Student ______________________

Teacher ______________________

BRIGIDINE COLLEGE RANDWICK

MATHEMATICS

HSC

HALF YEARLY

2010

(TIME - 2 HOUR)

Directions to candidates

- *Put your **name at the top** of this paper and on each of the **6 sections** that are to be collected.*
- *All 6 questions are to be attempted.*
- *All 6 questions are of equal value.*
- *All questions are to be answered on separate pages and will be collected in separate bundles at the end of this exam.*
- *All necessary working should be shown in every question.*
- *Full marks may not be awarded for careless or badly arranged work.*

Standard Integrals

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}}\,dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}}\,dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

Question 1 *(Start a new page)*

a. Calculate $\sqrt{\dfrac{35.5 + 8.24 \times 4}{3^2 - \pi}}$ (to 3 significant figures). 2 m

b. Completely factorise $8x^3 - 27$. 1 m

c. Solve the inequation $x^2 \geq 3x - 2$ 3 m

d. Solve for x if $|3x - 1| = 2$ 3 m

e. State the natural domain for the following $\dfrac{1}{x^2 - 1}$ 2 m

f. If α and β are the roots to the equation $2x^2 + 5x = 3$.

i. State the value of $\alpha + \beta$ and $\alpha\beta$ and hence find 1 m

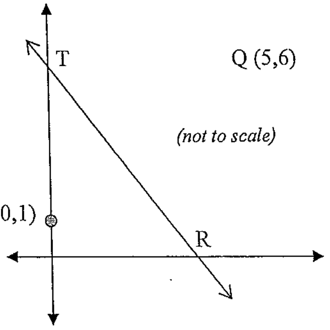
ii. $\dfrac{1}{\alpha} + \dfrac{1}{\beta}$ 1 m

iii. $\alpha^2 + \beta^2$ 2 m

Question 2 (start a new page)

a. In the diagram below, P and Q have coordinates (0,1) and (5,6) respectively. T lies on the y axis and R lies on the x axis.

The line through T and R has equation $3x + 2y - 12 = 0$.



√ i. Copy the diagram onto your answer page and find the length of PR. 2 m

ii. Show that the gradient of PQ is 1. 1 m

iii. Show that the equation of the line through P and Q is $x - y + 1 = 0$. 2 m

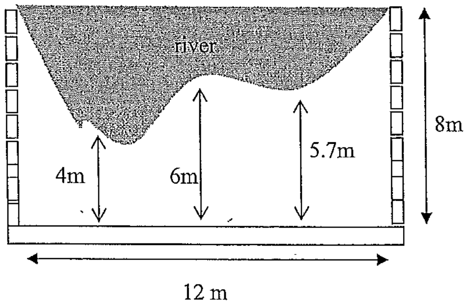
iv. Find the coordinates of the point at which the lines $x - y + 1 = 0$ and $3x + 2y - 12 = 0$ intersect. 3 m

v. Find the perpendicular distance of the point P from the line $3x + 2y - 12 = 0$. 3 m

vi. Find the angle which the line PQ makes with the positive direction of the x axis. 1 m

vii. On your diagram shade the region satisfying the inequality $x - y + 1 \geq 0$. 1 m

b.



A market garden is bounded on one side by a river and the other 3 sides by stones.

Its dimensions are shown in this diagram to the left.

By using Simpson's Rule with two applications, determine the area of this market garden.

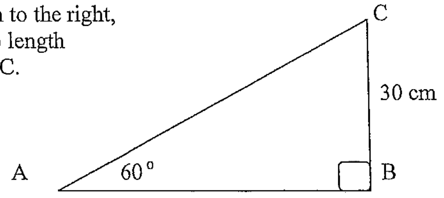
3 m

Question 3 *(Start a new page)*

a. Find the locus of all the points P (x,y) whose distance from A (1,4) is twice its distance from B (-3,5). 3 m

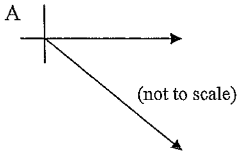
b. i. Show that the exact value of $\sin 60^\circ = \frac{\sqrt{3}}{2}$. 1 m

ii. Consider this diagram to the right, Determine the (exact) length of the hypotenuse AC. 2 m



c. If $0 \le x \le 360^\circ$, find all values of x if $\sin x = \frac{-1}{2}$. 3 m

d. Two ships sail from a starting point A. The first ship sails due east for 40 km. The second sails for 60 km in a southerly direction (as shown) until it is due south of the first ship. Determine the bearing of the second ship from the starting point A (nearest minute). 4 m



e. The table below lists the number of sales for a new product N over a period of time t (months). 2 m

t	1	2	3	4	5	6
N	5000	7500	8750	9375	9687	9849

Comment on these sales in terms of $\frac{dN}{dt}$ and $\frac{d^2N}{dt^2}$ justifying your answer in words.

