Student _____

Teacher _____

BRIGIDINE COLLEGE RANDWICK

MATHEMATICS

PRELIMINARY YEARLY

2006

(TIME - 2 HOUR)

Directions to candidates

* *Put your **name at the top** of this paper and on each of the **6 sections** that are to be collected.*
* *All 6 questions are to be attempted.*
* *All 6 questions are of equal value.*
* *All questions are to be answered on separate pages and will be collected in separate bundles at the end of this exam.*
* *All necessary working should be shown in every question.*
* *Full marks may not be awarded for careless or badly arranged work.*

Question 1 ***(Start a new page)***

a. After a discount of 20%, the cost of a DVD player sold for $252. Determine the original price marked on this DVD player. 2 m

b. Calculate $\frac{\sqrt{41.6 + 39.5}}{0.52 - 321}$ to 2 significant figures. 2 m

c. Completely simplify the following (leaving denominator Rational when necessary)

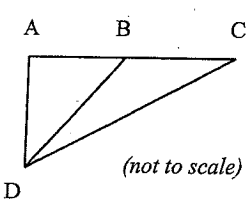
i. $4\sqrt{40} - 3\sqrt{10}$ 2 m

ii. $(\sqrt{3} - 2\sqrt{2})^2$ 2 m

iii. $\frac{2\sqrt{5} \times \sqrt{8}}{3\sqrt{80}}$ 2 m

d. Completely factorise $3x^2 + 7x + 2$ 1 m

e. i Copy the diagram below onto your exam page and fill in the following information 1 m



AB = AD, BC = BD
<ACD = x, <DAC = 90°

(not to scale)

ii. Find the value for x, giving reasons. 3 m

Question 2 *(Start a new page)*

a. Solve the following equations *(completely simplify when necessary)*

i. $5 - \frac{2x}{3} = 10$ 2 m

ii. $(x + 2)^2 = (x - 1)(x + 3)$ 2 m

iii. $3x^2 = 4x + 1$ 2 m

b. Completely Simplify 3 m

$$\frac{x^2 - 9}{x^2 - x - 12} \times \frac{x^2 - 9x + 20}{x^2 - 3x}$$

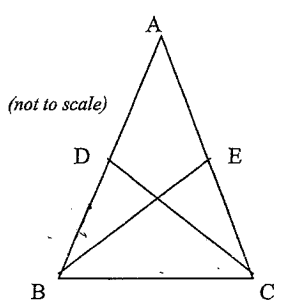
c. Solve the simultaneous equations 3 m

$2x + 3y = 5$ and $3x + 4y = 6$

d. Redraw the figure to the right onto your exam page and mark in the following information. 3 m

ΔABC is isosceles with AB = D and E are the midpoints of AB and AC, respectively.

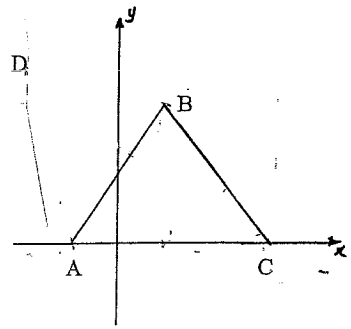
Prove that
$\Delta DBC \equiv \Delta ECB$



Question 3 *(Start a new page)*

a. The vertices of a triangle are A (-1,0)
B (1,4) and C lies on the x axis to the right of A,
such that $\angle BAC = \angle BCA = \vartheta$.
D is the point (-2,5).

not to scale



i. Redraw this figure and mark in this information. 1 m

ii. Find the gradient of AB and hence find ϑ, to the nearest degree. 2 m

iii. Show that the equation of line AB is $y = 2x + 2$.

