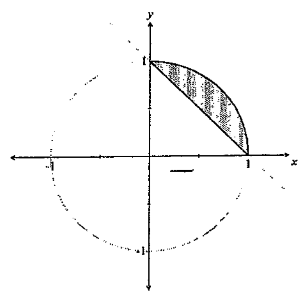
i.e. $-3x \ge -1$

(Divide by -3; switch $\ge$ to $\le$)

$x \le \dfrac{1}{3}$

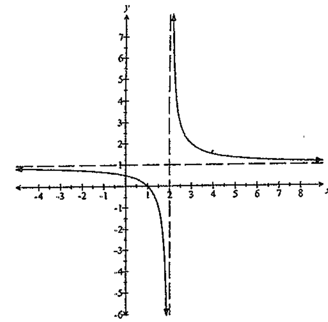
Domain: Real $x \le \dfrac{1}{3}$

(c)

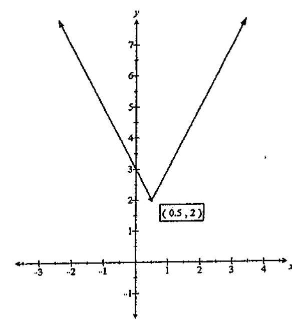


(d)

(i)



(ii)



YR11 ½ yrly 2008 maths (marking scheme)

Q2a) (3marks) $x = \frac{219}{99}$ or $\frac{73}{33}$ or $2\frac{7}{33}$

(2marks) $99x = 2\dot{1}\dot{9}$

(1mark) first 3 lines in solution lines 1 and 3 or fraction with a $\frac{\square}{99}$ or $\frac{219}{\square}$

b) i) (3marks) $x = -2$

(2marks) one mistake after correct expansion.

(1mark) correct expansion

ii) (3marks) $x = -\frac{31}{2}$ or $-15\frac{1}{2}$

(2marks) one mistake or $x = -14\frac{1}{2}$

(1mark) multiplying through by 3 correctly.

iii) (3marks) $x = -3\frac{1}{2}$

(2marks) both correct expansions

(1mark) one correct expansion

c) (3marks) $x = \frac{7}{8}$, $y = \frac{3}{4}$

(2marks) one correct answer ~~or on the~~

(1mark) correctly ~~find~~ subing in to get second value. or on the right track.

Q4a) (3marks) $x = 35{\cdot}2$

(2marks) correct sine rule

(1mark) $104°24'$ or correct sine rule from students diagram

b) i) (2marks) $\theta = 138°$

(1mark) correct cos rule

ii) (3marks) 12m (or correct from i)

(2marks) $\phi = 48$ and $h = 4.45\ldots$ or correct from students working.

(1mark) $\phi = 48$

c) i) (2marks) correctly showing 79° and labelling 2 correct bearings and 2 correct distances.

(1mark) correct labelling

ii) (3marks) AC=365 (with all 3 steps shown)

(2marks) 2 correct lines (in cos rule)

(1mark) one correct line. (in cos rule)

iii) (2marks) 344°

(1mark) on the right track.

Q5 a) (3marks) $x \leq 1$, $x \geq 2$

(2marks) $x \leq 1$ or $x \geq 2$

or incorrectly factorised quadratic but correctly solved (for both)

(1mark) correctly factorised

b) i) (3marks) $-\frac{\sqrt{3}}{2}$

(2marks) $\frac{\sqrt{3}}{2}$

(1mark) $-\cos 30$

ii) (3marks) $-\sqrt{2}$

(2marks) $-\frac{1}{\sin 45}$ or $\sqrt{2}$ or $-\frac{1}{\sqrt{2}}$

(1mark) $\frac{1}{\sin 225}$ or $\frac{1}{\sqrt{2}}$

c) (3marks) $\alpha = 210$ and $330°$

(2marks) $\alpha = 210$ or $330°$

(1mark) $\underbrace{180 + \square \text{ and } 360 - \square}_{\text{must show.}}$

Yr11 2yrly 2008 Mathematics
(marking scheme cont)

Q5d

(3marks) see sketch

(2marks) Sine curve + correct amplitude shown

or Sine curve + correct period shown

(1mark) correct amplitude or correct period shown on sketch.

or ~~sine curve~~ (positive sine curve starting on origin)

Q6 a) i) (1mark) $x^2+y^2=1$

ii) (1mark) $\cos\alpha = \frac{x}{1}$

must show

iii) (1mark) state $\sin\alpha = y$

then $\cos^2\alpha + \sin^2\alpha = 1$

or clearly linking $\sin\alpha$ to $\frac{y}{1}$

iv) (2marks) Proving LHS=RHS

(1mark) using statement

$\cos^2\alpha = 1-\sin^2\alpha$

or $\sin^2\alpha = 1-\cos^2\alpha$

or moving equ from left to right in a proof.

