Student ______________________

BRIGIDINE COLLEGE
RANDWICK

PRELIMINARY
EXTENSION 1
MATHEMATICS

YEARLY

2009

(Time - 90 minutes)

Directions to candidates

* *Put your* ***name at the top*** *of this paper and on each of the* ***5 sections*** *that are to be collected.*
* *All 5 questions are to be attempted.*
* *All 5 questions are of equal value.*
* *All questions are to be answered on separate pages and will be collected in separate bundles at the end of this exam.*
* *All necessary working should be shown in every question.*
* *Full marks may not be awarded for careless or badly arranged work.*

Question 1

a. Evaluate the following (leaving your answer in simplified surd form where necessary):

i. $\int \frac{dx}{\sqrt{x}}$ 2

ii. $\int (1-x)^4\,dx$ 2

iii. $\int \frac{x^2+1}{2x^2}\,dx$ 2

b. Differentiate the following with respect to x **(leave your answers fully simplified)** :

i. $y = \frac{1}{\sqrt{2x+1}}$ 2

ii. $y = \frac{x^2+1}{2x}$ 2

c. i. Show that the equation of the tangent of the curve $y = x^3 - 2x$ at $x = 2$ is given by $y = 10x - 16$ 2

ii. Find the equation of the normal. 1

iii. Determine the area of the triangle bounded by the y axis , the tangent and the normal (Do not attempt to sketch $y = x^3 - 2x$). 2

Question 2 (Start a new page)

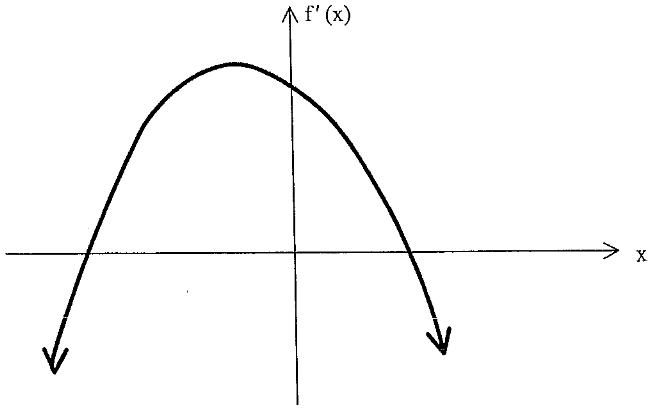
a. Find the area bound by the curve $y = x^2 + 1$ between $y = 1$ to $y = 3$. (leave your answer in simplified exact form) 3

b. Evaluate $\sum_{n=2}^{6} n^2$ 1

c. Consider the following statement : " The governments strategy to slow the increasing rate of swine flu infections on the population is taking effect".

Interpret the above statement with reference to $\frac{dP}{dt}$ and $\frac{d^2P}{dt^2}$, where P = population getting the virus, and t = time. 2

d.



i. Copy the above diagram onto your answer sheet. (Make it a large diagram please)

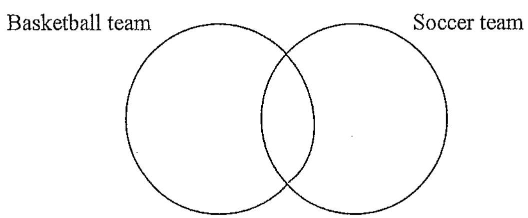
ii. On the same axes draw the graph of $y = f(x)$. 2

e. The equation of a parabola is given by $(x - 1)^2 = 8y$.

i. Write down the coordinates of the vertex. 1

ii. Find the focal length. 1

f. The Suns are a basketball team and the Hammers are a soccer team. A total of 18 people are members of one or the other or both teams. The soccer team has 14 members and the basketball team has 10 members. The diagram below represents the players in the teams.



i. Copy the diagram and place appropriate numbers in each section. 1

ii. A player is selected at random from the basketball team. What is the probability that the player is not in the soccer team? 1

g. i. How many terms are there in the sequence -20,–5,10,25,……., 955 ? 2

ii. Find the sum of the above terms. 1

Question 3 (Start a new page)

