[Differentiation + Intro Integration]

BRIGIDINE COLLEGE RANDWICK

Year 12 Mathematics | **Student**

6 December 2004 | **Teacher**

Time 45 Minutes

Show all necessary working.
Neatness may be taken into consideration in the awarding of marks.

There are 6 Questions.

1. Differentiate the following
(leaving answers completely simplified with positive indices)

a. $f(x) = 3x^5 - \frac{2}{x}$ *2 Marks*

b. $g(x) = \frac{x^2}{2\sqrt{x}}$ *2 Marks*

c. $h(x) = 3(2x^2 + 5)^6$ *2 Marks*

d. $j(x) = \frac{4x^2 - 2}{x^2 + 5}$ 3 Marks

2. Find the primitives of the following

a. $2x^3 + 11$ *2 Marks*

b. $6 - x^{\frac{2}{3}}$ *2 Marks*

3. Consider the curve given by $y = 3x^2 - x^3$.

a. Determine the x and y intercepts of this curve. *1 Mark*

b. Show that when $x = 0$ and $x = 2$ there are two stationary values and determine their nature. *3 Mark*

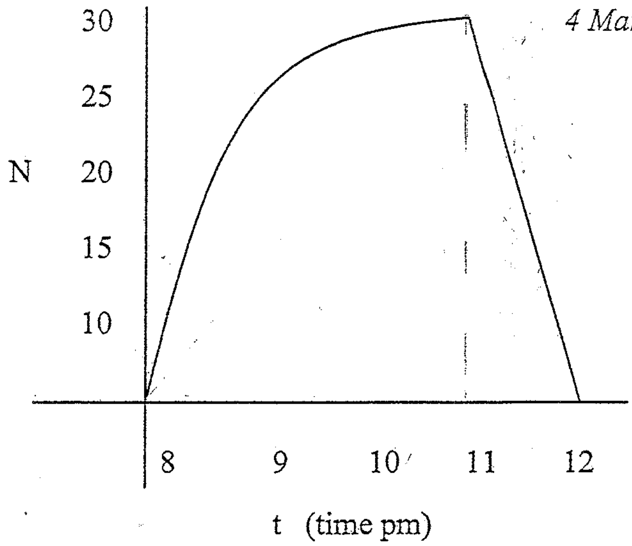
c. Determine any possible points of inflection. *1 Mark*

d. Sketch the curve, showing the above features and indicating where it crosses the x axis. *2 Mark*

e. Find the equation of the tangent to this curve when $x = -1$. *3 Mark*

4. To celebrate the end of the college year, Fran throws a party inviting all the Mathematics students and Staff, of course. To the right is N(t). This represents N (the number who came to Fran's party) with respect to time T.

4 Marks



a. By considering this sketch of N (t) comment on

$\frac{dN}{dt}$ and $\frac{d^2N}{dt^2}$ for $8 < t < 11$

b. State the value of $\frac{dN}{dt}$ and $\frac{d^2N}{dt^2}$ for $11 < t < 12$.

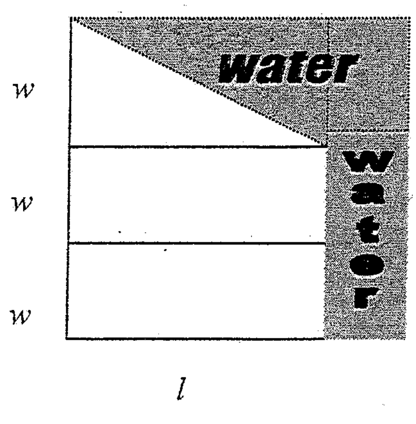
5. Given that $\frac{d^2y}{dx^2} = 6x$ and when $x = 2$, $\frac{dy}{dx} = 8$ and $y = 6$, *3 Mark*

find y in terms of x.

6. A farmer needs to separate his land into **3 sections.** These sections are made from 2 equal rectangles and a triangle bounded by water (as shown to the right)

Each section is to have equal widths w and length l.

The amount of fencing available to this farmer is P metres.



a. Show that the Area of this field may be given by $A = \frac{5}{6} P w - \frac{5}{2} w^2$. *2 Marks*

b. Show that if this farmer is to maximize the area of each section $w = l$. *4 Marks*

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1a) $f(x) = 3x^5 - \frac{2}{x}$
$= 3x^5 - 2x^{-1}$ ✓
$f'(x) = 15x^4 + 2x^{-2}$
$= 15x^4 + \frac{2}{x^2}$ ✓

b) $g(x) = \frac{x^2}{2\sqrt{x}} = \frac{1}{2}\cdot\frac{x^2}{x^{\frac{1}{2}}} = \frac{1}{2}x^{\frac{3}{2}}$
$g'(x) = \frac{3}{4}x^{\frac{1}{2}}$ ✓
$= \frac{3}{4}\sqrt{x}$ ✓

c) $h(x) = 3(2x^2+5)^6$
$= 3\cdot 6(2x^2+5)^5\cdot 4x$ ✓
$= 18(2x^2+5)^5 4x$ ✓
$= 72x(2x^2+5)^5$ ✓

d) $j(x) = \frac{4x^2-2}{x^2+5} = \cancel{\frac{vu' - uv'}{v^2}}$ $\frac{vu'-uv'}{v^2}$
$\frac{[(8x)(x^2+5)] - [(2x)(4x^2-2)]}{(x^2+5)^2}$
$= \frac{[8x^3+40x] - [8x^3-4x]}{(x^2+5)^2} = \frac{44x}{(x^2+5)^2}$
$\cancel{= \frac{-8x^3 - 40x + 8x^3 - 4x}{(x^2+5)^2}}$
$\cancel{= \frac{-44x}{(x^2+5)^2}}$