v) (2marks) see solutions

(1mark) $\div \cos^2\alpha$ in each term

* b) i) (2marks) see solutions

(1mark) stating $m=\frac{3}{2}$

or attempting to get equ with wrong grad but correctly subing in A or B

subing in A and B (1mark)

ii) (2marks) see solution

(1mark) ~~correctly~~

$|3\times1+-2\times0+6|$

or $\sqrt{3^2+2^2}$

iii) (2marks) $=4\frac{1}{2}$

(1mark) base $=3$

or height $=3$

or using height $\frac{9}{\sqrt{13}}$ in Δ formula correctly.

or $AB=\sqrt{13}$

iv) (2marks) $(-1,-3)$

(1mark) $(4,3)$

or $x=-1$ or $y=-3$

YR11 $\frac{1}{2}$ yrly 2008 Mathematics (Solutions)

Q2a) $x = 2.212121\ldots$ ①

$10x = 22.121212\ldots$ ②

$100x = 221.212121\ldots$ ③

③ − ① $99x = 219$

$x = \frac{219}{99}$ or $\frac{73}{33}$ or $2\frac{7}{33}$

b) i) $4\left(\frac{1}{x}+2\right) = 5 - \frac{2}{x}$

$\frac{4}{x} + 8 = 5 - \frac{2}{x}$

$4 + 8x = 5x - 2$

$3x = -6$

$x = -2$

ii) $5 - \frac{2x+1}{3} = 15$

$15 - (2x+1) = 45$

$15 - 2x - 1 = 45$

$-2x = 31$

$x = -\frac{31}{2}$ or $-15\frac{1}{2}$

iii) $(2x+3)^2 = (2x-1)(2x+5)$

$4x^2 + 12x + 9 = 4x^2 + 10x - 2x - 5$

$4x = -14$

$x = -3\frac{1}{2}$

c) $2x - y = 1 \longrightarrow y = 2x - 1 \cdots$ ①

$4x + 2y = 5 \cdots$ ②

sub ① in ②

$4x + 2y = 5$

$4x + 2(2x-1) = 5$

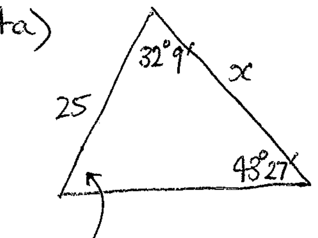
$4x + 4x - 2 = 5$

$8x = 7$

$x = \frac{7}{8}$ sub in ①

$y = 2 \times \frac{7}{8} - 1 = \frac{3}{4}$

Q4a)



$180 - 32°9' - 43°27'$

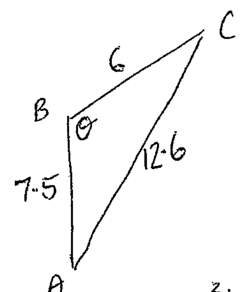
$= 104°24'$

$\frac{x}{\sin 104°24'} = \frac{25}{\sin 43°27'}$

$x = \frac{25 \sin 104°24'}{\sin 43°27'}$

$x = 35.2$

b)



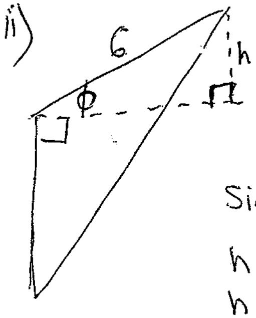
i) $\cos\theta = \frac{6^2 + 7.5^2 - 12.6^2}{2 \times 6 \times 7.5}$

$\cos\theta = -0.739$

$\theta = 137°39'$

$\theta = 138°$

ii)



$\phi = 138° - 90$

$\phi = 48°$

$\sin 48° = \frac{h}{6}$

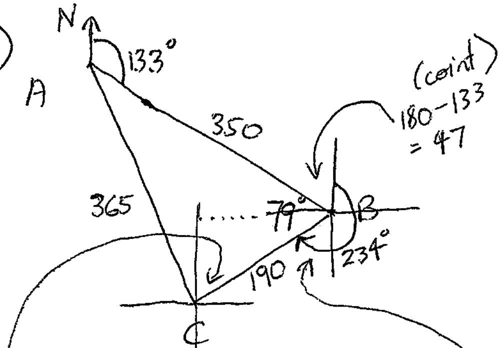
$h = 6 \sin 48$

$h = 4.45886\ldots$

$\therefore c = 7.5 + 4.45886\ldots$

$= 12$ m

c)



i) $360 - 47 - 234 = 79°$

ii) $AC^2 = 350^2 + 190^2 - 2 \times 350 \times 190 \times \cos 79$

$AC^2 = 133222.4036\ldots$

$AC = 365$ km

iii) ~~$\cos C = \frac{365^2 + 190^2 - 350^2}{2 \times 365 \times 190}$~~

~~$C = 70°16'$~~ ← YES

~~270 − 234~~ $234 - 180 = 54°$

also 54° (alt)

