Student ____________________

Teacher ____________________

BRIGIDINE COLLEGE RANDWICK

MATHEMATICS

(HSC)

YEAR 12

HALF YEARLY

2007

(TIME - 2 HOURS)

Directions to candidates

* *Put your* ***name and teacher's name at the top*** *of this paper and on each of the* ***6 sections*** *that are to be collected.*

* *All 6 questions are to be attempted.*

* *All 6 questions are of equal value.*

* *All questions are to be answered on separate pages and will be collected in separate bundles at the end of this exam.*

* *All necessary working should be shown in every question.*

* *Full marks may not be awarded for careless or badly arranged work.*

Question 1 *(Start a new page)*

a. Determine the value of $\dfrac{32.4 + \sqrt{8904.5}}{8.5 - 0.367^2}$ correct to the nearest hundredth. *2 m*

b. After a 10 % discount, Jana paid \$ 405 for a DVD player.
What was the original price marked on this DVD player? *2 m*

c. Completely factorise $8x^3 - 1$. *1 m*

d. Solve the inequation $2x^2 + 3x \geq 2$ *3 m*

e. Show that $\sqrt{3} \times 2\sqrt{6}$ may be expressed in the form $\sqrt{y}$ and state the value for y. *2 m*

f. Differentiate

i. $(6x^2 + 3x)^4$ *2 m*

ii. $6x^2 + \dfrac{2}{x} - \sqrt{x}$ *3 m*

Question 2 *(Start a new page)*

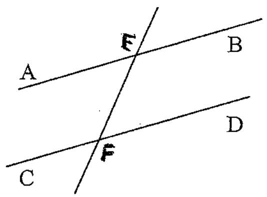
a. Solve for x if $|2x - 3| \leq x - 3$ *3 m*

b. Solve the simultaneous equations below *3 m*

$y = x^2 - 3x + 4$

$y = 2x - 2$

c. For the figure to the right AB // CD and the transversal XY is drawn through these parallel lines cutting AB and CD at E and F respectively.



i. Redraw this information onto your exam pages and mark in all the relevant information. *1 m*

ii. Prove that $\angle AEX = 180° - \angle XFD$ *2 m*

d. If α and β are the roots to the equation $x^2 + 2x - 3 = 0$.

State the value of $\alpha + \beta$ and $\alpha\beta$ and hence find

i. $\frac{1}{\alpha} + \frac{1}{\beta}$ *2 m*

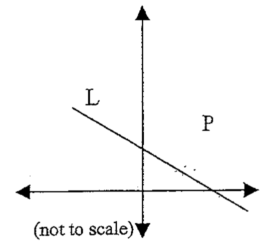
ii. $\alpha^2 + \beta^2$ *2 m*

e. If $0 \leq \alpha \leq 360°$, find all values of α of *2 m*

$\cos \alpha = \frac{-1}{2}$

Question 3 *(Start a new page)*

a. The line L has equation $x + 2y = 5$ and P is the point (2,4).
Note : P is not on line L



i. Make a large sketch of this information onto your exam page, marking in the origin O, the point P and the line L. *1 m*

ii. Find the midpoint M, of the interval OP. *1 m*

iii. Show that M lies on the line L. *1 m*

iv. Find the gradients of the line OP and the line L. *2 m*

v. Show that the line L is the perpendicular bisector of the interval OP. *1 m*

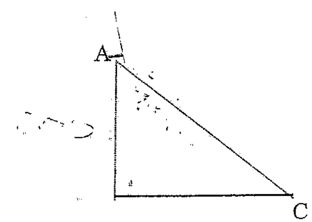
vi. Line L meets the x-axis at Q. Find the coordinates of Q. *1 m*

vii. A line is drawn through O parallel to PQ and it meets line L at R. Find the equation of OR. *2 m*

viii. Explain why PQOR is a rhombus. *1 m*

ix. Determine the area of PQOR *2 m*

b. An aircraft "A" flying at an altitude of 1500 metres notices that the angle of depression to a nearby city "C" is 42°20'. How far is this aircraft from this city? *3 m*



Question 4 *(Start a new page)*

Consider the curve given by $y = x^3 - 6x^2 + 9x + 1$.

