

Student : _____

Teacher : _____

BRIGIDINE COLLEGE
RANDWICK

MATHEMATICS
YEAR 11

HALF-YEARLY

2010

(TIME - 1.5 HOUR)

Directions To Candidates

- * Put your name at the top of this paper and on each of the 5 sections that are to be collected.
- * All 5 questions are to be attempted.
- * All 5 questions are of equal value.
- * All questions are to be answered on separate pages and will be collected in separate bundles at the end of this exam.
- * All necessary working should be shown in every question IN PEN.
- * Full marks may not be awarded for careless or badly arranged work.

Question 1

- a. Evaluate $\frac{\sqrt[3]{7} + \pi}{\sqrt{2^4} \div 5}$ to 3 significant figures. 2
- b. Solve $|2x - 6| \leq 4$ 2
- c. Solve for x : $2x - 4 = x - 2(6 - 3x) + 1$ 2
- d. Fully factorise the following :
- i. $m^2 - n^2 + 5m - 5n$ 3
- ii. $3x^2 + 15x - 72$ 3

Question 2 (Start a new page)

- a. Fully simplify the following :
- i. $\frac{2x^2}{x^2 - y^2} - \frac{2x}{x^2 + xy}$ 3
- ii. $\frac{a^2 + b^2 + 2ab}{a + b}$ 2
- b. Solve $x^2 - 6x - 5 = 0$ by completing the square. 3
- c. A projectile is fired from the ground. It's path is given by the equation $h = 10t - 2t^2$, where h = the projectile's vertical height in (metres) after any time t (seconds).
- i. Sketch the path of the projectile. 1
- ii. For how long is the projectile in the air? 1
- iii. What is its maximum height reached? 2

Question 3 (Start a new page)

- a. Solve these equations simultaneously :

$$x^2 + y^2 - 25 = 0 \quad y + x = 1 \quad 3$$

- b. Solve for x :

i. $(x - 4)^2 = \frac{11 - 3x}{2}$ 3

ii. $(5 - 4x)^3 = 64$ 2

- c. Simplify the following :

i. $\sqrt{32} + 2\sqrt{18} - 5\sqrt{28}$ 2

ii. $(2\sqrt{3} - \sqrt{5})^2$ 2

Question 4 (Start a new page)

- a. Sketch these graphs on separate number planes showing all important features :

i. $y = \sqrt{36 - x^2}$ 1

ii. $y = \sqrt{36 - x}$ 2

- b. Find the domain and range for $y = \sqrt{x - 2}$ 2

- c. Find the value of W in the following table if $y = f(x)$ represents

i. an even function. 1

ii. an odd function. 1

$y = f(x)$				
x	-5	-3	4	3
y	8	-1	14	W

- d. A function is given by the equation $g(x) = x^2 - 6x + 8$, find :

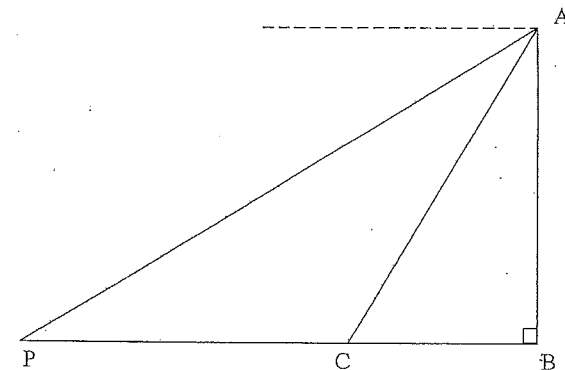
i. $g(a - 3)$. 2

ii. all the values of x for which $g(x) < 0$. 3

Question 5 (Start a new page)

- a. Find the exact value of $\operatorname{cosec} 45^\circ$. 2

- b. The angle of depression from the top of a 12m building A to a person P on the ground is 30° . When the person walks a certain distance toward the building to point C the angle of elevation from the person to the top of the building is 60° .