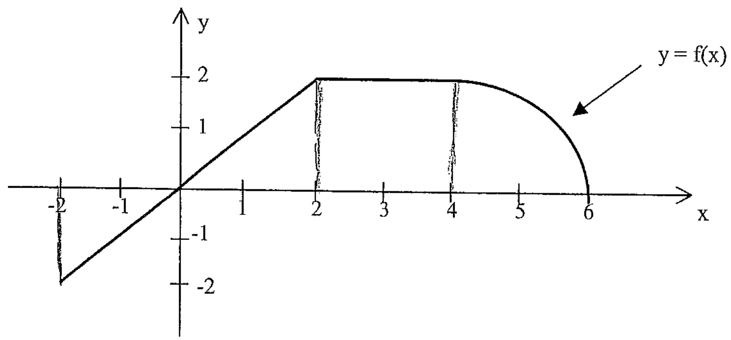
a. i. Without using calculus, sketch the graph of $y = 2x(x - 3)^2$ clearly indicating the x intercepts. 1

ii. Use 4 applications of the Tapeziodal rule to find the approximate area of the closed region bounded by the curve and the x-axis. 2

iii. Find the exact area of the closed region bounded by the curve and the x-axis. 2

iv. Determine the percentage error in area when using the trapezoidal rule. 1

b. The graph of the function $y = f(x)$ consists of a straight line segment from , a horizontal straight line segment from and a quarter circle.



i. Evaluate $\int_{-2}^{6} f(x)\,dx$. 2

iii. For what value(s) of x between $-2 < x < 6$ is the function not differentiable ? 1

c. The probability that any pregnant Australian woman will have twins is $\frac{1}{60}$.

i. What is the probability that a pregnant Australian woman will not have twins? 1

ii. Maria's three daughters are pregnant. One of Maria's daughters is pregnant with twin boys and the other two daughters are expecting single babies of unknown sex.
What is the probability that at least one of Maria's expected four grandchildren will be female? Assume that males and females are equally likely. 2

d. For what values of x does the function $y = x^4 + \frac{7}{3}x^3 - 5$ concave upwards? 3

Question 4 (Start a new page)

a. For the curve $f(x) = 4x^3 - x^4$:

i. Find the x intercepts of the curve. 1

ii. Find any stationary points and determine their nature. 3

iii. Find any points of inflexion. 2

iv. Sketch the curve, showing all of the above information as well as any other important features over the domain $-1 \le x \le 4$. Please draw a large enough diagram. 2

v. Solve $4x^3 - x^4 > 0$ over the given domain. 1

b. **Clearly** show that $y = \frac{1}{3-x}$ is increasing for all values of x where $x \neq 3$. 2

c. Consider the geometric series :

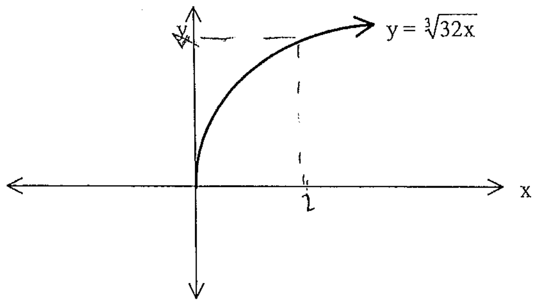
$$\frac{1}{2} + \frac{(x+1)}{4} + \frac{(x+1)^2}{8} + \ldots\ldots\ldots\ldots\ldots$$

i. Find the values of x for which this geometric series has a limiting sum. 2

ii. Given that this geometric series has a limiting sum, find an expression for the limiting sum in terms of x. 2

Question 5 (Start a new page)

a. The diagram below shows the graph of $y = \sqrt[3]{32x}$ for $x \ge 0$.



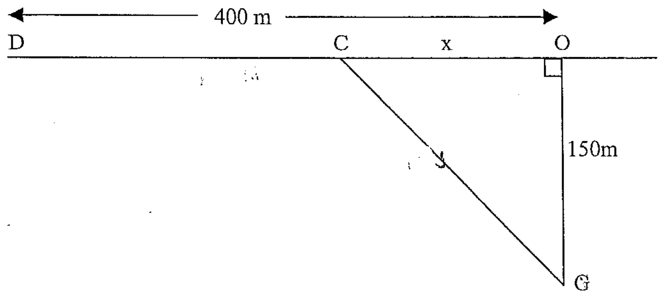
Calculate the volume of the solid formed when the section of the curve $y = \sqrt[3]{32x}$ bounded by the y axis and $x = 2$ is rotated about the y axis. 4

b. Davo is retiring next week and his Superannuation Fund contains $1 200 000. The fund is earning 6%p.a. compound interest, compounding monthly. Davo wants to withdraw a regular amount of $8000 per month to live on in his retirement. Let A represent the amount of money left in the account.

