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BRIGIDINE COLLEGE RANDWICK

MATHEMATICS

PRELIMINARY YEARLY

2005

(TIME - 2 HOUR)

Directions to candidates

* *Put your **name at the top** of this paper and on each of the **8 sections** that are to be collected.*
* *All 8 questions are to be attempted.*
* *All 8 questions are of equal value.*
* *All questions are to be answered on separate pages and will be collected in separate bundles at the end of this exam.*
* *All necessary working should be shown in every question.*
* *Full marks may not be awarded for careless or badly arranged work.*

Question 1 ***(Start a new page)***

a. Calculate $\dfrac{\sqrt{3.56 \times 20.93}}{8.5^2 + 6.1}$ to 3 significant figures. (2)

b. Completely simplify the following: **(leaving denominator Rational when necessary)**

i. $2\sqrt{8} - \sqrt{18} + 3\sqrt{2}$ (2)

ii. $\dfrac{2\sqrt{5} \times 3\sqrt{8}}{6\sqrt{80}}$ (2)

c. Completely factorise

i. $6 + 7x - 3x^2$ (2)

ii. $ax^2 - ax - x + 1$ (2)

d. A polygon has 20 sides. Determine the size of each interior angle. (2)

Question 2 ***(Start a new page)***

a. Solve the following equations

i. $3x - 5 = 8(x + 2)$ (1)

ii. $\dfrac{1}{3x} = 3 - \dfrac{3}{2x}$ (2)

iii. $12x + 6 = (2x + 1)^2$ (3)

... Question 2 continued

b. Simplify $\dfrac{x^2 - x - 20}{x^2 - 25} \div \dfrac{x + 4}{2x^2 + x - 1}$ (3)

c. Solve the simultaneous equations (3)

$xy = 8$ and $x + 2y = 8$

Question 3 *(Start a new page)*

a. Solve for x in the following $|4 + 5x| \geq 20$ (3)

b. If the $\sin\alpha = \dfrac{x}{y}$ determine the value of $\cos\alpha$. (where $0 \leq \alpha \leq 90^\circ$) (2)

c. Solve for α where $0 \leq \alpha \leq 360^\circ$, if (3)

$$2\sin\alpha + \sin\alpha = 1$$

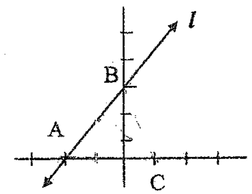
d. Determine the domain of these curves :

i. $y = \dfrac{2x}{x^2 - 4}$ (2)

ii. $y = \sqrt{1 - 3x}$ (2)

Question 4 *(Start a new page)*

a.



To the left is a diagram of line l which passes through the points A (-2,0) and B (0,3). *(not to scale)*

The point C is given by the point (1,0).

(6)

i. Show that the equation of line l may be given by: $3x - 2y + 6 = 0$.

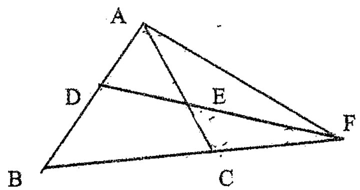
ii. Show that the perpendicular distance of the point C to the line l is $\dfrac{9}{\sqrt{13}}$ units.

iii. Determine the area of triangle ABC.

b. i. $\angle DAE = 30^\circ$, $\angle ABC = 58^\circ$,

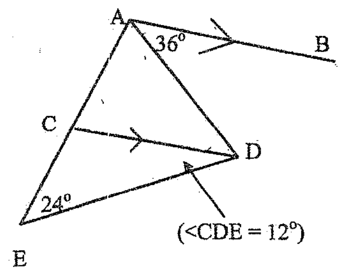
$\angle CFD = 40^\circ$ and $\angle CEF = x$

Redraw this figure on to your exam, marking in the relevant information.



ii. Determine the value for x, giving reasons. (3)

c.