a. Determine $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. *2 m*

b. Show that stationary values exist at $x = 1$ and $x = 3$ and determine their nature. *3 m*

c. Determine the coordinates of any points of inflection. *(be sure to show how this was found)* *1 m*

d. State the values of x for which this curve is increasing. *2 m*

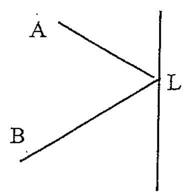
e. Neatly sketch this curve showing all relevant information. *2 m*

f. Determine the equation of the tangent to this curve $y = x^3 - 6x^2 + 9x + 1$ when $x = -1$. *2 m*

g. Determine the area trapped by this curve $y = x^3 - 6x^2 + 9x + 1$ and the ordinates $x = 0$ and $x = 3$. *3 m*

Question 5 *(Start a new page)*

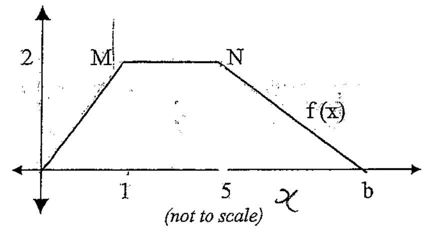
a. From a lighthouse L, two boats A and B are positioned such that A is 7 km N 23° W and B is 18 km S 59° W from this lighthouse.



i. Redraw this figure to the right on to your exam exam page and mark in these relative bearings. *1 m*

ii. Calculate the distance of B from A, to the nearest metre. *2 m*

b. Solve for x if $2\sin^2 x + \sin x - 1 = 0$ and $0° \le x \le 360°$ *3 m*

c. 

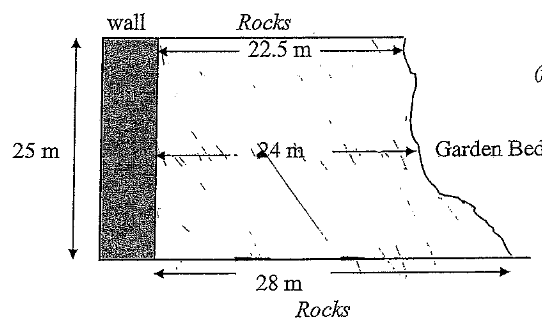
This figure to the left represents the continuous curve f (x). M and N are the points (1,2) and (5,2) respectively.

If $\int_1^b f(x)\,dx = 13$, find the value of b. *2 m*

d. "Although the amount of pollution is increasing, the Government policy to reduce pollution seems to be taking effect." *1 m*

Given that P is the amount of pollution, what statements can be made about $\frac{dp}{dt}$ and $\frac{d^2p}{dt^2}$ with regard to the above quote?

e. A school wanted to calculate the area of lawn in one of its quads. This area was surveyed and the diagram below shows a notepad sketch of the area. The quad is surrounded by a wall, two sets of rocks and a garden bed. Measurements were taken at either end of the quad and from the middle of the quad. *2 m*



(figure not to scale)

By using Simpson's Rule, with one application, determine the area of this quad.

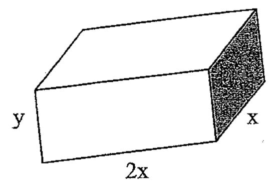
f. A parabola has the equation $x^2 = 8(y+2)$. Find the

i. x-intercepts of this parabola. *1 m*

ii. Coordinates of the vertex and focus. *3 m*

Question 6 *(Start a new page)*

a. Boxes in the shape of rectangular prisms are to be constructed from special materials.
The width (x metres) of the base is to be half the length of the base and each box is to hold a volume of 4 cubic metres.



Material that is used to make the base and the top costs \$ 15 per m^2 and a cheaper material at \$ 10^2 per is used for the four sides.

i. Show that the total cost of building each box is given by *2 m*

$$C = \$ (60x^2 + \frac{120}{x^2}).$$

ii. Determine the width of the cheapest box that can be constructed. *3 m*

b. The background for a logo of a company was designed by using the area trapped between the curves $y = 2x^2$ and $y = 12 - x^2$.

i. Show that these two curves intersect when $x = -2$ and $x = 2$ and on the same diagram sketch these two curves, showing their points of intersection. *2 m*

ii. Determine the area of the background for this logo. (that is the area trapped between the two curves) *2 m*

iii. If the area trapped between these two curves was to be rotated about the y-axis, determine the resultant volume. *3 m*

c. Patricia moves a load of soil for top dressing an oval by emptying barrow-loads in a line 20 m apart, with the first heap 20 metres from the load of soil. How far does she walk if she empties 24 barrowfulls and returns to the load each time? *3 m*