i. Show that after 3 months the amount left in his account A_3 is given by $A_3 = \$1\ 193\ 969.95$ 2

ii. Show that $A_n = 1\,600\,000 - 400\,000(1.005^n)$, hence find how long the money will last. 3

c. A girl on a surfboard is 150metres from the nearest point O of a straight beach. Her destination D,is 400metres along the beach from O. She can paddle at 15m/s and walk at 20m/s. The girl realises that the quickest way to get to D is to paddle to a point C, somewhere between O and D and then walk as shown in the diagram below.



i. If OC = x metres , show that GC is a distance of $\sqrt{22500 + x^2}$ metres. 1

ii. Explain why the time taken from G to C could be expressed as $\dfrac{\sqrt{22500 + x^2}}{15}$ seconds. 1

iii. Show that the total time , T seconds, to travel from G to C and then to D could be expressed as :

$$T = \frac{\sqrt{22500 + x^2}}{15} + \frac{400 - x}{20}$$ 1

iv. Use calculus to find the distance (to the nearest metre) her landing at point C should be from O so that she completes her trip in the least amount of time. 3

END OF EXAM

Q1a) i) $\int x^{-\frac{1}{2}}\,dx$

$\frac{x^{\frac{1}{2}}}{\frac{1}{2}} = 2\sqrt{x} + C$

ii) $\int (1-x)^4\,dx$

$\frac{(1-x)^5}{5x-1}$

$-\frac{(1-x)^5}{5} + C$

iii) $\int \frac{x^2+1}{2x^2}\,dx$

$\int \frac{x^2}{2x^2} + \frac{1}{2x^2}\,dx$

$\int \frac{1}{2} + \frac{1}{2}x^{-2}\,dx$

$\frac{1}{2}x + \frac{1}{2}\times\frac{x^{-1}}{-1}$

$\frac{1}{2}x - \frac{1}{2x} + C$

or $\frac{x^2-1}{2x} + C$

b) i) $y = (2x+1)^{-\frac{1}{2}}$

$y' = -\frac{1}{2}(2x+1)^{-\frac{3}{2}} \times 2$

$y' = -\frac{1}{(2x+1)^{\frac{3}{2}}}$

$= -\frac{1}{\sqrt{(2x+1)^3}}$

ii) $y = \frac{x^2+1}{2x}$

$y = \frac{x^2}{2x} + \frac{1}{2x}$

$y = \frac{1}{2}x + \frac{1}{2}\times x^{-1}$

$y' = \frac{1}{2} - \frac{1}{2}\times x^{-2}$

$y' = \frac{1}{2} - \frac{1}{2x^2}$

or $\frac{x^2-1}{2x^2}$

or $y' = \frac{2x(2x) - 2(x^2+1)}{(2x)^2}$

$= \frac{4x^2 - 2x^2 - 2}{4x^2}$

$= \frac{2x^2-2}{4x^2}$

$= \frac{2(x^2-1)}{4x^2}$

$= \frac{x^2-1}{2x^2}$

c) i) $y = x^3 - 2x$

$y' = 3x^2 - 2$

$m = 3\times 2^2 - 2$

$m = 10$

$x = 2$

$y = 2^3 - 2\times 2$

$y = 4$

$(2, 4)$

$y - 4 = 10(x-2)$

$y - 4 = 10x - 20$

$y = 10x - 16$

ii) $m_\perp = -\frac{1}{10}$ $(2, 4)$

$y - 4 = -\frac{1}{10}(x-2)$

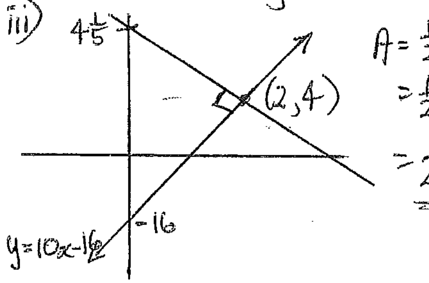
$y - 4 = -\frac{1}{10}x + \frac{1}{5}$

$y = -\frac{1}{10}x + 4\frac{1}{5}$

or $10y = -x + 42$

$0 = x + 10y - 42$

iii)

