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BRIGIDINE COLLEGE RANDWICK

MATHEMATICS

YEAR 11

HALF-YEARLY

2005

(TIME: 1½ HOURS)

Directions to candidates:

- Write your **name** at the top of this question paper and each of the **5 sections** that are to be handed in.

- All 5 questions are to be attempted

- All questions are to be answered on **separate pages** and will be collected **separately at the conclusion of this exam.**

- All necessary working should be shown for every question.

- Full marks may not be awarded for careless or badly arranged work.

QUESTION 1   *(Start a new page)*

(a) Find the value of $23^{-0.5}$ correct to two decimal places.   1

(b) Express $0.2\dot{3}$ as a fraction in its simplest form.   2

(c) Find the value of $\dfrac{5}{8+2\times5}$ correct to 3 significant figures   2

(d) In the formula $N = \dfrac{A}{(1+r)^n}$ find:

     (i) N if $A = 80\,000$, $r = 0.056$ and $n = 6$.   2

     (ii) r if $N = 5000$, $A = 9000$ and $n = 10$.   2

(e) A shop has a sale in which all items are sold with 12% off the marked price. A   2
blouse was purchased for \$158.40. What was the marked price?

(f) Expand $(3m + 2)(m + 1)$.   1

(g) Expand and simplify $(3x + 2y)(3x - 2y) - (2x + 3y)^2$.   2

(h) Rationalise the denominator of $\dfrac{4}{\sqrt{5}+2}$ 2

QUESTION 2 *(Start a new page)*

(a) Factorise fully:

    (i) $x^2 + 16x + 63.$ **1**

    (ii) $3x^2 - 16x + 5.$ **2**

    (iii) $5x^3 - 40$ **2**

(b) Simplify $3\sqrt{5} - \sqrt{20} + \sqrt{45}$.

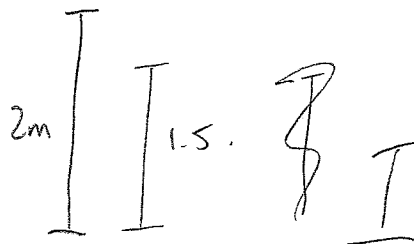(c) Simplify $\dfrac{x}{x^2 - 16} - \dfrac{2}{x - 4}$ **2**

(d) Solve

    (i) $\dfrac{5}{2 - x} = 3$ **1**

    (ii) $\dfrac{5m}{2} - \dfrac{m + 3}{5} = 4 - m$ **2**

    (iv) $6(2a - 9) - 5(a - 4) = 3a$ **2**

(e) A ball is dropped from a height of 2 metres onto a hard floor and bounces. After   **2** each bounce, the maximum height reached by the ball is 75% of the previous maximum height.

What is the maximum height reached after the third bounce?

QUESTION 3   *(Start a new page)*

(a) Solve $2x^2 - 4x - 7 = 0$ 2

(b) Solve $\dfrac{3}{x + 1} = 4x$ 3

(handwritten:) $15x + 5y = 130$
$-15x + 3y = -138$

(c) Solve the simultaneous equations   3
$$3x + y - 26 = 0$$
$$46 + y = 5x$$

(handwritten:)
$(3x + y = 26 \cdot)5$
$(-5x + y = -46)3$
$3x + y = 26$
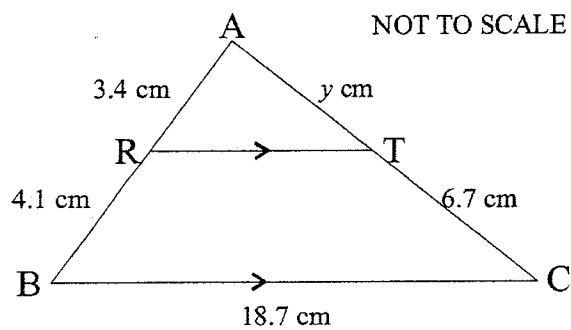$(-5x + y = -46)-1$ $5x - y = 46$

(d) (i) Solve $|x + 2| \leq 3$.   2

(handwritten:) $2x = 20$

   (ii) Graph your solution to (*i*) on a number line.   1

(e)

A   NOT TO SCALE

3.4 cm y cm

R   T

4.1 cm   6.7 cm

B   C

18.7 cm

ABC is a triangle.

RT is drawn parallel to BC.