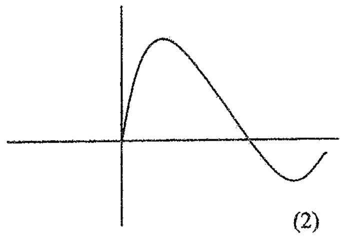


Prove that Δ ACD is isosceles (3)

Question 5 *(Start a new page)*

a. If y = f(x) is an even function where f(-2) = 3 and f(1) = -5, find f(-1) and f(2). (2)

b. The figure to the right is a sketch of part of the function f (x).



i. **Redraw this figure to the right on to your exam page.**

ii. Complete f(x), if f(x) is an odd function. (2)

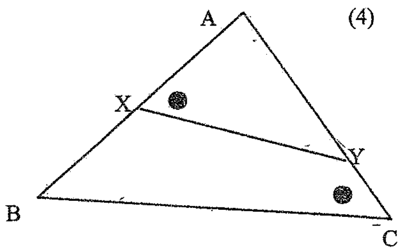
c. By rewriting the parabola $8y = x^2 - 6x + 17$ in an appropriate form determine its vertex and focus. (3)

d. Find the locus of all the points P (x,y) whose distance from A (1,4) is twice its distance from B (-3,5). (3)

e. Determine the $\lim\limits_{x \to -2} \dfrac{2x^2 + 3x - 2}{x + 2}$ (2)

Question 6 *(Start a new page)*

a. In this figure to the right, <AXY = < ACB. (4)



i. Prove that $\Delta AXY \;///\; \Delta ACB$.

ii. Hence, or otherwise, show that AB.AX = AC.AY

b. Prove that $\dfrac{\sin\alpha \sec\alpha}{\tan\alpha + \cot\alpha} = \sin^2\alpha$ (3)

... Question 6 continued

c. A function f(x) is defined by the rule

$$f(x) = \begin{cases} -x & \text{for } x \le 0 \\ -1 & \text{for } 0 < x \le 1 \\ x^2 & \text{for } x > 1 \end{cases}$$

i. Find f(2) (1)

ii. Find f(0.75) + f(-3) (1)

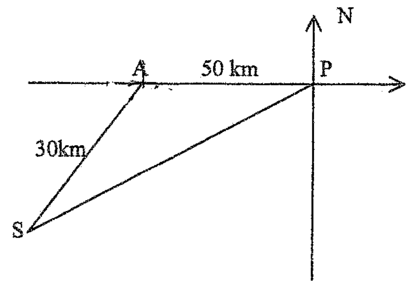
iii. Sketch f(x) (2)

iv. Find f '(0.5) (1)

Question 7 *(Start a new page)*

a. A ship S sails due west from port P for 50 km to A. It then changes course and sails on a bearing of 210 degrees for 30km. (5)

i. Copy the diagram below onto your answer sheet.



ii. Show that $\angle PAS = 120°$

iii. Find the direct distance from the ship to port

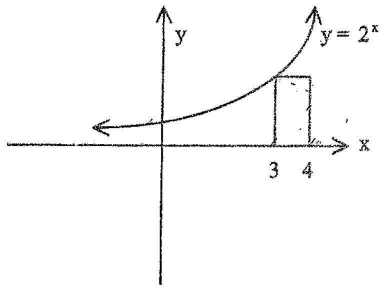
iv. Calculate the ships new bearing from port.

b. If $\frac{a}{b} = 3$, find the value of $\dfrac{2a + 5b}{a + b}$. (2)

c. What angle does the tangent to the curve $y = x^4 - x^3 + 7x$ make with the x-axis at the origin ? (2)

... Question 7 continued

d. The rectangle below is rotated about the x-axis (3)



i. What is the name of the solid shape formed when the rectangle is rotated about the x-axis?

ii. Calculate the volume of the solid when the rectangle is rotated about the x-axis.