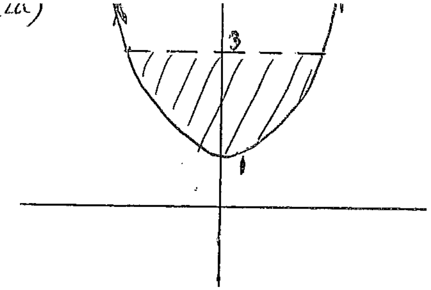


$A = \frac{1}{2}\times b\times h$

$= \frac{1}{2}\times(20\frac{1}{5})\times 2$

$= 20\frac{1}{5}$

Q2a)



$y = x^2 + 1$

$x^2 = y - 1$

$x = \pm\sqrt{y-1}$

$A = 2\int_1^3 \sqrt{y-1}\,dy$

$= 2\int_1^3 (y-1)^{\frac{1}{2}}\,dy$

$= 2\left[\frac{2(y-1)^{\frac{3}{2}}}{3}\right]_1^3$

$= 2\left[\frac{2(3-1)^{\frac{3}{2}}}{3} - 0\right]$

$= \frac{4}{3}[2]^{\frac{3}{2}}$

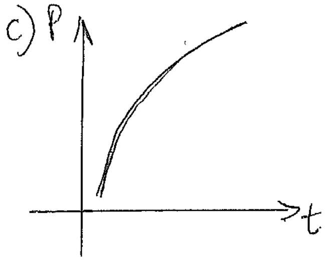
$= \frac{4}{3}\times\sqrt{2^3}$

$= \frac{4}{3}\times\sqrt{8}$

$= \frac{4}{3}\times 2\sqrt{2} = \frac{8\sqrt{2}}{3}$

b) $2^2 + 3^2 + 4^2 + 5^2 + 6^2 = 90$

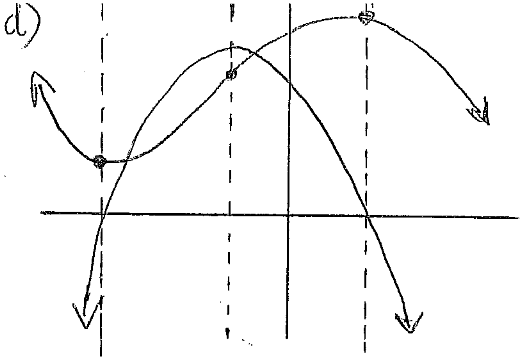
c)



$\frac{dP}{dt} > 0$

$\frac{d^2P}{dt^2} < 0$

d)



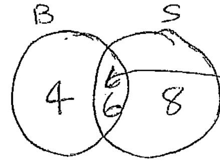
e) i) $(x-1)^2 = 8y$

i) $(1, 0)$

ii) $4a = 8$

$a = 2$

f) i



$14 + 10 - 18 = 6$

ii) $\frac{4}{10}$ or $\frac{2}{5}$

g) i) $a = -20$

$d = 15$

$T_n = a + (n-1)d$

$955 = -20 + (n-1)\times 15$

$975 = 15(n-1)$

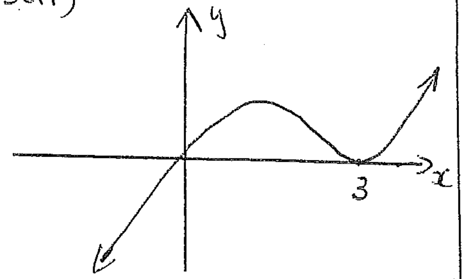
$65 = n - 1$

$n = 66$

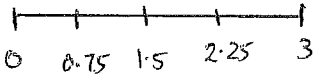
ii) $S_{66} = \frac{66}{2}[-20 + 955]$

$= 30855$

Q5a) i)



ii)



$A \doteqdot \frac{0.75}{2}\left[0 + 2\times[2\times0.75\times(0.75-3)^2 + 2\times1.5\times(1.5-3)^2 + 2\times2.25\times(2.25-3)^2] + 0\right]$

