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BRIGIDINE COLLEGE RANDWICK

PRELIMINARY **EXTENSION 1** MATHEMATICS

YEARLY

2005

(Time - 90 minutes)

Directions to candidates

- *Put your **name at the top** of this paper and on each of the **5 sections** that are to be collected.*
- *All 5 questions are to be attempted.*
- *All 5 questions are of equal value.*
- *All questions are to be answered on separate pages and will be collected in separate bundles at the end of this exam.*
- *All necessary working should be shown in every question.*
- *Full marks may not be awarded for careless or badly arranged work.*

Question 1 ***(Start a new page)***

a. If α and β are the roots to the equation $x^2 = 5 + 2x$.

i. State the value of $\alpha + \beta$ and $\alpha\beta$ *1 m*

and hence find

ii. $\dfrac{1}{\alpha} + \dfrac{1}{\beta}$ *1 m*

iii. $\alpha^2 + \beta^2$ *2 m*

b. Solve the inequation $2x^2 + 9x \geq 5$ *3 m*

c. The first term of a geometric series is 32 and the sixth term is 1. *3 m*

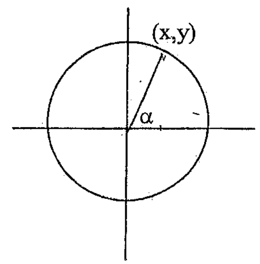
i. Find the common ratio.

ii. Find the limiting sum of this series.

d. Expand and **simplify fully** $2(2^k - 1) + 2^{k+1}$ *2m*

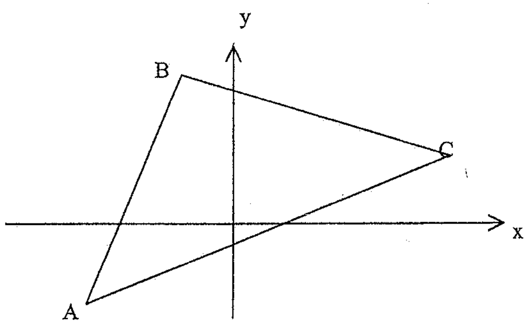
e. i. By considering the **unit** circle to the right, show that x may be represented by cos α. *1m*

ii. Hence deduce that $\tan^2\alpha + 1 = \sec^2\alpha$. *2m*



Question 2 *(Start a new page)*

a. The diagram below shows points A(-3,-2) , B(-1,4) , C(5,2)



i. Redraw this figure onto your exam page and find the gradient of AC. *1 m*

ii. P is the midpoint of AC. Show that the coordinate of P is (1,0). Mark P on the diagram above. *2 m*

iii. Show that the equation of the line perpendicular to AC and passing through P is $2x + y - 2 = 0$ *2 m*

iv. Show that B lies on the line $2x + y - 2 = 0$ *1 m*

v. Show that the length of BP is $2\sqrt{5}$ *2 m*

vi. Find the area of Δ ABC . *3 m*

b. i. Factorise the following expression $2p^2 - 7p + 3$ *1m*

ii. Hence or otherwise solve the following equation for x : *3m*

$$2(\log_3 x)^2 - 7(\log_3 x) + 3 = 0$$

Question 3 *(Start a new page)*

a. Plants infected with a fungus are sprayed with a newly developed fungicide. In field tests with this fungicide, it was found that the probability that the fungus was eliminated was 4 in 5.
Two plants are selected at random from a group of sprayed plants. What is the probability that : *6m*

i. the fungus was eliminated from both plants?

ii. exactly one plant is still infected with the fungus?

iii. both plants are still infected with the fungus?

iv. at least one plant is still infected with the fungus?

b. A woman invests \$ 50 at the beginning of each month. Interest is compounded monthly at 1 % per month. *4 m*

i. Determine the size of her first investment at the end of one month.

ii. Show that at the end of 10 years her accumulated value of her investments may be given by \$ $5050\,(1.01^{120} - 1)$

iii. Determine the accumulated value of her investment.

c. Jessica negotiated a loan for \$100 000. The interest rate is charged at 18% pa compounded monthly over a 25 year period. Interest is charged on the amount owing at the end of each month, before any repayments are made. Let Q = the amount of each monthly repayment. *5 m*

i. Show that the amount owing at the end of the first month is \$101 500 - Q.

ii. Show that at the end of three months, an expression for the amount Jessica owes may be given by $A_3 = 100\,000 \times 1.015^3 - Q\,(1 + 1.015 + 1.015^2)$ where A_3 represents the amount owing after 3 months.

iii. Find the amount of each monthly repayment.