iv. Explain why BC has a gradient of -2. 1 m

v. Find the perpendicular distance of D from line AB. 2 m

vi. Hence, or otherwise, calculate the area of the quadrilateral ADBC. 3 m

b. Find the Exact Value of the following

i. $\cos 30°$ 2 m

ii. $\operatorname{cosec} 240°$ 2 m

Question 4 *(Start a new page)*

a. Show that $99.\overline{9}$ % (ie 99.9999....) represents a whole number. 2 m

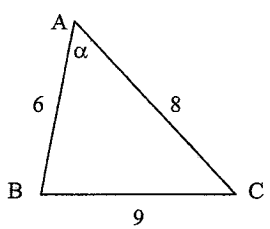
b. Determine the natural domain of $\frac{1}{x^2 - 9}$ 2 m

c. Solve for x in the following $|6 - 2x| \leq 12$ 3 m

d. Solve for α where $0 \leq \alpha \leq 360^\circ$, if 3 m

$\sqrt{3} \tan \alpha = 1$

e. i. Find the value of α in this figure below to the nearest minute. 2 m

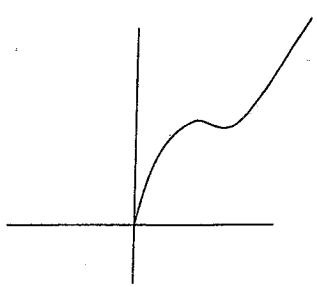


ii. Hence, or otherwise, find the area of triangle ABC. (2 decimal places) 1 m

f. Copy this figure to the right onto your exam page.

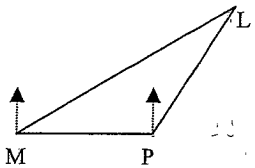
i. Using a dotted line, complete f(x) if f(x) is an even function. 1 m

ii. Using a solid line, complete f(x) if f(x) is a n odd function. 1 m



Question 5 *(Start a new page)*

a. The bearing of a lighthouse L from a ship at M is N55°E. The ship sails due East from M to a point P which is 10 km from L. The bearing of the lighthouse from P is N25°E.



i. Copy this diagram onto your answer page and show that < MLP = 30°. 1 m

ii. Show that MP = 5 cosec 35° and hence find MP correct to the nearest metre. 3 m

iii. The ship continues to sail due East. Determine the closest distance it will come to this lighthouse L (nearest km). 2 m

b. If α & β are the roots to $2x^2 + 4x + 6 = 0$, find

i. $\alpha + \beta$ and $\alpha\beta$ 1 m

ii. $\frac{1}{\alpha} + \frac{1}{\beta}$ 1 m

iii. $(\alpha - 1)(\beta - 1)$ 2 m

iv. $(\alpha - \beta)^2$ 2 m

c. Find the locus of all the points P (x,y) whose distance from A (1,4) is twice its distance from B (-3,5). 3 m

Question 6 *(Start a new page)*

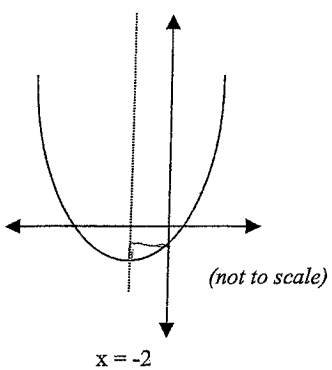
a. Solve for x if $2x^2 + 5x \geq 3$ 3 m

b. Prove that $\dfrac{\sin\alpha \sec\alpha}{\tan\alpha + \cot\alpha} = \sin^2\alpha$ 3 m

c. A (1,4) and B (5,2) are two fixed points in a plane. The point P moves so that < APB is a right angle.

Show that the locus of P may be represented by 3 m

$x^2 + y^2 - 6x - 6y + 18 = 0.$

d. 

The parabola to the left has 3 m

x = -2 as its axis, vertex (-2,-4)

and crosses the y-axis at y = -1 .

Write down the equation of this parabola.

e. Solve for x if $9^x - 10(3^x) + 9 = 0$ 3 m

- end of exam -

yr11 2U yrly 2006
marking scheme.

Q2a i) 1mark: on the right track without making the Q easier.

2marks: $x = -7\frac{1}{2}$

ii) 1mark: * correct expansion of both sides
or * correct answer from incorrect expansion

2marks: $x = -3\frac{1}{2}$

i) 1mark: * applying the quadratic formula correctly
* correctly simplifying a surd
* finding answer from factorised answer.