AR = 3·4 cm; RB = 4·1 cm; AT = y cm; BC = 18·7 cm; and TC = 6·7 cm

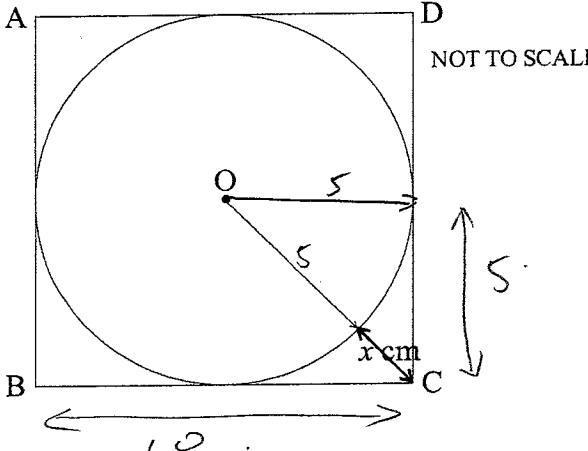   (i) Prove $\triangle$ART and $\triangle$ABC are similar.   2

   (ii) Find y, correct to one decimal place.   2

QUESTION 4 *(Start a new page)*
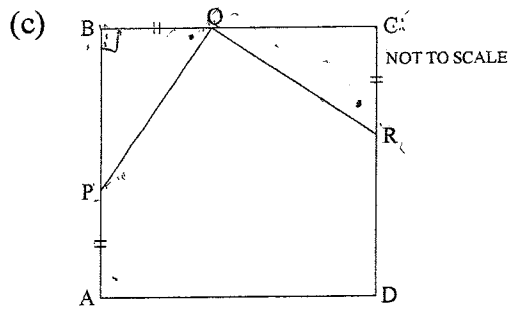
(a) Evaluate, correct to two decimal places $\sqrt{\dfrac{5^2 + 11^2}{208 - 11^2}}$

2

(b)

3

NOT TO SCALE

$5^2 + 5^2 = (x+5)^2$

$50 = x^2 + 10x + 25$

A circle of radius 5 cm is drawn in a square ABCD as shown.
O is the centre of the circle.
OC = (5 + x) cm.
Find the exact value of x.

(c)

NOT TO SCALE

In the diagram; ABCD is a square; P, Q and R lie on AB, BC and CD respectively such that AP = BQ = CR.
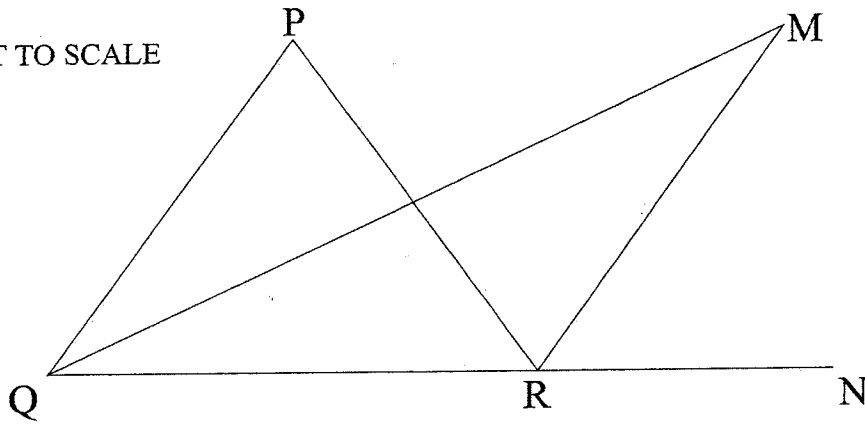
    (i)   Prove $\triangle$PBQ and $\triangle$QCR are congruent.　　2

    (ii) Prove $\angle$PQR is a right angle.　　2

Question 4 is continued on the next page

QUESTION 4 (*Continued*)

(d)

NOT TO SCALE

ΔPBR is an isosceles triangle in which:

    PQ = PR;

    $\angle$QPR = 104°;

    QR is produced to N;

    QM bisects $\angle$PQR;

    RM bisects $\angle$PRN

  (*i*)   Draw a neat sketch and mark on it all the given information.   1

 (*ii*)   Find the size of $\angle$PQR. Give reasons for your answer.   2

(*iii*)   Find the size of $\angle$QMR. Give reasons for your answer.   2

QUESTION 5   *(Start a new page)*

(a) If $f(x) = 2x^2 - x$ find $f(1)$ 1

(b) If $g(x) = \begin{cases} 2-x^2 & \text{if } x \geq 0 \\ 3-x & \text{if } x < 0 \end{cases}$ find $g(4)$.   1