See yellow sheet

BRIGIDINE COLLEGE RANDWICK

YR 12 HALF-YEARLY 2007 – MATHEMATICS: Solutions and Marking Scheme Q2, Q4, Q6

QUESTION 2

(a) $|2x-3| \le x-3$

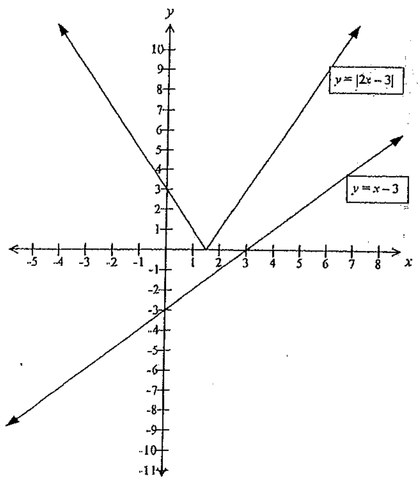
Then

$2x-3 \le x-3$ and	$2x-3 \ge -(x-3)$
$x-3 \le -3$	$2x-3 \ge -x+3$
$x \le 0$	$3x \ge 6$
	$x \ge 2$
Test $x=-1$	Test $x=3$
LHS $= \|2x-3\|$	LHS $= \|2x-3\|$
$= \|2\times -1-3\|$	$= \|2\times 3-3\|$
$= 5$	$= 3$
RHS $= -1-3$	RHS $= 3-3$
$= -4$	$= 0$
$x \le 0$ fails test	$x \ge 2$ fails test
Reject	Reject

There is no solution

Alternative graphical solution

The solution of $|2x-3| \le x-3$ is the set of x values for which the graph $y=|2x-3|$ either cuts or is below the graph of $y=x-3$



From the graph there is no solution.

(b) $y = x^2 - 3x + 4$ (1)

$y = 2x - 2$ (2)

Substitute for y from (2) into (1)

$2x - 2 = x^2 - 3x + 4$

$x^2 - 3x + 4 = 2x - 2$

$\quad -2x + 2 \quad -2x + 2$

$x^2 - 5x + 6 = 0$

$(x-2)(x-3) = 0$

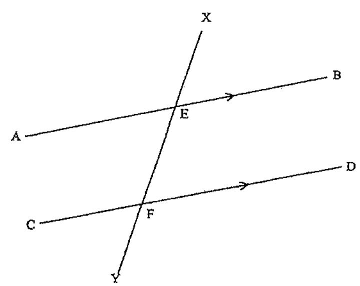
$x = 2, \qquad x = 3$

$y = 2\times 2 - 2 \qquad y = 2\times 3 - 2$

$= 2 \qquad\qquad = 4$

(c)

(*i*)



(*ii*) RTP $\angle AEX = 180° - \angle XFD$

$\angle AEX = \angle BEF$ (Vertically opposite angles)

$\angle BEF + \angle XFD = 180°$

(co-interior angles formed by parallel lines)

Thus

$\angle AEX + \angle XFD = 180°$

$\angle AEX = 180° - \angle XFD$

(d) $x^2 + 2x - 3 = 0$

$\alpha+\beta = -\frac{b}{a} \qquad \alpha\beta = \frac{c}{a}$

$= -\frac{2}{1} \qquad = -\frac{3}{1}$

$= -2 \qquad = -3$

OR

$x^2 + 2x - 3 = 0$

$(x+3)(x-1) = 0$

$x = -3$ or $x = 1$

i e.

$\alpha = -3 \quad \beta = 1$

$\alpha+\beta = -3+1 \qquad \alpha\beta = -3\times 1$

$= -2 \qquad = -3$

(*i*) $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta}$

$= \frac{-2}{-3}$

$= \frac{2}{3}$

(*ii*) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= (-2)^2 - 2\times(-3)$

$= 4 + 6$

$= 10$

(e) $\cos\alpha = -\frac{1}{2} \qquad 0° \le \alpha \le 360°$

α is in the 2nd or 3rd quadrants.