Question 4 *(Start a new page)*

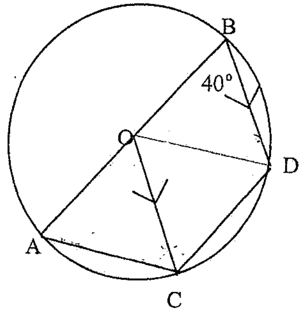
a. Differentiate the following with respect to x.
(leaving answers completely simplified with positive indices when necessary)

i. $x\sqrt{x}$ *2 m*

ii. $(5x^3 - 2)^3$ *2 m*

iii. $\dfrac{4x^2 - 2}{x^2 + 5}$ *2 m*

b. AB is the diameter of the circle, center O. BD and OC are parallel, and angle OBD is 40 degrees. Find the size of angle OCD giving reasons. *3m*

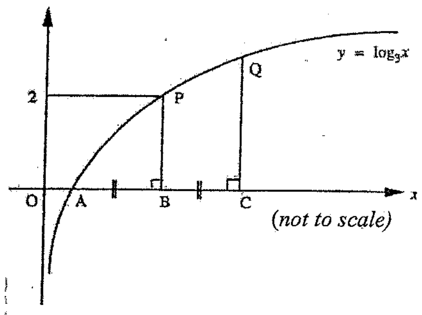


c. Given that $\log_a b = 2.75$ and $\log_a c = 0.25$ *2 m*

find the value of $\log_a (bc)^2$

d. i. To the right is a sketch of $y = \log_3 x$. Determine the x coordinate of P. *1m*

ii. If AB = BC and the length of CQ is $\log_3 m$, find the value of m. *3m*



(not to scale)

Question 5 *(Start a new page)*

a. If $3^x = 2$ find the value of 9^{2x}. *1m*

b. A parabola has a focal point of (3,-7) and a directrix of $y = 5$. *3m*
Determine the

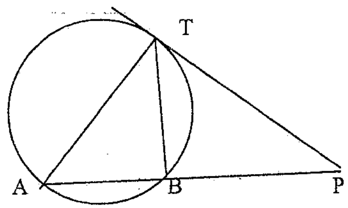
i) Coordinates of the vertex.

ii) Equation of this parabola.

c. Find the set of values for k for which the quadratic expression $x^2 - (k+1)x + (3k-5)$ is positive definite. *3m*

d. Redraw this figure onto your exam page and by considering similar triangles prove $(PT)^2 = AP.PB$
Note : PT is a tangent at T
AT and BT are chords.
ABP are collinear.

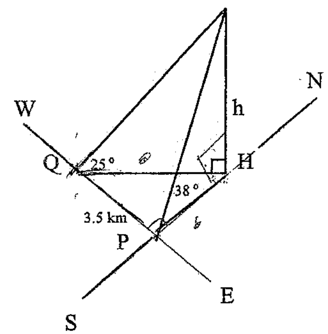
4 m



e. The angular elevation of a mountain at a place P due south of it is 38° and at a place Q due west of P the elevation is 25°. *4 m*

If the distance from P to Q is 3.5 km, find the height "h" of the mountain, to 2 dec. pl.

(P and Q are both at sea level.)