Question 8 *(Start a new page)*

a. Differentiate the following with respect to x :

i. $y = 3x^5 - 4x + 7$ (2)

ii. $f(x) = \dfrac{1}{x^3}$ (2)

iii. $g(x) = \dfrac{5x}{2\sqrt{x}}$ (2)

b. If $f(x) = \dfrac{(2x+1)(2x+1)}{x}$, find $f'(x)$ (3)

c. Find the value(s) of k if the normal to the curve $y = kx^2 - x - 3$ is parallel to the line $5x - 3y + 2 = 0$ at the point where x = 1. (3)

- end of exam -

Q1a) 0.110 ✓✓ 1 mark for $0.110171842\ldots$ or 0.11

b) i) $2\sqrt{8} - \sqrt{18} + 3\sqrt{2}$

$2 \times \sqrt{4} \times \sqrt{2} - \sqrt{9} \times \sqrt{2} + 3\sqrt{2}$ ✓

$4\sqrt{2} - 3\sqrt{2} + 3\sqrt{2}$

$4\sqrt{2}$ ✓

ii) $\dfrac{2\sqrt{5} \times 3\sqrt{8}}{6\sqrt{80}}$

$\dfrac{6\sqrt{40}}{6\sqrt{80}}$ ✓ Correctly attempting to rationalise denominator (1 mark)

$\dfrac{1}{\sqrt{2}} \times \dfrac{\sqrt{2}}{\sqrt{2}}$

$= \dfrac{\sqrt{2}}{2}$ ✓

c) i) $6 + 7x - 3x^2$

2 +3x

3 −1x

$(2+3x)(3-x)$ ✓ ✓

ii) $ax^2 - ax - x + 1$

$ax(x-1) - 1(x-1)$

$(ax-1)(x-1)$ ✓ ✓

d) int ∠ sum $= (n-2) \times 180$

$= (20-2) \times 180$

$= 3240$ ✓

$3240 \div 20 = 162°$ ✓

Q2a) i) $3x - 5 = 8(x+2)$

$3x - 5 = 8x + 16$ ✓

$-21 = 5x$ 1

$x = \frac{-21}{5}$ or $-4\frac{1}{5}$ ✓

-4.2

ii) $\dfrac{1}{3x} = 3 - \dfrac{3}{2x}$ $\times 6x$

$2 = 18x - 9$ ✓

$11 = 18x$ 2

$x = \frac{11}{18}$ ✓

Q2a) iii) $12x + 6 = (2x+1)^2$

$12x + 6 = 4x^2 + 4x + 1$

$0 = 4x^2 - 8x - 5$ ✓

$0 = (2x+1)(2x-5)$ 3

$x = -\frac{1}{2}$, $x = \frac{5}{2}$ ✓ ✓

b) $\dfrac{x^2 - x - 20}{x^2 - 25} \div \dfrac{x+4}{2x^2 + x - 1}$

$\dfrac{(x-5)(x+4)}{(x-5)(x+5)} \times \dfrac{(2x-1)(x+1)}{(x+4)}$

$\dfrac{(2x-1)(x+1)}{x+5}$ ✓ 3

1 mark = changing to x and reciprocal.
1 mark = correctly factorising
1 mark = correct answer.

c) $xy = 8$ ---①

$x + 2y = 8$ ---② → $x = 8 - 2y$ sub in ①

$(8-2y) \times y = 8$

$8y - 2y^2 = 8$

$0 = 2y^2 - 8y + 8$ ✓

$0 = y^2 - 4y + 4$ 3

$0 = (y-2)(y-2)$

$y = 2$ sub in ① ✓

$x = 4$ ✓

Q2 $= 12$.