Now

$\cos 60° = \frac{1}{2}$

So

2nd quadrant: $\alpha = 180° - 60°$

$= 120°$

3rd quadrant: $\alpha = 180° + 60°$

$= 240°$

QUESTION 4

$y = x^3 - 6x^2 + 9x + 1$

(a) $\frac{dy}{dx} = 3x^2 - 12x + 9$

$\frac{d^2y}{dx^2} = 6x - 12$

(b) For a stationary value $\frac{dy}{dx} = 0$

i.e. $3x^2 - 12x + 9 = 0$

$3(x^2 - 4x + 3) = 0$

$x^2 - 4x + 3 = 0$

$(x-1)(x-3) = 0$

$x = 1, 3$

Nature of values

Method 1:

At $x = 1$

x	1^-	1	1^+
Sign of $\frac{dy}{dx}$	+	0	–
Tangent	/	—	\

$\Rightarrow$ a maximum turning point at $x = 1$

At $x = 3$

x	3^-	3	3^+
Sign of $\frac{dy}{dx}$	–	0	+
Tangent	\	—	/

$\Rightarrow$ a minimum turning point at $x = 3$

At

$x = 1$ there is a maximum turning point;

$x = 3$ there is a minimum turning point.

Method 2:

$\frac{d^2y}{dx^2} = 6x - 12$

$x = 1 \qquad \frac{d^2y}{dx^2} = 6\times 1 - 12 = -6 < 0 \rightarrow$ *Max*

$x = 3 \qquad \frac{d^2y}{dx^2} = 6\times 3 - 12 = 6 > 0 \rightarrow$ *Min*

At

$x = 1$ there is a maximum turning point;

$x = 3$ there is a minimum turning point.

Comment:

Method 1 works for all functions. However, Method 2 will not work for all functions e.g. $y = x^4$

Be aware of this when choosing a method for determining the nature of stationary points.

(c) For a point of inflection $\frac{d^2y}{dx^2} = 0$ and changes sign (concavity).

i.e. $6x - 12 = 0$

$6x = 12$

$x = 2$

x	2^-	2	2^+
Sign of $\frac{d^2y}{dx^2}$	–	0	+

At $x = 2$ a point of inflection exists and:

$y = x^3 - 6x^2 + 9x + 1$

$= 2^3 - 6 \times 2^2 + 9 \times 2 + 1$

$= 3$

(2, 3) is the point of inflection

Note: At a point of inflexion the second derivative is zero and changes sign. It is necessary to analyse the sign of the second derivative in the immediate neighbourhood to confirm a point of inflexion.

(d) The curve is increasing for $\frac{dy}{dx} > 0$

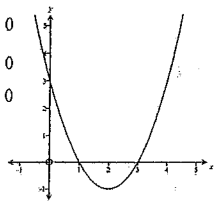
i.e.

$3(x^2 - 4x + 3) > 0$

$x^2 - 4x + 3 > 0$

$(x - 1)(x - 3) > 0$

$x < 1$ or $x > 3$



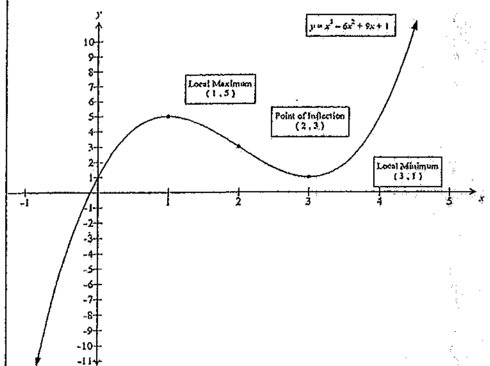
(e) For $y = x^3 - 6x^2 + 9x + 1$

y-intercept = 1

(1, 5) is a local maximum

(2, 3) is a point of inflection

(3, 1) is a local minimum



(f) At $x = -1$

$\frac{dy}{dx} = 3x^2 - 12x + 9$

$= 3 \times (-1)^2 - 12 \times (-1) + 9$

$= 24$

$= m$ (Gradient of the tangent at $x = -1$)

$y = x^3 - 6x^2 + 9x + 1$

$= (-1)^3 - 6 \times (-1)^2 + 9 \times (-1) + 1$

$= -15$

Tangent at (-1, -15) is given by

$y - y_1 = m(x - x_1)$

$y - (-15) = 24(x - (-1))$

$y + 15 = 24(x + 1)$

$y + 15 = 24x + 24$

$y = 24x + 9$

(g) Area $= \int_0^3 (x^3 - 6x^2 + 9x + 1)dx$

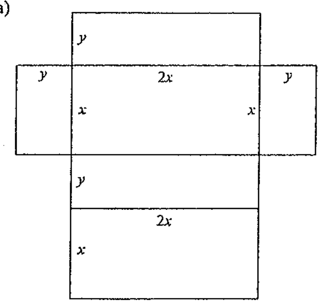
$= \left[\frac{x^4}{4} - \frac{6x^3}{3} + \frac{9x^2}{2} + x\right]_0^3$