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wrong order

1 d) $2(2^k-1)+2^{k+1} = 2\times 2^k - 2 + 2^{k+1} = 2^{k+1} - 2 + 2^{k+1}$ ✓

$= 2\times 2^{k+1} - 2$

$= 2^{k+2} - 2$ ✓

a) $x^2-2x-5=0$

i) $\alpha+\beta = \frac{--2}{1} = 2$ $\alpha\beta = \frac{-5}{1} = -5$ ✓ (need both)

ii) $\frac{1}{\alpha}+\frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{2}{-5}$ ✓ (or correct working from incorrect ~~answer~~ numbers in (i) above = 1 mark)

iii) $\alpha^2+\beta^2 \Rightarrow (\alpha+\beta)^2 = \alpha^2+2\alpha\beta+\beta^2$

$\alpha^2+\beta^2 = (\alpha+\beta)^2-2\alpha\beta$ ✓

$= (2)^2 - 2\times -5$

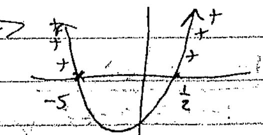
$= 14$ ✓

b) $2x^2+9x-5 \geq 0$

$2x \quad -1$

$x \quad +5$

$(2x-1)(x+5) \geq 0$ ✓



$x \leq -5,\ x \geq \frac{1}{2}$ ✓ ✓

c) i) $a=32$ $T_6=1$

$T_n = ar^{n-1}$

$T_6 = 32\times r^{6-1} = 1$ ✓

$r^5 = \frac{1}{32}$

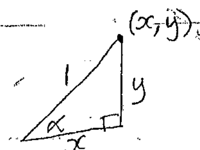
$r = \frac{1}{2}$ ✓

ii) $S = \frac{a}{1-r}$

$= \frac{32}{1-\frac{1}{2}}$

$= 64$ ✓

e) i)



$\cos\alpha = \frac{x}{1} = x$ ✓

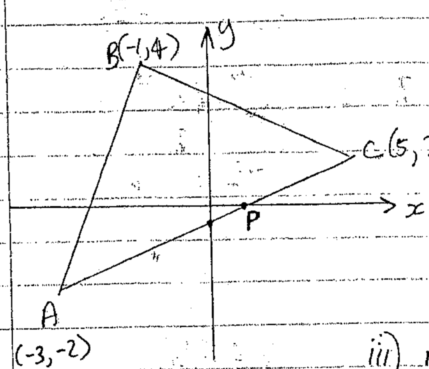
ii) $\sin\alpha = y$ $\therefore x^2+y^2=1$

$\cos^2\alpha+\sin^2\alpha=1$ ✓

$\frac{\cos^2\alpha}{\cos^2\alpha}+\frac{\sin^2\alpha}{\cos^2\alpha}=\frac{1}{\cos^2\alpha}$

$1+\tan^2\alpha=\sec^2\alpha$ ✓

Q2a)



i) $m = \frac{2--2}{5--3} = \frac{1}{2}$ ✓

ii) $M = \left(\frac{5-3}{2}, \frac{2-2}{2}\right)$ ✓

$= \left(\frac{2}{2}, \frac{0}{2}\right)$

$= (1, 0)$ ✓

iii) $m_\perp = -2$ ✓, $(1, 0)$

$y-0 = -2(x-1)$

$y-0 = -2x+2$

$2x+y-2=0$ ✓

iv) $B(-1,4) \rightarrow 2\times -1+4-2=0$ ✓

$-2+4-2=0$

$0=0$

$\therefore$ TRUE

v) $B(-1,4)$ $P(1,0)$

$d = \sqrt{(1--1)^2+(0-4)^2}$ ✓

$= \sqrt{20}$

$= \sqrt{4}\times\sqrt{5}$

$= 2\sqrt{5}$ ✓

vi) $A(-3,-2)$ $C(5,2)$

$d = \sqrt{(5--3)^2+(2--2)^2}$

$= \sqrt{64+16}$

$= \sqrt{80}$ ✓

$A = \frac{1}{2}\times 2\sqrt{5}\times\sqrt{80}$ ✓

$A = 20$ ✓

b i) $2p^2-7p+3$

$2p \quad -1$

$p \quad -3$

$(2p-1)(p-3)$ ✓

ii) $2(\log_3 x)^2 - 7(\log_3 x)+3=0$

let $p = \log_3 x$

$\therefore 2p^2-7p+3=0$

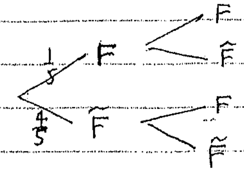
$(2p-1)(p-3)=0$

$p=\frac{1}{2}$ $p=3$ ✓ (for both answers)