$A = 12.65625$

iii) $A = \int_0^3 2x(x-3)^2$

$A = 2\int_0^3 x(x^2-6x+9)$

$= 2\int_0^3 x^3 - 6x^2 + 9x$

$= 2\left[\frac{x^4}{4} - \frac{6x^3}{3} + \frac{9x^2}{2}\right]_0^3$

$= 2\left[\left(\frac{3^4}{4} - \frac{6\times3^3}{3} + \frac{9\times3^2}{2}\right) - 0\right]$

$= 13.5$

iii) error $= 13.5 - 12.65625$
$= 0.84375$

$\frac{0.84375}{13.5}\times100 = 6.25\%$

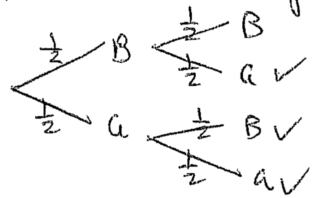
b) i) $A = -(\frac{1}{2}\times2\times2) + (\frac{1}{2}\times2\times2) + 2\times2 + \frac{1}{4}\times\pi\times2^2$

$= 4 + \pi$ or 7.14

ii) $x = 2$

c) i) $\frac{59}{60}$

ii) The daughter with twin boys is not used because the sex are boys. We want to know about girls



$3\times(\frac{1}{2}\times\frac{1}{2}) = \frac{3}{4}$

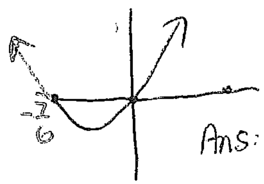
d) $y = x^4 + \frac{7}{3}x^3 - 5$

$y' = 4x^3 + 7x^2$

$y'' = 12x^2 + 14x > 0$

$6x^2 + 7x > 0$

$x(6x+7) > 0$



Ans: $x < -\frac{7}{6}, x > 0$

a) i) $f(x) = 4x^3 - x^4$

$4x^3 - x^4 = 0$

$x^3(4-x) = 0$

$x = 0 \quad x = 4$

ii) $y' = 12x^2 - 4x^3 = 0$

$3x^2 - x^3 = 0$

$x^2(3-x) = 0$

$x = 0 \qquad x = 3$

$y = 0 \qquad y = 27$

Test $(0,0)$ $\qquad$ $(3,27)$

$y'' = 24x - 12x^2$ $\qquad$ $y'' = 24\times3 - 12\times3^2$

$y'' = 24\times0 - 12\times0^2$ $\qquad$ $y'' = -36 < 0$

$y'' = 0$ $\qquad$ $\therefore$ MAX at $(3,27)$

$(0,0)$ Horizontal pt of inflex

iii) $y'' = 0 \quad 24x - 12x^2 = 0$

$2x - x^2 = 0$

$x(2-x) = 0$

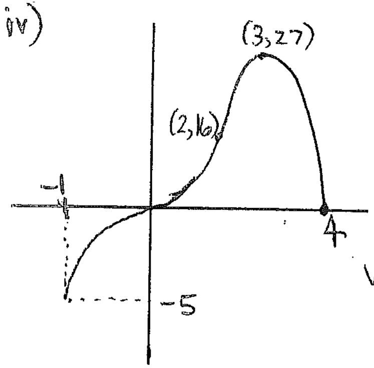
$x = 0 \qquad x = 2$

$x = 0 \qquad x = 2$

$y = 0 \qquad y = 16$

$(0,0) \qquad (2,16)$

iv)



v) from graph $0 < x < 4$

b) $y = (3-x)^{-1}$

$y' = -1(3-x)^{-2}\times-1$

$y' = \frac{1}{(3-x)^2}$

increasing if $y' > 0$ and since

$y' = \frac{1}{(3-x)^2}$ ← + ← + $\therefore y' > 0$

$\therefore$ increasing for all x.