$= \frac{1}{4} \times 3^4 - 2 \times 3^3 + \frac{9}{2} \times 3^2 + 3$

$= 9\frac{3}{4}$ unit^2

QUESTION 6

(a)



(i)

Area of Top + Bottom $= 2 \times 2x^2$

$= 4x^2\ m^3$

Area of Sides $= 2 \times xy + 2 \times 2xy$

$= 6xy\ m^3$

Total surface area $= (4x^2 + 6xy)m^3$

Thus

Cost of material $= \$(4x^2 \times 15 + 6xy \times 10)$

$= \$(60x^2 + 60xy)$

Also

Volume of box $= 4\ m^3$

and

Volume of box $= 2x \times x \times y$

$= 2x^2y\ m^3$

$\therefore\ 2x^2y = 4$

$y = \frac{4}{2x^2}$

$= \frac{2}{x^2}$

Thus C, the cost of the material in the box is given by

$C = \$\left(60x^2 + 60 \times x \times \frac{2}{x^2}\right)$

$= \$\left(60x^2 + \frac{120}{x}\right)$

where x is the width of the box.

(ii)

Using the cost function in the question

$C = 60x^2 + \frac{120}{x^2}$

$C = 60x^2 + 120x^{-2}$

$\frac{dC}{dx} = 120x - 240x^{-3}$

$= 120x - \frac{240}{x^3}$

For the minimun value of C

$\frac{dC}{dx} = 0$

$120x - \frac{240}{x^3} = 0$

$120x^4 - 240 = 0$

$120x^4 - 240 = 0$

$x^4 - 2 = 0$

$x^4 = 2$

$x = \pm\sqrt[4]{2}$

As x is a length it is positive

$x = \sqrt[4]{2}$ or $2^{\frac{1}{4}}$

At $x = \sqrt[4]{2}$

x	←	$\sqrt[4]{2}$	→
Sign of $\frac{dy}{dx}$	–	0	+
Tangent	\	__	/

The minimum value of C occurs when the width of the box is $\sqrt[4]{2}\ m$

Alternatively, using the cost function from above

$C = \$\left(60x^2 + \frac{120}{x}\right)$

and following the same process, the minimum cost of the box occurs when $x = 1$; i.e. the width is 1 metre.

(b)

(i) $y = 2x^2$ (1)

$y = 12 - x^2$ (2)

Substitute $2x^2$ from (1) for y in (2)

$2x^2 = 12 - x^2$

$3x^2 = 12$

$x^2 = 4$

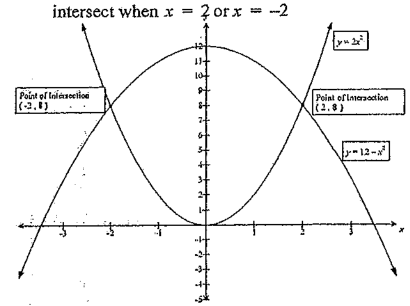
$x = \pm 2$

Thus the graphs of

$y = x^2$

and $y = 12 - x^2$

intersect when $x = 2$ or $x = -2$



(*ii*) The area trapped between the curves is given by

$$\text{Area} = \int_{-2}^{2}(12 - x^2)dx - \int_{-2}^{2} 2x^2 dx$$

$$= 2\int_{0}^{2}(12 - x^2 - 2x^2)dx$$

$$= 2\int_{0}^{2}(12 - 3x^2)dx$$

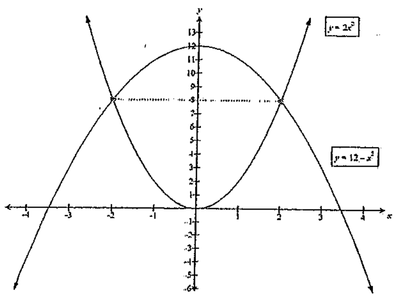
$$= 2\left[12x - x^3\right]_0^2$$

$$= 2\left[12 \times 2 - 2^3\right]$$

$$= 32\,\text{unit}^2$$

(*iii*) For the volume, V, of solid of revolution about the y-axis

$$V = \pi\int_{a}^{b} x^2 dy$$



From the diagram the required volume is made is created when:

the area bounded by $y = 2x^2$ from $y = 0$ to $y = 8$ is rotated; and

the area bounded by $y = 12 - x^2$ from $y = 8$ to $y = 12$;

are rotated respectively about the y-axis.