$\log_3 x = \frac{1}{2}$ $\log_3 x = 3$

$x=\sqrt{3}$ ✓ $x=27$ ✓

Q3a)



i) $P(\tilde{F}\tilde{F}) = \frac{4}{5} \times \frac{4}{5} = \frac{16}{25}$ ✓

ii) $P(F\tilde{F} \text{ or } \tilde{F}F) = \frac{1}{5} \times \frac{4}{5} + \frac{4}{5} \times \frac{1}{5} = \frac{8}{25}$ ✓ ✓

iii) $P(FF) = \frac{1}{5} \times \frac{1}{5} = \frac{1}{25}$ ✓

iv) $1 - P(\tilde{F}\tilde{F})$

$1 - \frac{4}{5} \times \frac{4}{5} = \frac{9}{25}$ ✓✓

b) i) $A_1 = 50 + 50 \times \frac{1}{100} = \$50.50$ ✓

ii) $A_1 = 50(1.01)$

$A_2 = A_1 \times 1.01 = 50(1.01)^2$

$A_3 = 50(1.01)^3$

$\vdots$

$10\text{yrs} = 10 \times 12 = 120$ investments.

$A_{120} = 50(1.01)^{120}$

$\text{Total} = 50(1.01) + 50(1.01)^2 + \cdots\cdots \quad 50(1.01)^{120}$

$= 50\underbrace{[1.01 + 1.01^2 + \cdots\cdots 1.01^{120}]}$

$S_{120} = \frac{1.01[1.01^{120} - 1]}{1.01 - 1}$ ✓

$= 101[1.01^{120} - 1]$

$\text{Total} = 50 \times 101[1.01^{120} - 1]$

$= 5050[1.01^{120} - 1]$ ✓

iii) $\$11616.95$ ✓

Q3c) $18\% \div 12 = 1.5\%$ per month

$n = 25 \times 12 = 300$

i) $A_1 = 100000 \times \frac{1.5}{100} + 100000 = \$101500 - Q$ ✓

ii) $A_1 = 100000(1.015) - Q$

$A_2 = A_1(1.015) - Q$

$= [100000(1.015) - Q]1.015 - Q$

$= 100000 \times (1.015)^2 - Q(1.015) - Q$ ✓

$A_3 = A_2(1.015) - Q$

$= [100000 \times (1.015)^2 - Q(1.015) - Q](1.015) - Q$

$= 100000 \times 1.015^3 - Q(1.015)^2 - Q(1.015) - Q$

$= 100000 \times 1.015^3 - Q[1 + 1.015 + 1.015^2]$ ✓

iii) $A_{300} = 100000 \times 1.015^{300} - Q\underbrace{[1 + 1.015 + 1.015^2 + \cdots\cdots 1.015^{299}]}$ ✓

$A_{300} = 0$

$S_{300} = \frac{1[1.015^{300} - 1]}{1.015 - 1}$

$= 5737.253308$

$\therefore 0 = 100000 \times 1.015^{300} - Q \times 5737.253308$

$Q \times 5737.253308 = 100000 \times 1.015^{300}$

$Q = \$1517.43$ ✓

Q4a i) $x\sqrt{x} = x^{\frac{3}{2}}$ ✓

$\frac{d}{dx}(x^{\frac{3}{2}})$

$= \frac{3}{2}x^{\frac{1}{2}}$

$= \frac{3\sqrt{x}}{2}$ ✓

ii) $\frac{d}{dx}(5x^3 - 2)^3$

$3(5x^3 - 2)^2 \times 15x^2$ ✓

$= 45x^2(5x^3 - 2)^2$ ✓

iii) $\frac{d}{dx}\left(\frac{4x^2 - 2}{x^2 + 5}\right)$

$\frac{8x(x^2+5) - 2x(4x^2 - 2)}{(x^2+5)^2}$ ✓

$\frac{8x^3 + 40x - 8x^3 + 4x}{(x^2+5)^2}$

$\frac{44x}{(x^2+5)^2}$ ✓

b) $\angle AOC = 40°$ (corresponding $\angle$'s =)

$OA = OC$ (radii)

$\therefore \triangle AOC$ is isosceles ($OA = OC$)