(c) For the function $f(x) = x^2 + 3x$ find:

    (i) $f(x + h)$ 1

    (ii) $f(x + h) - f(x)$ 1

(d) For each of the following functions write down the domain:

    (i) $y = x^2 + 2$ 1

    (ii) $y = \dfrac{1}{x}$ 1

    (iii) $y = \sqrt{9-x}$ 1

    (iv) $y = |x-5|$ 1

(e) Sketch the graph of each function and state the range

    (i) $y = x^2 + 5$ 2

    (ii) $y = -3x^2 + 6x$ 2

    (iii) $y = \dfrac{1}{x-3}$ 2
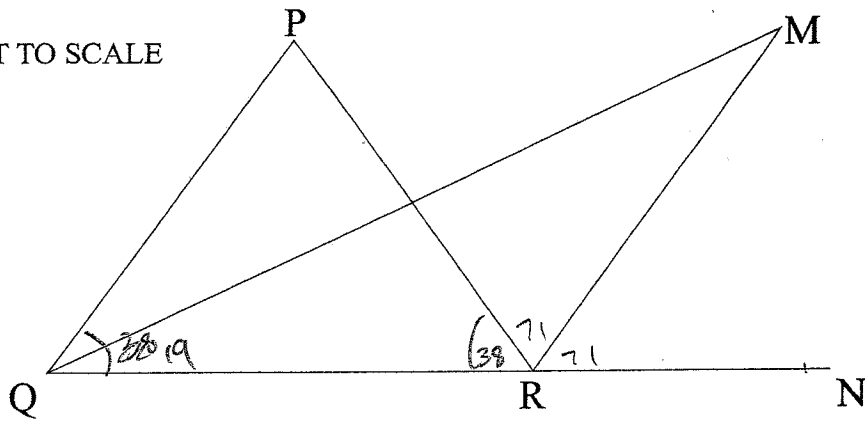
    (iv) $y = |2x - 3|$ 2

QUESTION 4 (*Continued*)

(d)

NOT TO SCALE

ΔPQR is an isosceles triangle in which:

PQ = PR;

∠QPR = 104°;

QR is produced to N;

QM bisects ∠PQR;

RM bisects ∠PRN

(*i*)   Copy the above diagram and mark on it all the given information.   1

(*ii*)   Find the size of ∠PQR. Give reasons for your answer.   2

(*iii*)   Find the size of ∠QMR. Give reasons for your answer.   2

QUESTION 2

(i) 1 mark for correct answer

$x^2 + 16x + 63$

$\times \begin{array}{c} 63 \\ + 16 \\ \hline 7, 9 \end{array}$

$= (x+7)(x+9)$

(ii) 1 mark for process
1 mark for answer

$3x^2 - 16x + 5$ $\times \begin{array}{c} 3\times 5 = 15 \\ + -16 \\ \hline -1, -15 \end{array}$

$= \dfrac{(3x-15)(3x-1)}{3}$

$= \dfrac{3(x-5)(3x-1)}{3}$

$= (x-5)(3x-1)$

OR $3x^2 - 16x + 5$ $\begin{array}{c} 3x \ \diagdown \ 1 \\ x \ \diagdown \ 5 \end{array}$

$= (3x-1)(x-5)$

) 1 mark for common factor
1 mark for diff'ce of two cubes

$5x^3 - 40 = 5(x^3 - 8)$

$= 5(x^3 - 2^3)$

$= 5(x-2)(x^2 + 2x + 4)$

:) 1 mark for correct common denominator
1 mark for answer

$\dfrac{x}{x^2 - 16} - \dfrac{2}{x-4}$

$= \dfrac{x}{(x+4)(x-4)} - \dfrac{2}{(x-4)} \times \dfrac{(x+4)}{(x+4)}$

$= \dfrac{x - 2(x+4)}{(x+4)(x-4)}$

$= \dfrac{x - 2x - 8}{(x+4)(x-4)}$

$= \dfrac{-x - 8}{(x+4)(x-4)}$

(d)(i) 1 mark for correct answer

$\dfrac{5}{2-x} = 3$

$5 = 3(2-x)$

$5 = 6 - 3x$

$3x = 1$

$x = \dfrac{1}{3}$

(ii) 1 mark for
$25m - 2(m+3) = 10(4-m)$
1 mark for answer.