$y = 2x^2$, is rewritten as $x^2 = \frac{1}{2}y$

$y = 12 - x^2$, is rewritten as $x^2 = 12 - y$

Thus

$$V = \pi\int_{0}^{8}\frac{1}{2}y\,dy + \pi\int_{8}^{12}(12 - y)dy$$

$$= \pi\left[\frac{1}{4}y^2\right]_8^0 + \pi\left[12y - \frac{1}{2}y^2\right]_8^{12}$$

$$= 16\pi + 8\pi$$

$$= 24\pi\,\text{unit}^3$$

(c) $x = 3\sin\theta$ (1)
$y = 3\cos\theta - 2$ (2)

From (2)
$3\cos\theta = y + 2$

Thus

$$x^2 + (y+2)^2 = (3\sin\theta)^2 + (3\cos\theta)^2$$

$$= 9\sin^2\theta + 9\cos^2\theta$$

$$= 9(\sin^2\theta + \cos^2\theta)$$

$$= 9$$

i.e. $x^2 + (y + 2)^2 = 9$

MARKING SCHEME

QUESTION 2

(a)	Correct answer supported by appropriate working	3
	Two correct inequalities with solved correctly; or Two correct graphs	2
	One correct inequality solved correctly; or One correct graph	1
(b)	Correct answer supported by appropriate working.	2
	Correct quadratic equation; or One correct (x, y) pair; or Two correct x solutions; or Two correct y solutions	1
(c) (*i*)	Correct diagram; must have the lines AB and CD marked with arrows to indicate they are parallel.	1
(*ii*)	Appropriate pairing of ∠AEX with another angle giving the correct reason	1
	Correctly arriving at the required result with an appropriate reason.	1
(d) (*i*)	Correct answer	2
	Correctly deriving the result $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta}$; or Correctly substituting for α and β	1
(*ii*)	Correct answer	2
	Correctly deriving the result $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$; or Correctly substituting for α and β	1
(e)	Both values of α found using correct methods.	2
	Correctly determining on value of α.	1

QUESTION 4

(a)	$\frac{dy}{dx}$ correct.	1
	$\frac{d^2y}{dx^2}$ correct.	1
(b)	Correct answer supported by appropriate working.	3
	Stating $\frac{dy}{dx} = 0$ for a stationary value ; or Correct solutions for $\frac{dy}{dx} = 0$; or Correctly determining the nature of the stationary values.	1
(c)	Correctly determining the coordinates of the point of inflection with appropriate working.	1
(d)	Correct answer supported by appropriate working.	3
	$\frac{dy}{dx} > 0$ when the curve is increasing; One correct solution; Correct graph	1
(e)	The diagram has clearly marked the following points determined in prior sections of the question: • Local maximum • Local minimum • Point of inflection plus • y-intercept and is drawn correctly to reflect these.	2
	Correctly show at least two of the above correctly.	1
(f)	Correctly determining the equation of the required tangent	2
	Determining the gradient of the tangent and the y-coordinate of the point correctly.	1
	Using the appropriate method to find the equation of the tangent.	1
(g)	Correct answer supported by appropriate working.	3
	Correct definite integral with correct limits	1
	Correct substitution of the limits to evaluate the definite integral.	1
	Correct evaluation of definite integral.	1

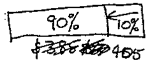
QUESTION 6

(a) (*i*)	Correctly expressing y in terms of x.	1
	Correctly deriving $\text{Cost} = \$(60x^2 + 60xy)$; or Correctly deriving $\text{Cost} = \$\left(60x^2 + 60 \times x \times \frac{2}{x^2}\right)$ or its equivalent.	1
	Note: the correct answer is $\text{Cost} = \$\left(60x^2 + \frac{120}{x}\right)$	
(*ii*)	Correct answer supported by appropriate working.	3
	Correctly finding $\frac{dy}{dx}$; Correctly solving $\frac{dy}{dx} = 0$; Correctly determining the nature of the stationary value;	1
(b) (*i*)	Solving simultaneous equations to show there are only two points of intersection.	1
	Correctly drawn graph with at least one of the functions correctly labelled and: • points of intersection with coordinates; • y-intercepts; clearly marked.	1
(*ii*)	Correct answer obtained with appropriate working.	2
	Correct integration with correct limits.	1
	Correctly evaluating the definite integral.	1
(*iii*)	Correct answer with correct working.	3
	Showing understanding that the required volume is found using $V = \pi\int_a^b x^2 dy$	1
	Each sub-volume correctly determined.	1
	Adding the sub-volumes	1
(c)	Correct answer with appropriate working	3
	Expressing $\sin\theta$ and $\cos\theta$ in terms of x and y respectively.	1
	Correctly applying the result $\sin^2\theta + \cos^2\theta = 1$	1