2marks: * $x = \frac{2 \pm \sqrt{7}}{3}$

ii) 1mark: * correct manipulation of equations
* subing ~~=~~ incorrect value ● correctly to find the 2nd value.

2marks: * finding $x = -2$ correctly
or finding $y = 3$ correctly

3marks: $x = -2$
$y = 3$

d) 1mark: * (S) – –
(A) – –
(S) – – without reasons.
* correct conclusion from 2 students working
* 2 correct lines with reasons

2marks:
* any 2 correct lines of SAS + reasons with conclusion
* SAS + reasons without conclusion

3marks: SAS + reasons + (SAS) conclusion.

b) 1mark: * any 2 correct factorising

2marks: * all 4 correctly factorised
* 3 correctly factorised + cancelled

3marks: $\frac{x-5}{x}$

Q4a) 1mark: $9x = 900$

2marks: $x = 100\%$ or 1 whole

b) 1mark: $x^2 - 9 \neq 0$
or $x \neq 3$
or $x \neq -3$
or $x = 3$ and -3

2marks: $x \neq \pm 3$

c) 1mark: $-12 \leq 6 - 2x \leq 12$
or $x \leq -3$
or $x \geq 9$

2marks: $-18 \leq -2x \leq 6$
or $x \geq -3$
or $x \leq 9$
} from correct working. MUST BE

3marks: $9 \geq x \geq -3$
$-3 \leq x \leq 9$

d) 1mark: * $\tan \alpha = \frac{1}{\sqrt{3}}$
* incorrect "α" + 180°.

2marks: $\alpha = 30°$
or $\alpha = 210°$

3marks: $\alpha = 30°$ and $210°$

e i) 1mark: * $\cos \alpha = \frac{8^2 + 6^2 - 9^2}{2 \times 8 \times 6}$

2marks: $\alpha = 78° 35'$

ii) $A = 23 \cdot 5$ or $A = \frac{1}{2} \times 6 \times 8 \times \sin$ ↑ angle in part (i).

f) i) } See answers
ii) } (one mark for each).

1mark for "f" if BOTH
* correct diagrams but
* NOT LABELED
or * incorrectly labelled.

Q6a) 1mark: $(2x-1)(x+3)$
or 1 inequality correct from students incorrect working.

2marks: * $x \leq -3$
or * $x \geq \frac{1}{2}$
or * $-3 \leq x \leq \frac{1}{2}$
or * correct solution from incorrect working.

3marks: $x \leq -3$ and $x \geq \frac{1}{2}$

b) 1mark: line 1 or equivalent

2marks: * line 2 or equivalent

3marks: showing $\sin^2 \alpha$

c) 1mark: $\frac{y-4}{x-1} \times \frac{y-2}{x-5} = -1$

2marks: $\frac{y^2 - 6y + 8}{x^2 - 6x + 5} = -1$
or equivalent.

3marks: $x^2 + y^2 - 6x - 6y + 13 = 0$
or getting to the 2nd last line in the solutions.

2U yrly 2006 (Solutions)