$\dfrac{5m}{2} - \dfrac{m+3}{5} = 4 - m$

$\times 10 \qquad \qquad \times 10$

$\dfrac{5m}{2} \times 10 - \dfrac{(m+3)}{5} \times 10 = (4-m) \times 10$

$25m - 2(m+3) = 10(4-m)$

$25m - 2m - 6 = 40 - 10m$

$23m - 6 = 40 - 10m$

$33m = 46$

$m = \dfrac{46}{33}$ or $1\dfrac{13}{33}$

or 1.39

Q2 cont'd

(iv) 1 mark for $7a - 34 = 3a$
or equivalent expansion and simplification
1 mark for answer.

$6(2a-9) - 5(a-4) = 3a$

$12a - 54 - 5a + 20 = 3a$

$7a - 34 = 3a$

$4a = 34$

$a = \dfrac{34}{4}$

$= \dfrac{17}{2}$ or $8\dfrac{1}{2}$

(e) 1 mark for correctly calculating height after 1st bounce
1 mark for height after 3rd bounce.

Height after 1st bounce
$= 75\%$ of $2m$
$= 1.5 m$

After 2nd bounce: 75% of $1.5m$
$= 1.125 m$

After 3rd bounce: 75% of $1.125m$
$= 0.84375 m$

OR $\dfrac{27}{32}$ m

or equivalent in cm.

QUESTION 4

(a) 2 marks for 1.30
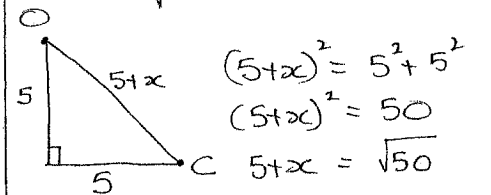1 mark for 1.29
1 mark for "correct to 2 d.p."

(b) 3 marks for $5\sqrt{2}-5$, $-5+5\sqrt{2}$
$5(\sqrt{2}-1)$, $\sqrt{50}-5$, $-5+\sqrt{50}$

2 marks for $-5 \pm \sqrt{50}$, $-5 \pm 5\sqrt{2}$
– negative value must be discarded as x is a "length".

2 marks for "decimal approximation" to correct answer.
i.e. 2.071067812

1 mark for theorem of Pythagoras correctly applied in context of question.
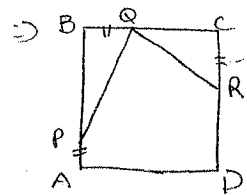

$(5+x)^2 = 5^2 + 5^2$

$(5+x)^2 = 50$

$5+x = \sqrt{50}$

$x = \sqrt{50} - 5$

$= 5\sqrt{2} - 5$

QUESTION 4 cont.

:)

(i) 2 marks requires:
- correct congruence test SAS.
- Showing why BP=CQ
- Other correct elements of SAS with reasons.

1 mark if showing BP=CQ is omitted or inadequate.

As
ABCD is a square
$AP = BQ = CR$ (Given)
Then
$AB = AP + PB$
i.e $PB = AB - AP$
Similarly
$QC = BC - BQ$
$= AB - AP$ (BC, AB sides of sq)
$= PB$

In $\triangle PBQ$ and $\triangle QCR$
$BQ = CR$ (Given) S
$\angle PBQ = \angle QCR$ (Angles of a square) A
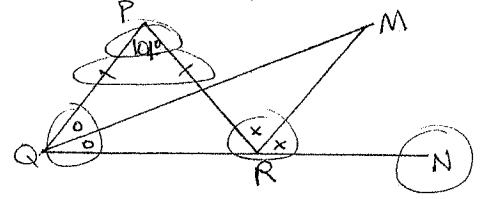$PB = QC$ (See above) S

$\angle PBQ$ and $\angle QCR$ are included angles
$\therefore \triangle PBQ \equiv \triangle QCR$ (SAS)

(ii) As $\triangle PBQ \equiv \triangle QCR$
$\angle BPQ = \angle RQC$ (Corresp angles in congr. $\triangle$s)

Also $\angle BPQ + \angle PQB = 90°$
(Complementary angles in rt. angled $\triangle PBQ$)

Thus $\angle RQC + \angle PQB = 90°$

$\angle BQP + \angle PQR + \angle RQC = 180°$
(Str. angle)

i.e $\angle PQR + 90° = 180°$

$\therefore \angle PQR = 90°$

(d)(i) 1 mark for 3 out of 5 items correctly indicated

(ii) 1 mark for 38°
1 mark for reasons — must include
- sum of angles of a $\triangle$
- angles opp. equal sides (base angles of isos $\triangle$ accepted)

In $\triangle PQR$
$\angle PQR = \angle PRQ$ (opp. equal sides in isosceles $\triangle$)

QUESTION 4 cont.