c)

i) $-1 < r < 1$

$r = \frac{\frac{(x+1)}{4}}{\frac{1}{2}}$

$r = \frac{2(x+1)}{4}$

$r = \frac{x+1}{2}$

$-1 < \frac{x+1}{2} < 1$

$-2 < x+1 < 2$

$-3 < x < 1$

ii) $S_\infty = \frac{a}{1-r}$

$= \frac{\frac{1}{2}}{1 - \frac{x+1}{2}}$

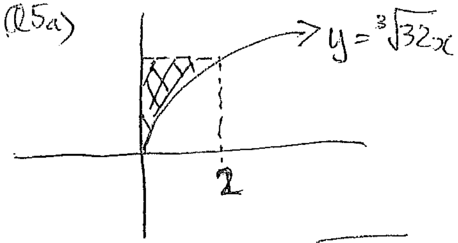
$= \frac{\frac{1}{2}}{\frac{2-x-1}{2}}$

$= \frac{1}{2} \div \frac{1-x}{2}$

$= \frac{1}{2}\times\frac{2}{1-x}$

$S_\infty = \frac{1}{1-x}$

Q5a)



if $x=2$, $y = \sqrt[3]{32 \times 2}$
$= 4$

$y = \sqrt[3]{32x}$
$y^3 = 32x$
$x = \frac{y^3}{32}$
$x^2 = \left(\frac{y^3}{32}\right)^2 = \frac{y^6}{1024}$

$V = \pi \int_0^4 \frac{y^6}{1024}\, dy$
$= \frac{\pi}{1024} \int_0^4 y^6\, dy$
$= \frac{\pi}{1024} \left[\frac{y^7}{7}\right]_0^4$
$= \frac{\pi}{1024} \times \frac{4^7}{7}$
$= 2\frac{2}{7}\pi$ or $\frac{16\pi}{7}$

bi) $r = 6 \div 12 = 0.5\%$

$A_1 = 1200000 \times 1.005 - 8000$
$A_2 = A_1 \times 1.005 - 8000$
$A_2 = (1200000 \times 1.005 - 8000) \times 1.005 - 8000$
$A_2 = 1200000 \times 1.005^2 - 8000 \times 1.005 - 8000$

$A_3 = 1200000 \times 1.005^3 - 8000 \times 1.005^2 - 8000 \times 1.005 - 8000$
$A_3 = \$1193969.95$

ii) $A_n = 1200000 \times 1.005^n - 8000[1 + 1.005 + 1.005^2 + \dots + 1.005^{n-1}]$

$a = 1$
$r = 1.005$
$n = n$
$S_n = \frac{1(1.005^n - 1)}{1.005 - 1}$
$= 200(1.005^n - 1)$

$A_n = 1200000 \times 1.005^n - 8000 \times 200(1.005^n - 1)$
$= 1200000 \times 1.005^n - 1600000 \times 1.005^n + 1600000$

$A_n = 1600000 - 400000 \times 1.005^n$
$A_n = 0$ (no money left)
$0 = 1600000 - 400000 \times 1.005^n$
$1.005^n = 4$
$\log(1.005)^n = \log 4$
$n = \frac{\log 4}{\log 1.005}$
$n = 277.95$ months

c) i) $CA^2 = x^2 + 150^2$
$CA^2 = x^2 + 22500$
$CA = \sqrt{x^2 + 22500}$

ii) $S = \frac{D}{T}$
$\therefore T = \frac{D}{S} = \frac{\sqrt{x^2+22500}}{15}$

iii) $DC = 400 - x$
$T_{DC} = \frac{400 - x}{20}$

$T_{TOTAL} = \frac{\sqrt{x^2+22500}}{15} + \frac{400-x}{20}$

$T = \frac{1}{15}(x^2 + 22500)^{\frac{1}{2}} + 20 - \frac{1}{20}x$
$T' = \frac{1}{30}(x^2 + 22500)^{-\frac{1}{2}} \times 2x - \frac{1}{20}$
$T' = \frac{x}{15\sqrt{x^2+22500}} - \frac{1}{20}$
st pts: $T' = 0$
$\frac{x}{15\sqrt{x^2+22500}} - \frac{1}{20} = 0$
$\frac{x}{15\sqrt{x^2+22500}} = \frac{1}{20}$
$20x = 15\sqrt{x^2+22500}$
$400x^2 = 225(x^2 + 22500)$
$400x^2 = 225x^2 + 5062500$
$175x^2 = 5062500$
$x^2 = 28928.57\dots$
$x = 170$ m

test

x	170^-	170	170^+
T'	−	0	+

\ _ /

Time is a minimum when $x = 170$ m