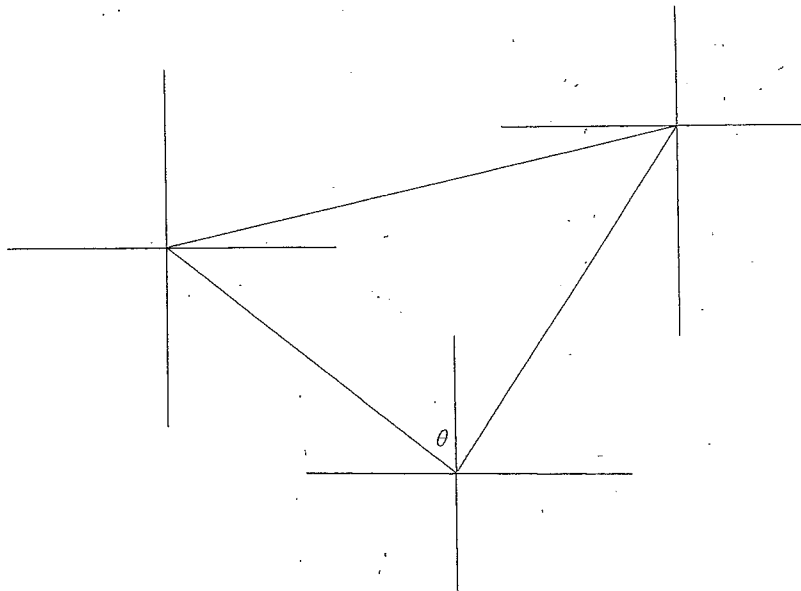


- i. Copy and complete the diagram with the information provided. 1

ii. Show that $CB = 4\sqrt{3}$. 1

iii. Calculate the exact distance that the person has walked towards the building. (i.e. find PC) 2

- c. A ship leaves port and sails on a bearing of $S70^{\circ}E$ for 50km to another port. It then turns and sails on a bearing of 020° for 90km.



- | | | |
|------|--|---|
| i. | Copy and complete the diagram with all the information provided. | 1 |
| ii. | Explain why $\theta = 70^{\circ}$. | 1 |
| iii. | Calculate the distance that the ship is from its starting point. | 1 |
| iv. | Find the bearing of the starting point from its final position. | 3 |

END OF EXAM

Q1a) 2.83

2 marks: 2.83

1 mark: 2.82 or 2.07 or 2.826

or 2.825564723
or write out calculator display
then correctly round from it

2 marks: $1 \leq x \leq 5$

1 mark:

$x \geq 1$ ~~or a silly mistake~~

2 marks: $\frac{7}{5}$

1 mark: correctly expand brackets
or one mistake

i) 3 marks: $(m-n)(m+n+5)$

2 marks: $(m-n)(m+n) + 5(m-n)$

1 mark: $(m-n)(m+n)$ or $5(m-n)$

ii) 3 marks: $3(x+8)(x-3)$

2 marks: $(3x+24)(x-3)$ or
 $(x+8)(3x-9)$

1 mark: $3(x^2+5x-24)$

Q2 i) $\frac{2x^2}{x^2-y^2} - \frac{2x}{x^2+xy}$

$\frac{2x}{x(x-y)(x+y)} - \frac{2x}{x(x+y)} \times \frac{(x-y)}{(x-y)}$

$\frac{2x^3 - 2x^2 + 2xy}{x(x-y)(x+y)}$

$\frac{2x^2 - 2x + 2y}{(x-y)(x+y)}$

ii) $\frac{a^2+b^2+2ab}{a+b}$

$\frac{a^2+2ab+b^2}{a+b}$

$\frac{(a+b)(a+b)}{a+b} = a+b$

b) $x^2 - 6x = 5$

$x^2 - 6x + (\frac{6}{2})^2 = 5 + (\frac{6}{2})^2$

$(x-3)^2 = 14$

$x-3 = \pm\sqrt{14}$

$x = 3 \pm\sqrt{14}$

2a) 3 marks:

$\frac{2x^2 - 2x + 2y}{(x-y)(x+y)}$

2 marks:

$\frac{2x^3 - 2x^2 + 2xy}{x(x-y)(x+y)}$

1 mark: $\frac{2x^2}{(x-y)(x+y)}$

ii) 2 marks: $a+b$

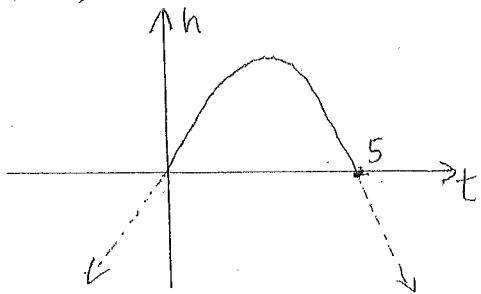
1 mark: numerator = $(a+b)(a+b)$

b) 3 marks: $3 \pm\sqrt{14}$

2 marks: $(x-3)^2 = 14$

1 mark: $x^2 - 6x + 9 = 5 + 9$

Q2c) i) $h = 10t - 2t^2$
 (same as $y = 10x - 2x^2$)
 $0 = 10t - 2t^2$
 $0 = 2t(5 - t)$
 $t = 0, t = 5$