$104° + 2\angle PQR = 180°$ (Angle sum of $\triangle$)
$2\angle PQR = 76°$
$\angle PQR = 38°$

(iii) $\angle PRN = \angle PQR + \angle QPR$
(Ext'l angle of $\triangle PQR$)
$= 38° + 104°$
$= 142°$
$\angle PRM = \frac{1}{2} \times \angle PRN$ (Given)
$= 71°$
$\angle QRM = \angle QRP + \angle PRM$
$= 38° + 71°$
$= 109°$
$\angle MQR = \frac{1}{2} \times \angle PQR$ (Given)
$= 19°$

In $\triangle MQR$
$\angle QMR = 180° - (\angle QRM + \angle MQR)$
(Angle sum of $\triangle$)
$= 180° - (109° + 19°)$
$= 52°$

yr11 2005 ½ yrly

2a) 0.21 ✓

b) $x = 0.2323\ldots$
$100x = 23.2323\ldots$ ✓
$99x = 23$
$x = \frac{23}{99}$ ✓

students needed to stop
at $100x = 23.23\ldots$ for the mark

c) 0.278 ✓✓
* 0.277 ✓

d)
i) $N = \frac{80000}{(1+0.056)^6}$ ✓
$= 57690.7889$ ✓

ii) $5000 = \frac{9000}{(1+r)^{10}}$
$(1+r)^{10} = \frac{9000}{5000}$
$(1+r)^{10} = 1.8$
$1+r = \pm\sqrt[10]{1.8}$
$r = -1 + \sqrt[10]{1.8}$ or $-1 - \sqrt[10]{1.8}$
$r = 0.0605\ldots$ ✓ or $-2.0605\ldots$

[showing me $\sqrt[10]{}$ somewhere = 1 mark]

Note: full marks were given for 0.0605
but not in future!!!

e)

88%	12%

158.40 ←
$\therefore 88\% = 158.40$ ✓
$1\% = 1.8$
$100\% = \$180$ ✓

f) $(3m+2)(m+1)$
$3m^2 + 3m + 2m + 2$
$3m^2 + 5m + 2$ ✓

g) $(3x+2y)(3x-2y) - (2x+3y)^2$
$(9x^2 - 4y^2) - (4x^2 + 12xy + 9y^2)$
$9x^2 - 4y^2 - 4x^2 - 12xy - 9y^2$ [1 mark for correctly expanding one binomial]
$5x^2 - 12xy - 13y^2$ ✓✓

h) $\frac{4}{\sqrt{5}+2} \times \frac{\sqrt{5}-2}{\sqrt{5}-2}$ ✓

$\frac{4\sqrt{5}-8}{5-4}$

$= 4\sqrt{5} - 8$ ✓

Q3
a) $2x^2 - 4x - 7 = 0$

$2x \times x \quad 7,2 \quad$ does not
$\quad\quad 2,7 \quad$ work so try formula.

$x = \frac{4 \pm \sqrt{4^2 - 4 \times 2 \times -7}}{2 \times 2}$ ✓

$x = \frac{4 \pm \sqrt{72}}{4}$ ✓ Note question should have been completely simplified in future!

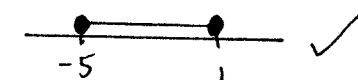$x = \frac{4 \pm \sqrt{36} \times \sqrt{2}}{4}$

$x = \frac{4 \pm 6\sqrt{2}}{4}$

$x = \frac{2(2 \pm 3\sqrt{2})}{4}$

$x = \frac{2 \pm 3\sqrt{2}}{2}$ ← correct answer.

b) $\frac{3}{x+1} = 4x$
$3 = 4x(x+1)$
$3 = 4x^2 + 4x$
$0 = 4x^2 + 4x - 3$ ✓
$2x \times -1$
$2x \times +3$
$(2x-1)(2x+3) = 0$ ✓
$x = \frac{1}{2}$ or $x = \frac{-3}{2}$ ✓

c) $3x + y - 26 = 0 \cdots ①$
$46 + y = 5x \cdots ②$
① $y = 26 - 3x$ sub in ②
$46 + (26 - 3x) = 5x$ ✓
$72 - 3x = 5x$
$72 = 8x$
$x = 9$ ✓
$y = 26 - 3 \times 9 = -1$ ✓
(correct procedure = 1mark)

d) i) $|x+2| \leq 3$
$-3 \leq x+2 \leq 3$ ✓
$-5 \leq x \leq 1$ ✓

ii)