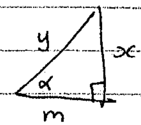
Q3a) $|4 + 5x| \geq 20$

$4 + 5x \leq -20$ $4 + 5x \geq 20$ ✓

$5x \leq -24$ $5x \geq 16$

$x \leq -\frac{24}{5}$ ✓ $x \geq \frac{16}{5}$ ✓

b) $\sin\alpha = \dfrac{x}{y}$



$y^2 = x^2 + m^2$

$m = \sqrt{y^2 - x^2}$ ✓

$\therefore \cos\alpha = \dfrac{\sqrt{y^2 - x^2}}{y}$ ✓

c) $2\sin\alpha + \sin\alpha = 1$

$3\sin\alpha = 1$

$\sin\alpha = \frac{1}{3}$ ✓

$\alpha = 19°28'$, $180 - 19°28'$

$\alpha = 19°28'$, $160°32'$ ✓ ✓

d) i) $y = \dfrac{2x}{x^2 - 4}$

$x^2 - 4 \neq 0$ ✓

$x^2 \neq 4$

$x \neq \pm 2$ ✓

ii) $1 - 3x \geq 0$ ✓

$-3x \geq -1$

$x \leq \frac{1}{3}$ ✓

$A = 0 \quad B = -2 \quad C = 6$

i) copy → no mark

Q4a) i) $m = \frac{\text{rise}}{\text{run}} = \frac{3}{2}$, point $= (0, 3)$ ← y-int 1

$\therefore y = \frac{3}{2}x + 3$

$2y = 3x + 6$

$0 = 3x - 2y + 6$ 1

(2)

iii) $d = \frac{|3 \times 1 + -2 \times 0 + 6|}{\sqrt{3^2 + (-2)^2}}$ $(1, 0)$ 1

$d = \frac{9}{\sqrt{13}}$ 1

(2)

iv) $A = \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 3 \times 3$

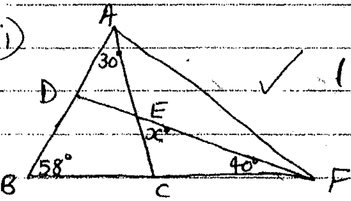
$= \frac{9}{2}$ or $4\frac{1}{2}$

1 mark for base = 3 1

1 mark for height = 3 1

(2)

b) i)



1

(1)

ii) $\angle ACF = 30 + 58$ (ext $\angle$ Δ) 1

$\angle ACF = 88$

$x + 40 + 88 = 180$ ($\angle$ sum Δ) 1

$x = 52$

(2)

c) $\angle ACD = 24 + 12$ (ext $\angle$ Δ) 1

$\angle ACD = 36$

$\angle ADC = 36$ (alt $\angle$'s = in // lines) 1

$\therefore \Delta ACD$ is isosceles (base $\angle$'s =) 1

(3)

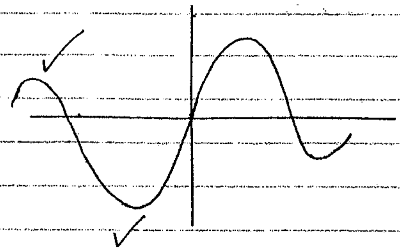
12

Q5a) If even → symmetrical

so $f(-2) = 3 \quad f(2) = 3$

$f(1) = -5 \quad f(-1) = -5$

b) i) no mark ii)



(1 mark for drawing an even function.)

Q5c) $8y = x^2 - 6x + 17$

$8y - 17 = x^2 - 6x$

$8y - 17 + \left(\frac{6}{2}\right)^2 = x^2 - 6x + \left(\frac{6}{2}\right)^2$

$8y - 8 = x^2 - 6x + 9$

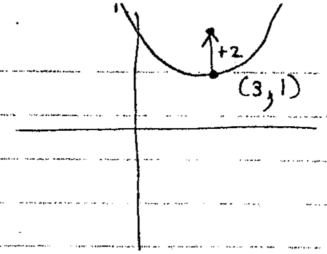
$8(y - 1) = (x - 3)^2$

$4a(y - k) = (x - h)^2$

$\therefore$ vertex $= (3, 1)$

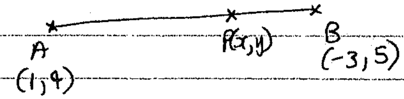
focal length $= 4a = 8$

$a = 2$



$\therefore$ Focus $= (3, 3)$

d)