(MATHEMATICS)

1a) 15·15

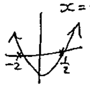
b)



~~$385~~ 405
90% = 385 ~~$450~~ 405
10% = ~~42.7~~ $45
100% = $450

c) $8x^3-1$
$(2x)^3-(1)^3$
$(2x-1)(4x^2+2x+1)$

d) $2x^2+3x \geq 2$
$2x^2+3x-2 \geq 0$
$(2x-1)(x+2) \geq 0$
$x=\frac{1}{2}$ $x=-2$



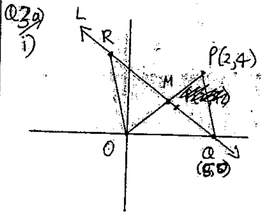
$x \leq -2, x \geq \frac{1}{2}$

e) $\sqrt{3}\times 2\sqrt{6}$
$=2\sqrt{18}$
$=\sqrt{4}\times\sqrt{18}$
$=\sqrt{72}$ $\therefore y=72$

$\frac{d}{dx}(6x^2+3x)^4$
$4(6x^2+3x)^3\times(12x+3)$
$4(12x+3)(6x^2+3x)^3$

ii) $\frac{d}{dx}\, 6x^2+2x^{-1}-x^{\frac{1}{2}}$
$12x-2x^{-2}-\frac{1}{2}x^{-\frac{1}{2}}$

Q3a) i)



ii) $M=\left(\frac{2+0}{2},\frac{4+0}{2}\right)=(1,2)$

iii) $x+2y=5$
$M(1,2) \longrightarrow 1+2\times2=5$
TRUE

iv) $m_{OP}=\frac{4}{2}=2$
$L \rightarrow x+2y=5$
$2y=-x+5$
$y=-\frac{1}{2}x+\frac{5}{2}$
$\therefore m_L=-\frac{1}{2}$

$m_{OP}\times m_L=-1$
$2\times-\frac{1}{2}=-1$
$\therefore$ OP ⊥ line L
and since M = midpoint of OP
then L is ⊥ bisector of OP

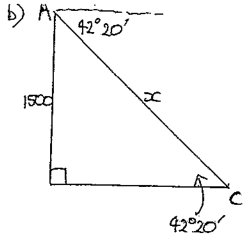
vi) $y=0$
$x+2y=5$
$x+2\times0=5$
$x=5$
$\therefore Q(5,0)$

vii) $m_{PQ}=\frac{4-0}{2-5}=-\frac{4}{3}$
$\therefore$ equ OR = $y-0=-\frac{4}{3}(x-0)$
$y=-\frac{4}{3}x$

viii) see marking scheme

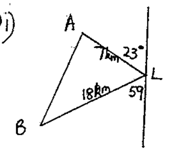
ix) Area $\Delta_{OPQ}=\frac{1}{2}\times5\times4$
$=10$
$\therefore$ Area Rhombus = 2 × 10
= 20

4b)



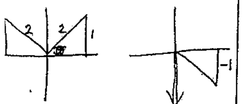
$\sin 42°20' = \frac{1500}{x}$
$x=\frac{1500}{\sin 42°20'}$
$x=2227.36$

5)a) i)



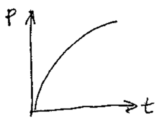
ii) $180-23-59=98°$
$AB^2=7^2+18^2-2\times7\times18\times\cos 98$
$=408.0716\ldots$
AB = 20.2km
Note 20m = 1mark

5b)
$2\sin^2x+\sin x-1=0$
$(2\sin x-1)(\sin x+1)=0$
$\sin x=\frac{1}{2}$ $\sin x=-1$