$\therefore \angle OCA + \angle OAC + 40 = 180$ ($\angle$ sum $\triangle$)

$2 \times \angle OCA + 40 = 180$

$2 \times \angle OCA = 140$

$\therefore \angle OCA = 70$ ✓

$\angle ACD + 40 = 180$ (opp $\angle$'s cyclic quad suppl) ✓

$\angle ACD = 140°$

$\therefore \quad 140 = 70 + \angle OCD$

$\angle OCD = 70°$ ✓

c) $\log_a (bc)^2 = 2\log_a bc = 2[\log_a b + \log_a c]$ ✓

$= 2[2.75 + 0.25]$

$= 2 \times 3$

$= 6$ ✓

d) i) $2 = \log_3 x$

$x = 3^2 = 9$ ✓

ii) coordinate of A:

when $y=0$ $\quad 0 = \log_3 x$

$x = 3^0$

$x = 1$ ✓

$\therefore AB = 9 - 1 = 8$ ✓

$BC = 8$ ✓

$\therefore$ x-coord of C = $OB + BC$

$= 9 + 8$

$= 17$

$\therefore x = 17 \quad y = \log_3 17$

$CQ = \log_3 m$

$m = 17$ ✓

Q5a) $3^x = 2$

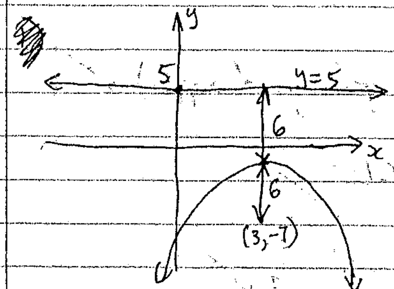
$9^{2x} = (3^2)^{2x}$

$= (3^x)^4$

$= 2^4$

$= 16$ ✓

b)



i) vertex $= (3, -1)$ ✓

ii) $(x-h)^2 = 4a(y-k)$

$a = 6$ ✓

$(x-3)^2 = -24(y+1)$ ✓

(realise negative parabola equation (mark))

c) positive defenite $\Delta < 0$ ✓

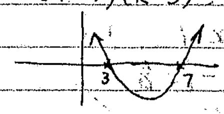
$b^2 - 4ac < 0$

$(k+1)^2 - 4 \times 1 \times (3k-5) < 0$

$k^2 + 2k + 1 - 12k + 20 < 0$

$k^2 - 10k + 21 < 0$

$(k-7)(k-3) < 0$ ✓



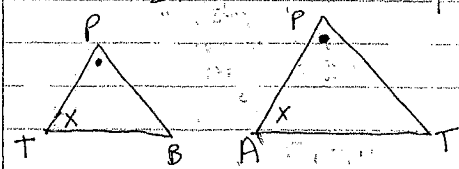
$3 < k < 7$ ✓

d) Prove $\triangle PTB \,|||\, \triangle PTA$

$\angle P$ is common

$\angle PTB = \angle TAP$ ($\angle$'s in alt segment) ✓

$\therefore \triangle PTB \,|||\, \triangle PTA$ equiangular ✓



$\frac{PT}{PA} = \frac{PB}{PT}$ ✓

$(PT)^2 = PA.PB$ ✓

Q5e) $\tan 25^\circ = \frac{h}{QH}$ $\tan 38^\circ = \frac{h}{PH}$

$QH = \frac{h}{\tan 25^\circ}$ $PH = \frac{h}{\tan 38^\circ}$ ✓

QHP is a right ∠Δ

so using pythagoras:

$$QH^2 = QP^2 + PH^2$$

$$\left(\frac{h}{\tan 25}\right)^2 = 3.5^2 + \left(\frac{h}{\tan 38}\right)^2$$ ✓

$$\frac{h^2}{\tan^2 25} - \frac{h^2}{\tan^2 38} = 3.5^2$$

$$h^2\left[\frac{1}{\tan^2 25} - \frac{1}{\tan^2 38}\right] = 3.5^2$$

$$h^2 = \frac{3.5^2}{\left[\frac{1}{\tan^2 25} - \frac{1}{\tan^2 38}\right]}$$ ✓

$$h^2 = 4.13759\ldots$$

$$h = 2.03$$ ✓