$\therefore 2PB = PA$

$2\sqrt{(x+3)^2 + (y-5)^2} = \sqrt{(x-1)^2 + (y-4)^2}$

$4\left[x^2 + 6x + 9 + y^2 - 10y + 25\right] = x^2 - 2x + 1 + y^2 - 8y$

$4x^2 + 24x + 36 + 4y^2 - 40y + 100 = x^2 - 2x + y^2 - 8y + 17$

$3x^2 + 26x + 3y^2 - 32y + 119 = 0$

e) $\lim_{x \to -2} \frac{2x^2 + 3x - 2}{x + 2} = \lim_{x \to -2} \frac{(2x-1)(x+2)}{x+2}$

$= \lim_{x \to -2} 2x - 1 = 2 \times -2 - 1 = -5$

Q6a) i) $\angle AXY = \angle ACB$ (given) 1

$\angle A$ (common) 1

$\therefore \Delta AXY \mathbin{|||} \Delta ACB$ equiangular.

ii) $\frac{AX}{AC} = \frac{AY}{AB}$ 1

$AX \cdot AB = AY \cdot AC$ 1

(4)

b) $\frac{\sin\alpha \sec\alpha}{\tan\alpha + \cot\alpha} = \sin^2\alpha$

LHS: $\frac{\sin\alpha \times \frac{1}{\cos\alpha}}{\frac{\sin\alpha}{\cos\alpha} + \frac{\cos\alpha}{\sin\alpha}}$ 1

$\frac{\frac{\sin\alpha}{\cos\alpha}}{\frac{\sin^2\alpha + \cos^2\alpha}{\sin\alpha \cos\alpha}}$ 1

$\frac{\sin^2\alpha \cos\alpha}{\cos\alpha(\sin^2\alpha + \cos^2\alpha)}$

$= \sin^2\alpha$ 1

There are variations to the procedure

~~1 mark for each correct step~~

(3 marks allocated.)

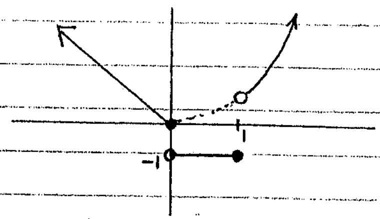
(3)

Q6c) $f(x) = \begin{cases} -x & \text{for } x \le 0 \\ -1 & \text{for } 0 < x \le 1 \\ x^2 & \text{for } x > 1 \end{cases}$

i) $f(2) = 2^2 = 4$ ✓ 1

ii) $f(0.75) + f(-3) = -1 + -(-3) = 2$ ✓ 1

iii)



1 mark for any one correct part of sketch

+ 1 mark for correct sketch. ③

iv) $f'(0.5)$

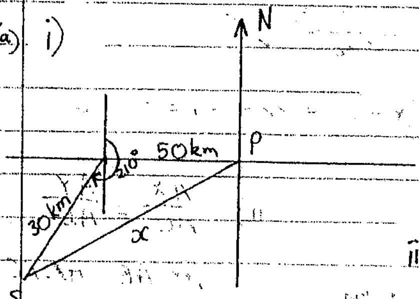
$f(x) = -1$ for $0 < x \le 1$

$\therefore f'(x) = 0$

$f'(0.5) = 0$ ✓ 1

total ⑤.