$x=30°, 150°$ $x=270°$

Trapezium = $\frac{1}{2}\times2\times(4+(b-1))$
$=4+b-1$
$4+b-1=13$
$b=10$



$\frac{dP}{dt}>0$ $\frac{d^2P}{dt^2}<0$

e) $h=\frac{25}{2}=12.5$
$A \doteqdot \frac{12.5}{3}[22.5+4\times24+28]$
$=610.41\dot{6}$

f) $x^2=8(y+2)$
i) $y=0 \rightarrow x^2=16$
$x=\pm4$

ii) vertex = $(0,-2)$
focal length = $4a=8$
$a=2$
$(0,-2)$ ↑+2
$\therefore$ focus = $(0,0)$

Marking scheme

1a) 15·15 (2marks)
~~15·2 (1mark)~~
15·1534902... (1mark) any correct rounding
14·78 (1mark) no brackets on denominator
43·68 (1mark) no brackets on numerator.

b) (2marks): $450
(1mark): $45, or $4.50

c) (1mark): $(2x-1)(4x^2+2x+1)$

d) (3marks) $x \leq -2, x \geq \frac{1}{2}$
(2marks): $x \geq -2$, $x \leq \frac{1}{2}$ ($-2 \leq x \leq \frac{1}{2}$)
or one correct.

(1mark): $(2x-1)(x+2)$
Correct inequation answer from incorrect factorising.

e) (2marks) $y=72$
(1mark) $y=\sqrt{72}$
or $y=\sqrt{\text{wrong}}$ and $y=$ wrong

f) i) (2marks): $4(12x+3)(6x^2+3x)^3$ or equivalent.
(1mark) $4(6x^2+3x)^3\times$ anything except 0

ii) (3marks): $12x-2x^{-2}-\frac{1}{2}x^{-\frac{1}{2}}$ or equivalent.
(2marks): any 2 correct derivative
(1mark): any 1 correct derivative

Q3a) i) (1mark) needs a sketch with P(2,4) and O marked on the origin, and L.
ii) (1mark): (1,2)
iii) (1mark): showing correct substitution

iv) (2marks) $m_{OP}=2$ and $m_L=-\frac{1}{2}$
(1mark) any one correct gradient.
v) (1mark) Need to show $m_{OP}\times m_L=-1$ and mention midpoint.
vi) (1mark) Q(5,0)
vii) (2marks): $y=-\frac{4}{3}x$ or equivalent.
(1mark): $m_{PQ}=-\frac{4}{3}$ or (−3,4)
or substituting (0,0) but using wrong gradient.
viii) see end of paper.
ix) (2marks): Area = 20
(1mark): Area of ΔPOQ = 10
or Area of ΔOMQ = 5

b) (3marks) AC = 2227·36
(2marks) } 1mark deducted
(1mark) } for each mistake
$x=1646.55$ (1mark) if no diagram is provided.

Q5 a) i) (1mark) correctly mark 23° and 59°
ii) (2marks): 20·2
(1mark): correct expression for AB or incorrect ∠ALB but correctly solved for AB
Note 20 metres = 1mark if 20·2 is not shown first.
b) (3marks) $x=30, 150, 270$
(2marks): any 2 correct ∠'s, no other incorrect ∠'s.
(1mark): $\sin x=\frac{1}{2}$ or $\sin x=-1$
or correct ∠'s from incorrect $\sin x$ solutions

c) (2marks): b=10
(1mark): Area of Trap = 13 or statement with "incorrect area = 4+(b−1)
or b=9

Q5d (1mark): need both $\frac{dP}{dt}>0$, $\frac{d^2P}{dt^2}<0$

e) (2marks): A = 610·4
(1mark): h = 12·5
or correct formula but incorrect h (Note $\frac{25}{3}$[correct] = $\frac{25}{9}$[correct] (1mark)) with substitutions

f) i) (1mark) need both $x=\pm4$
ii) (3marks): vertex (0,−2) and focus = (0,0)
(2marks): focus (0,0)
or focal length = 2 and vertex (0,−2)
(1mark): focal length = 2
or vertex (0,−2)
or incorrectly label vertex (0,0) or focus (0,−2)

Q3a) viii) No marks were given for students stating properties of a rhombus without working as a backup. "Diagonals bisect each other – proven in iii" is incorrect – only one diagonal was shown.
→ (1mark) Showing all sides = and diagonals bisecting at right ∠'s.
or Showing opp sides parallel and diagonals bisect at right ∠'s.