Q2a) i) $5 - \frac{2x}{3} = 10$

$15 - 2x = 30$

$-2x = 15$

$x = -7\frac{1}{2}$

ii) $(x+2)^2 = (x-1)(x+3)$

$x^2+4x+4 = x^2+3x-x-3$

$4x+4 = 2x-3$

$2x = -7$

$x = -3\frac{1}{2}$

iii) $3x^2 = 4x+1$

$3x^2-4x-1=0$

$x = \frac{4 \pm \sqrt{16-4\times3\times-1}}{2\times3}$

$x = \frac{4 \pm \sqrt{28}}{6}$

$x = \frac{4 \pm 2\sqrt{7}}{6}$

$x = \frac{2 \pm \sqrt{7}}{3}$

b) $\frac{x^2-9}{x^2-x-12} \times \frac{x^2-9x+20}{x^2-3x}$

$\frac{(x-3)(x+3)}{(x-4)(x+3)} \times \frac{(x-5)(x-4)}{x(x-3)}$

$= \frac{x-5}{x}$

c) $2x+3y=5 \quad \cdots ①$

$3x+4y=6 \quad \cdots ②$

$①\times3 \quad 6x+9y=15 \quad \cdots ③$

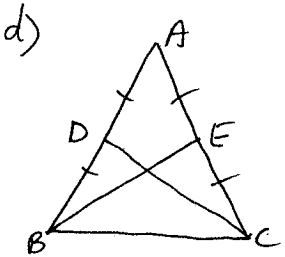
$②\times2 \quad 6x+8y=12 \quad \cdots ④$

$③-④ \quad y=3$ sub in ①

$2x+3\times3=5$

$x=-2$

d)



$AD = DB$ (midpt)

$AE = EC$ (midpt)

$\therefore EC = DB$ since $AB = AC$

$\therefore$ (S) $EC = DB$ (proven above for midpoints of = sides)

(A) $\angle DBC = \angle ECB$ (isos $\triangle$)

(S) BC common.

$\therefore$ by SAS $\triangle DBC \equiv \triangle ECB$

Q4a) $x = 99.999\ldots \quad ①$

$10x = 999.999\ldots \quad ②$

$② - ① \quad 9x = 900$

$x = 100\% = 1$ whole.

b) $\frac{1}{x^2-9}$ D: $x^2-9 \neq 0$

$x^2 \neq 3$

$x \neq \pm 3$

c) $|6-2x| \leq 12$

$-12 \leq 6-2x \leq 12$

$-18 \leq -2x \leq 6$

$9 \geqslant x \geqslant -3$

better written as:

$-3 \leqslant x \leqslant 9$

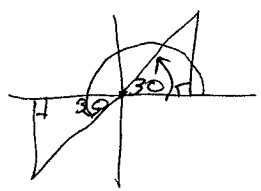
d) $\sqrt{3}\ \mathrm{Tan}\alpha = 1$

$\mathrm{Tan}\alpha = \frac{1}{\sqrt{3}}$

$\alpha = 30°$

$\therefore$ Ans $= 30°, 180+30$

$= 30°, 210°$



e i) $\mathrm{Cos}\alpha = \frac{8^2+6^2-9^2}{2\times8\times6}$

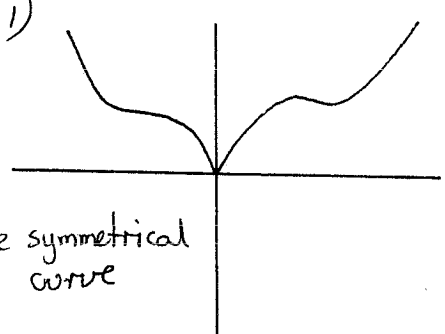
$\mathrm{Cos}\alpha = 0.1979\ldots$

$\alpha = 78°35'$

ii) $A = \frac{1}{2}\times6\times8\times\mathrm{Sin}\,78°35'$

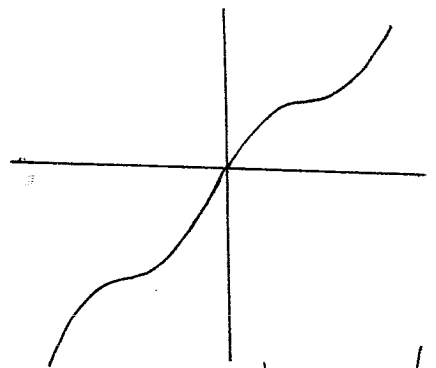
$= 23.5\ u^2$

f) i)



ie symmetrical curve

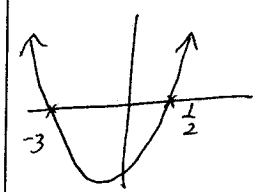
ii)



ie has point symmetry.