ii) $t = 5$ seconds

iii) $t = 2.5$ seconds

$h = 10 \times 2.5 - 2 \times 2.5^2$
 $= 12.5 \text{ m}$

Q3a) $x^2 + y^2 - 25 = 0$

$x + y = 1 \rightarrow y = 1 - x$

~~$x^2 + (1-x)^2 - 25 = 0$~~

$x^2 + 1 - 2x + x^2 - 25 = 0$

$2x^2 - 2x - 24 = 0$

$x^2 - x - 12 = 0$

$(x-4)(x+3) = 0$

$x = 4$ $x = -3$

$y = -3$ $y = 4$

i) 1 mark: see sketch

ii) 1 mark: 5 seconds or correct from student's sketch

iii) 2 marks: 12.5 m

1 mark: $t = \frac{1}{2} \times \text{part (ii)}$
 time

Q3a) 3 marks: $x = 4$ $x = -3$
 $y = -3$ $y = 4$

2 marks: $x = 4, -3$
 or $y = -3, 4$

1 mark: sub incorrect x -values to get correct y values from student's work. This MUST be shown or solving a quadratic correctly.

b) i) $(x-4)^2 = \frac{11-2x}{2}$

$x^2 - 8x + 16 = \frac{11-3x}{2}$

$2x^2 - 16x + 32 = 11 - 3x$

$2x^2 - 13x + 21 = 0$

$(2x-7)(x-3) = 0$

$x = \frac{7}{2}, 3$

ii) $(5-4x)^3 = 64$

$5-4x = \sqrt[3]{64} = 4$

$4x = 1$

$x = \frac{1}{4}$

c) i) $\sqrt{32} + 2\sqrt{18} - 5\sqrt{28}$

$4\sqrt{2} + 2 \times 3\sqrt{2} - 5 \times 2\sqrt{7}$

$4\sqrt{2} + 6\sqrt{2} - 10\sqrt{7}$

$10\sqrt{2} - 10\sqrt{7}$

ii) $(2\sqrt{3} - \sqrt{5})^2$

$(2\sqrt{3})^2 - 2 \times 2\sqrt{3} \times \sqrt{5} + (\sqrt{5})^2$

$12 - 4\sqrt{15} + 5$

$17 - 4\sqrt{15}$

b) 3 marks: $x = 2, 3$

2 marks: $2x^2 - 13x + 21 = 0$

1 mark: expanding $(x-4)^2$ correctly or correctly solving student's quadratic.

ii) 2 marks: $x = \frac{1}{4}$

1 mark: $\sqrt[3]{64}$

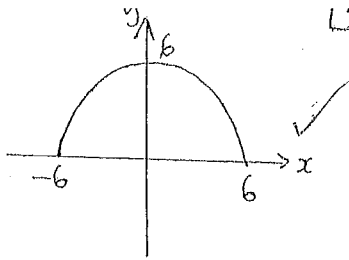
c) i) 2 marks: $10\sqrt{2} - 10\sqrt{7}$

1 mark: 2 correct surd simplification i.e. $4\sqrt{2}$ or $6\sqrt{2}$ or $-10\sqrt{7}$

ii) 2 marks: $17 - 4\sqrt{15}$

1 mark: correct expansion or correct simplification.

Q4 a) i)

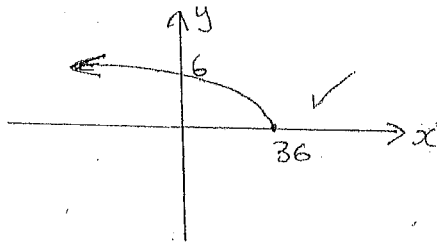


ii)

$$y = \sqrt{36-x}$$

$$y^2 = 36-x$$

$$x = 36-y^2$$



b) $y = \sqrt{x-2}$

D: $x-2 \geq 0$ R: if $x \geq 2$

$x \geq 2$ ✓ $x=2, y=\sqrt{2-2}$
 $y=0$

R: $y \geq 0$

c) i) $w = -1$ ✓

ii) $w = 1$ ✓

d) i) $g(a-3) = (a-3)^2 - 6(a-3) + 8$
 $= a^2 - 6a + 9 - 6a + 18 + 8$
 $= a^2 - 12a + 35$ ✓

ii) $x^2 - 6x + 8 < 0$
 $(x-4)(x-2) < 0$ ✓

i) 1 mark must be correct

ii) 2 mark if sketched
 $x = 36 - y^2$ must show where crosses x and y correctly

1 mark if showed eqn $x = 36 - y^2$ and incorrect graph

i) 1 mark if -ve half drawn
ii) 1 mark if whole parabola
iii) 1 mark if arrow extending on wrong side
iv) must have correct x-y intercept to get marks