Q7a) i)



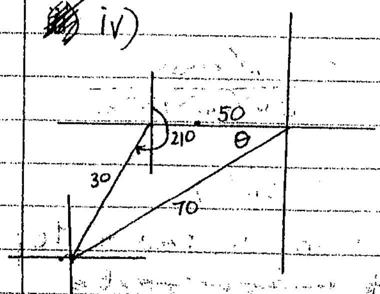
ii) $210° - 90° = 120°$ ✓

iii) $x^2 = 50^2 + 30^2 - 2 \times 50 \times 30 \times \cos 120°$ ✓

$x^2 = 4900$

$x = 70$ km ✓

iv)



$\cos\theta = \dfrac{50^2 + 70^2 - 30^2}{2 \times 50 \times 70}$

$\cos\theta = 0.9285\ldots$

$\theta = 21°47'$ ✓

$\therefore 270 - 21°47'$

$= 248°13'$ ✓

b) $\dfrac{a}{b} = 3$ $\quad\therefore\quad \dfrac{2(3b) + 5b}{3b + b}$ ✓

$a = 3b$ $\qquad \dfrac{11b}{4b}$

$= \dfrac{11}{4}$ or $2\frac{3}{4}$ ✓

c) $y = x^4 - x^3 + 7x$

$y' = 4x^3 - 3x^2 + 7$

when $x = 0$

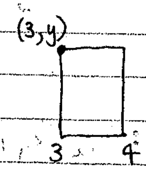
$m = 7$ ✓

$\therefore \tan\theta = 7$

$\theta = 81°52'$ or $180 - 81°52' = 98°8'$ ✓ (either answer is OK) you don't need both.

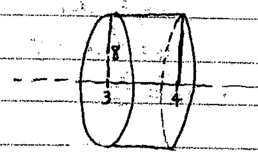
d) i) cylinder ✓

ii)



$(3, y) \rightarrow$ when $x = 3$ $\quad y = 2^3 = 8$ ✓

$(3, 8)$



$V = \pi r^2 h$

$V = \pi \times 8^2 \times 1$

$V = 64\pi$ or 201.1 units^3 ✓

Q8) i) $y = 3x^5 - 4x + 7$

$\dfrac{dy}{dx} = 15x^4 - 4$ ✓ ✓

$\boxed{2}$

ii) $f(x) = \dfrac{1}{x^3}$

$f(x) = x^{-3}$ ✓

$f'(x) = -3x^{-4}$ ✓

$\boxed{2}$

iii) $g(x) = \dfrac{5x}{2\sqrt{x}}$

$g(x) = \dfrac{5x^1}{2x^{\frac{1}{2}}}$

$g(x) = \dfrac{5x^{\frac{1}{2}}}{2}$ ✓

$g'(x) = \dfrac{5}{2} \times \dfrac{1}{2} x^{-\frac{1}{2}}$

$= \dfrac{5}{4} x^{-\frac{1}{2}}$

$= \dfrac{5}{4\sqrt{x}}$ ✓

$\boxed{2}$

Q8b) $f(x) = \dfrac{(2x+1)(2x+1)}{x}$

$f(x) = \dfrac{4x^2+4x+1}{x}$ ✓ 1

$f(x) = \dfrac{4x^2}{x} + \dfrac{4x}{x} + \dfrac{1}{x}$

$f(x) = 4x + 4 + x^{-1}$ ✓ 1

$f'(x) = 4 - x^{-2}$ ✓ 1

or $= \dfrac{4x^2-1}{x^2}$.

$\boxed{3}$

c) $y = kx^2 - x - 3$

$\dfrac{dy}{dx} = 2kx - 1$

when $x = 1$

m of tang $= 2 \times k \times 1 - 1$

gradient of tangent $= 2k - 1$ ✓

m of normal $= -\dfrac{1}{2k-1}$ ✓ ⟵ or 1 mark for ⟶ : $5x - 3y + 2 = 0$

① or

$3y = 5x + 2$

① $y = \frac{5}{3}x + \frac{2}{3}$

$\therefore m = \boxed{\frac{5}{3}}$

since normal is parallel then

$-\dfrac{1}{2k-1} = \dfrac{5}{3}$

$-3 = 5(2k-1)$

$-3 = 10k - 5$

$2 = 10k$

$k = \frac{1}{5}$ ✓ 1

$\boxed{3}$