Q6a) $2x^2+5x-3 \geqslant 0$

$(2x-1)(x+3) \geqslant 0$



$\therefore x \leqslant -3$

$x \geqslant \frac{1}{2}$

b) $\frac{\mathrm{Sin}\alpha\,\mathrm{Sec}\alpha}{\mathrm{Tan}\alpha+\mathrm{Cot}\alpha}$

$= \frac{\mathrm{Sin}\alpha \times \frac{1}{\mathrm{Cos}\alpha}}{\frac{\mathrm{Sin}\alpha}{\mathrm{Cos}\alpha}+\frac{\mathrm{Cos}\alpha}{\mathrm{Sin}\alpha}}$ ← line 1

$= \frac{\frac{\mathrm{Sin}\alpha}{\mathrm{Cos}\alpha}}{\frac{\mathrm{Sin}^2\alpha+\mathrm{Cos}^2\alpha}{\mathrm{Sin}\alpha\,\mathrm{Cos}\alpha}}$ ← line 2

$= \frac{\mathrm{Sin}\alpha \times \mathrm{Sin}\alpha\,\mathrm{Cos}\alpha}{\mathrm{Cos}\alpha\,(\mathrm{Sin}^2\alpha+\mathrm{Cos}^2\alpha)}$ ← line 3

$= \mathrm{Sin}^2\alpha$

d) 1mark: any correct single letter in quadratic
ie $a=\frac{3}{4}$ or $b=3$ or $c=-1$ or $y=(x+2)^2-4$

incorrect answer (letter) but uses it correctly to find 2nd letter.

2mks: any 2 correct letters correctly
finding 2 letters from incorrect first or second letter.

3mks: $y=\frac{3}{4}x^2+3x-1$

Method 2:

4mk: $*\ (x+2)^2=4a(y+4)$
finding a correct "a" from incorrect substitution of $(-2,-4)$

5mks: $*\ a=\frac{1}{3}$
writing the correct equation in the form $(x\ )^2=4a(y\ )$, need to show where focal length came from.
from an incorrect "a" or

5marks: $(x+2)^2=\frac{4}{3}(y+4)$

e) 1mark:
$*\ (3^x)^2-10(3^x)+9=0$
or $M^2-10M+9=0$

2marks: $x=2$ or $x=0$

3marks: $x=2$ and $x=0$

c) A (1,4), B (5,2), P (x,y)

$m_{AP}\times m_{PB}=-1$

$\frac{y-4}{x-1}\times\frac{y-2}{x-5}=-1$

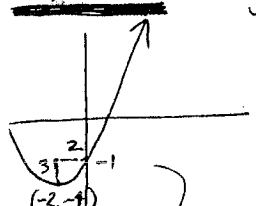
$\frac{y^2-2y-4y+8}{x^2-5x-x+5}=-1$

$\frac{y^2-6y+8}{x^2-6x+5}=-1$

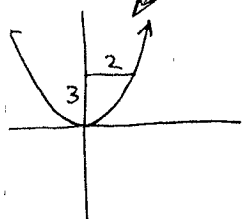
$y^2-6y+8=-x^2+6x-5$

$x^2+y^2-6x-6y+13=0$

Method 1: $y=ax^2+bx+c$



$*\ C=-1$



$y=ax^2$
$3=a\times 2^2$
$a=\frac{3}{4}$

$\therefore (-2,-4)$
$\Rightarrow y=\frac{3}{4}x^2+bx-1$
$-4=\frac{3}{4}(-2)^2-2b-1$
$4=3-2b-1$
$-6=-2b$
$b=3$
$y=\frac{3}{4}x^2+3x-1$

Method 2: Vertex $=(-2,-4)$
$(x+2)^2=4a(y+4)$
at $(0,-1)$
$(0+2)^2=4a(-1+4)$
$4=4a\times 3$
$a=\frac{1}{3}$
$\therefore (x+2)^2=\frac{4}{3}(y+4)$

e) $9^x-10(3^x)+9=0$
$(3^2)^x-10(3^x)+9=0$
$(3^x)^2-10(3^x)+9=0$
let $M=3^x$
$M^2-10M+9=0$
$(M-9)(M-1)=0$
$M=9 \quad M=1$
$3^x=9 \quad 3^x=1$
$x=2$, $x=0$