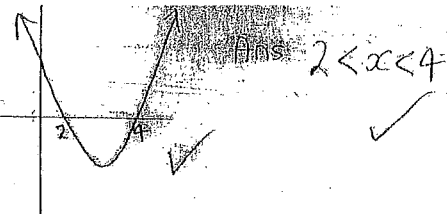
1 mark domain

1 mark range (must have \geq)

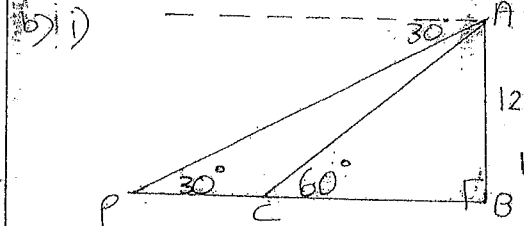
c) i) 1 mark each answer must have numbers not fcs

d) i) 1 mark substituting $x = a - 3$
1 mark expanding and simplifying

ii) 1 mark factorising
1 mark solutions $x = 2, x = 4$
1 mark inequality $2 < x < 4$



5a) $\operatorname{cosec} 45 = \frac{1}{\sin 45} = \frac{1}{\frac{1}{\sqrt{2}}} = \sqrt{2}$

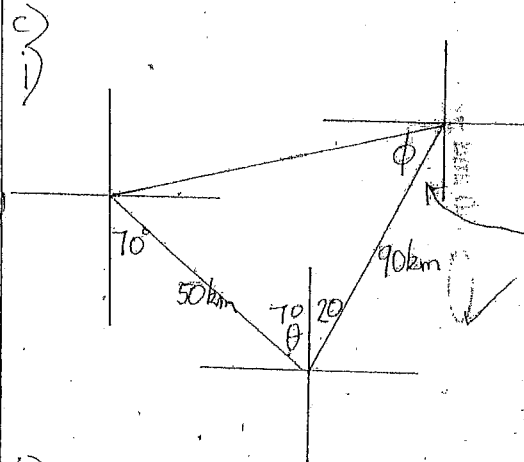


Tan 60 = $\frac{12}{CB}$
 $\frac{\sqrt{3}}{1} = \frac{12}{CB}$

$CB = \frac{12}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$
 $= 4\sqrt{3}$

ii) Tan 30 = $\frac{12}{PB}$
 $\frac{1}{\sqrt{3}} = \frac{12}{PB}$
 $PB = 12\sqrt{3}$

So $PC = 12\sqrt{3} - 4\sqrt{3} = 8\sqrt{3}$ ✓



ii) $\theta = 70^\circ$ (alternate angles) // lines ✓

Quest 5. marks

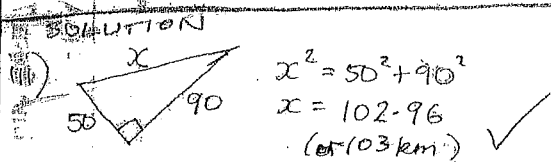
a) must show $\sin 45 = \frac{1}{\sqrt{2}}$ 1 mark and state $\operatorname{cosec} 45 = \frac{1}{\sin 45} = \sqrt{2}$ for 2 marks

b) i) 1 mark labelled diagram

ii) must show $CB = \frac{12}{\sqrt{3}}$ and rationalise denominator to show $= 4\sqrt{3}$ for 1 mark

iii) Show $PB - CB = PC$ 2 marks must be exact value not a decimal.

iv) 1 diagram
v) alternate angles // lines 1 mark



iv) Tan $\phi = \frac{50}{90}$
 $\phi = 29^\circ 3'$ ✓
20 (alternate) ✓

So bearing is $180 + 20 + 29^\circ 3'$
 $= 229^\circ 3'$ ✓

iii) find angle ϕ 1 mark add 20° and 180° 2 marks