2a) $\int 2x^3+11$ ✓
$= \frac{2x^4}{4} + 11x + c$ ✓
$= \frac{x^4}{2} + 11x + c$

b) $\int 6 - x^{\frac{2}{3}}$
$= 6x - \frac{3x^{\frac{5}{3}}}{5} + c$ ✓✓

3a) $y = 3x^2 - x^3$
$3x^2 - x^3 = 0$ ✓
$x^2(3-x) = 0$ ✓
$x = 0, 3$
sub $x = 3$ into eq'n
$y = 27 - 27$
$= 0$
$x, y = 3, 0$ ✓

b) $y' = 6x - 3x^2$
$6x - 3x^2 = 0$
$3x(2-x) = 0$
$x = 2, 0$

$y'' = 6 - 6x$
$y''(2) = -6 < 0$ ∩ max
$y''(0) = 6 > 0$ ∪ min ✓
$y(2) = 12 - 6 = 6$ ✓
$y(0) = 0$

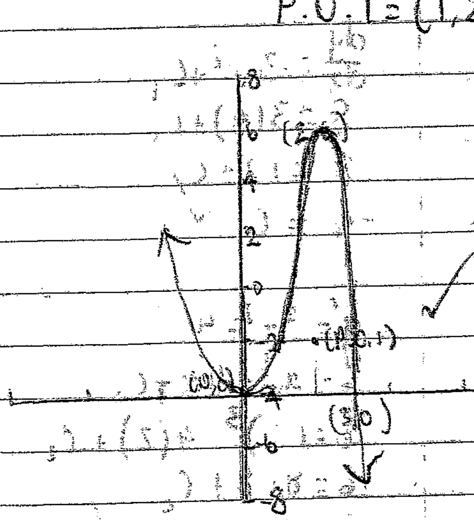
max @ $(2, 6)$ ✓
min @ $(0, 0)$ ✓

c) $y'' = 6 - 6x$
$6 - 6x = 0$
$-6x = -6$
$x = 1$

x	0.5	1	1.5
y''	3	0	−3

$y(1) = 3(1) - 1$
$= 2$ ✓
P.O.I $= (1, 2)$ ✓

d)



✓

e) $y' = 6x - 3x^2$
$= 6(-1) - 3(-1)^2$
$= -6 - 3$
$m = -9$

$y = 3(-1)^2 - (-1)^3$
$= 3 + 1$
$= 4$

$y - y_1 = m(x - x_1)$

$= y - 4 = -9(x+1)$
$y - 4 = -9x - 9$
$y = -9x - 5$
$\therefore -9x - y - 5 = 0$

4a) $\frac{dN}{dt} > 0$

$\frac{d^2N}{dt^2} < 0$

b) $\cancel{(8,0)}$ $(12,0)$ $(11,30)$

$\frac{dN}{dt} = \frac{rise}{run} = -5$ ✗ $\qquad \frac{d^2N}{dt^2} = 0.$

$\frac{dN}{dt} = \frac{30-0}{11-12} = -30$ Also $N - 0 = -30(t - 12)$

$N = -30t + 360$

$N' = -30$

5) $\frac{d^2y}{dx^2} = 6x$ when $x = 2$ $\quad y' = 8$
$y = 6$

$\frac{dy}{dx} = 3x^2 + C_1$
$8 = 3(4) + C_1$
$8 = 12 + C$
$-4 = C$

$y' = 3x^2 - 4$
$y = x^3 - 4x + C_2$
$6 = (2)^3 - 4(2) + C_2$
$6 = 8 - 8 + C_2$
$6 = C_2$
$\therefore y = x^3 - 4x + 6$

6a) $A = \frac{1}{2}wl + wl + wl \qquad P = 3w + 3l$
$= \frac{wl}{2} + 2wl \qquad P - 3w = 3l$
$= \frac{5wl}{2} \qquad \frac{P - 3w}{3} = l$

$A = \frac{5w\left(\frac{P-3w}{3}\right)}{2} = \frac{\frac{5wP - 15w^2}{3}}{2} = \frac{5wP}{6} - \frac{15w^2}{6}$
$= \frac{5wP}{6} - \frac{5w^2}{2}$
$= RHS$

b) $\frac{dA}{dw} = \frac{5P}{6} - 5w = 0$

$= \frac{5(3w+3l)}{6} - 5w = 0$
$\frac{15w + 15l}{6} - 5w = 0$
$\frac{15w}{6} + \frac{15l}{6} - \frac{30w}{6} = 0$
$-\frac{15w}{6} + \frac{15l}{6} = 0$
$-\frac{15w}{6} = -\frac{15l}{6}$
$w = l$