$\therefore$ so $70°16 - 54° = 16°16'$

$360 - 16°16' = 343°44'$

$= \underline{344°}$

Q5

a) $x^2 - 3x + 2 \geq 0$

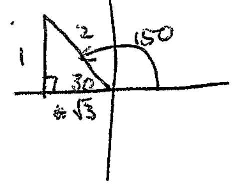
$(x-2)(x-1) \geq 0$

$x \leq 1$, $x \geq 2$

b) i) $\cos 150$

$-\cos 30$

$-\frac{\sqrt{3}}{2}$

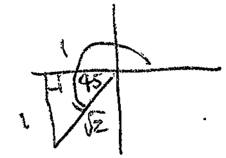


ii) $\operatorname{cosec} 225$

$\frac{1}{\sin 225}$

$\frac{1}{-\sin 45}$

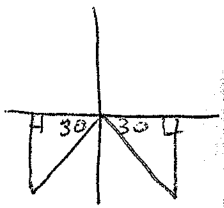
$\frac{1}{-\frac{1}{\sqrt{2}}} = -\sqrt{2}$



c) $\sin\alpha = -\frac{1}{2}$

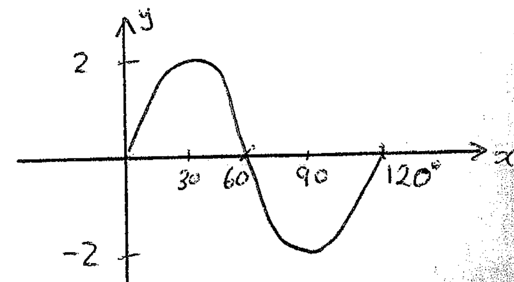
$\alpha = 180 + 30, 360 - 30$

$= 210, 330$

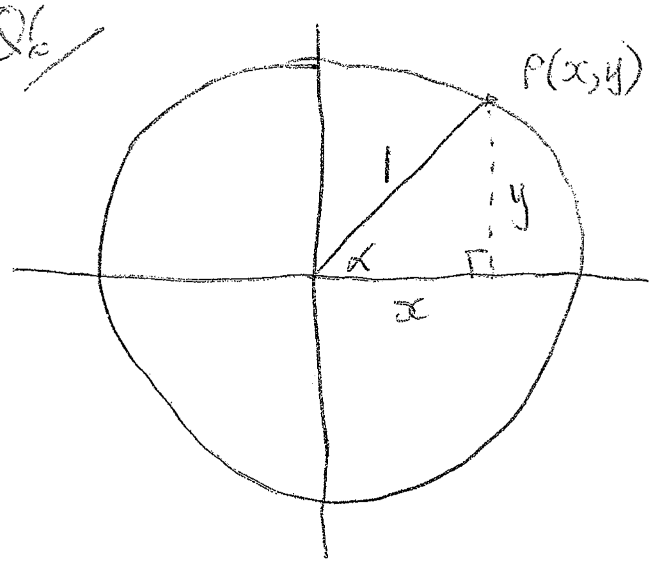


d) $y = 2\sin 3x$

period $= \frac{360}{3} = 120°$



Q6



a) i) $x^2+y^2=1$

ii) $\cos\alpha = \frac{x}{1}$

$\cos\alpha = x$

iii) $\sin\alpha = \frac{y}{1}$

$\sin\alpha = y$

$\therefore \quad x^2+y^2=1$

$\cos\alpha^2 + \sin^2\alpha = 1$

iv) $3\cos^2\alpha - 2 = 1 - 3\sin^2\alpha$

LHS $= 3\cos^2\alpha - 2$

$\cos^2\alpha = 1 - \sin^2\alpha$

$3(1-\sin^2\alpha) - 2$

$3 - 3\sin^2\alpha - 2$

$1 - 3\sin^2\alpha$

$=$ RHS

v) $\sin^2\alpha + \cos^2\alpha = 1$

$\div \cos^2\alpha \longrightarrow$

$\frac{\sin^2\alpha}{\cos^2\alpha} + \frac{\cos^2\alpha}{\cos^2\alpha} = \frac{1}{\cos^2\alpha}$

$\tan^2\alpha + 1 = \sec^2\alpha$

$\tan^2\alpha = \sec^2\alpha - 1$

as required.

b) i) $m = \frac{3}{2}$, $b = 3$

$y = \frac{3}{2}x + 3$

$2y = 3x + 6$

$0 = 3x - 2y + 6$

ii) $d = \frac{|3\times 1 + -2\times 0 + 6|}{\sqrt{3^2+2^2}}$

$= \frac{9}{\sqrt{13}}$

iii) $A = \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times (AC) \times$ height

$= \frac{1}{2} \times 3 \times 3$

$= 4\frac{1}{2}$

iv) $(-1, 3)$