(mark awarded if correctly drawn from part (i) answer)

e)
i) $\angle ART = \angle ABC$ (corresp $\angle$'s = ✓ in // lines)
$\angle A$ common ✓
$\therefore \triangle ART \,|||\, \triangle ABC$
(any 2 correct angle reasons got the marks)

ii) $\frac{y}{3.4} = \frac{y+6.7}{3.4+4.1}$ ✓
$\frac{y}{3.4} = \frac{y+6.7}{7.5}$
$7.5y = 3.4(y+6.7)$
$7.5y = 3.4y + 22.78$
$4.1y = 22.78$
$y = 5.6$ ✓

Q5a) $f(1) = 2 \times 1^2 - 1 = 1$ ✓
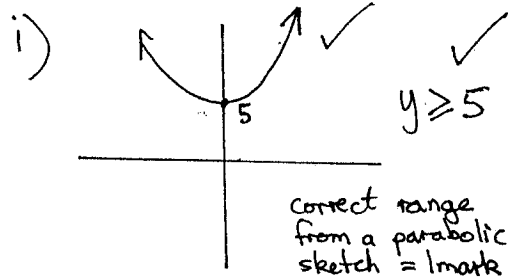
b) $g(4) = 2 - 4^2 = -14$ ✓

c) i) $f(x+h) = (x+h)^2 + 3(x+h)$
$= x^2 + 2xh + h^2 + 3x + 3h$ ✓

ii) $f(x+h) - f(x)$
$x^2 + 2xh + h^2 + 3x + 3h - (x^2 + 3x)$
$x^2 + 2xh + h^2 + 3x + 3h - x^2 - 3x$
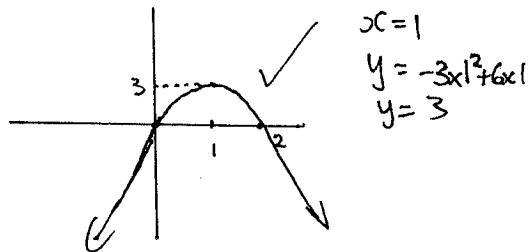$h^2 + 2xh + 3h$ ✓

for part ii) If students used incorrect answer from part i to then correctly simplify. Mark was awarded.

d)i) $x \in \mathbb{R}$ ✓   iv) $x \in \mathbb{R}$ ✓
ii) $x \neq 0$ ✓
iii) $9 - x \geq 0$ ✓
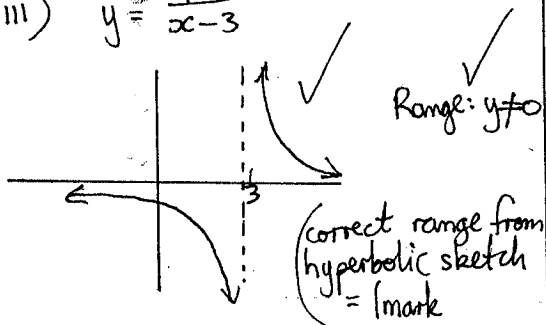$x \leq 9$

Q5e yr 11 2005 $\frac{1}{2}$ yrly.

i)

✓ ✓

$y \geqslant 5$

5

correct range
from a parabolic
sketch = 1 mark

ii) $y = -3x^2 + 6x$

cuts x-axis when $y = 0$

$0 = -3x^2 + 6x$

$3x^2 - 6x = 0$

$3x(x-2) = 0$

$x = 0 \quad x = 2$

✓

$x = 1$

$y = -3 \times 1^2 + 6 \times 1$

$y = 3$

∴ range : $y \leqslant 3$ ✓

(correct range from a parabolic
sketch = 1 mark)

iii) $y = \frac{1}{x-3}$

✓ ✓

Range: $y \neq 0$

(correct range from
hyperbolic sketch
= 1 mark

iv) $y = |2x-3|$

$2x - 3 = 0$

$x = 1\frac{1}{2}$

Range : $y \geqslant 0$ ✓

✓

$1\frac{1}{2}$

(correct range from absolute
value sketch = 1 mark)

Note : range needs
to be correct from
students sketches