Question 4 *(Start a new page)*

a. Given that the $\log_a 2 = 0.23$ and $\log_a 3 = 0.33$, find the $\log_a 54$. 2 m

b. Write down the equation of the tangent to the curve $y = \sqrt{x} + 1$ when $x = 4$ in general form. 3 m

c. The gradient function of a curve f (x) is given by $3x^2 - 4x + 7$. The curve f (x) passes through the point (1,-1). 3 m

Find the equation of f (x).

d. The tangent to the curve $y = 2x^2 + \frac{a}{x^2}$ has a turning point at $x = 3$. Find the constant a. 3 m

e. Consider $f(x) = |x - 2| + 3$

i. Neatly sketch f(x). 2 m

ii. Evaluate $\int_2^4 f(x)\ dx$. 2 m

Question 5 *(Start a new page)*

a. Differentiate

i. $x \log x$ — 1 m

ii. $e^{\sqrt{x}}$ — 2 m

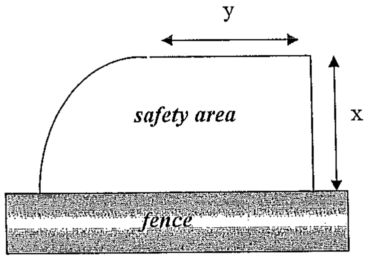
b. Evaluate

i. $\int_0^1 x^2 e^{x^3}\, dx$ — 2 m

ii. $\int_1^2 \frac{1}{x+1}\, dx$ — 2 m

c. A council decides to provide a safety area for a park by using an existing fence and roping off the rest of the area.

This safety area is made up of a quadrant of a circle and a rectangle as shown to the right. *(not to scale)*



i. If there is 33 metres of rope available, show that $y = 33 - (\frac{\pi}{2} + 1)x$. — 1 m

ii. Show that the area to be roped off is given by $A = 33x - (\frac{\pi}{4} + 1)x^2$. — 2 m

iii. Find in simplest form the exact value for x for which the maximum area can be roped off. — 3 m

d. Find the volume of the solid generated when the curve $y = \frac{1}{\sqrt{x}}$ is rotated about the x-axis between $x = 1$ and $x = 2$. — 2 m

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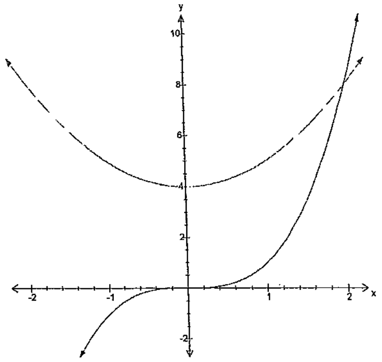
Question 6 *(Start a new page)*

a. To the right is s sketch of the curves $y = x^3$ and $y = x^2 + 4$



Verify that the curves $y = x^3$ and $y = x^2 + 4$ intersect at the point (2, 8).

Therefore

Find the area of the region bounded by the curves $y = x^3$ and $y = x^2 + 4$ in the first quadrant. — 4 m

b. Consider the curve $f(x) = \frac{1}{2}x^4 - 4x^2$

i. Show that this curve represents an even function. — 1 m

ii. Show that this curve has x intercepts at $x = \pm 2\sqrt{2}$ and $x = 0$. — 2 m

iii. Determine $f'(x)$ and $f''(x)$. — 2 m

iv. Show that there exists Stationary Values at $x = 0$ and $x = \pm 2$ and determine their nature. — 2 m

v. Determine the points of inflection for this curve. — 2 m

vi. Sketch this curve showing all the above features. — 2 m

- end of exam –

Q1

a) $\sqrt{\dfrac{35.5+8.24\times4}{3^2+\pi}}$ 3sf

$= \sqrt{\dfrac{68.46}{5.85\ldots}} = 11.685876$ ✓

$= \sqrt{11.7}$

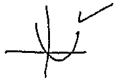
$= 3.42$ ✓ 2

b) $8x^3-27$

$(2x-3)(4x^2+6x+9)$ 1

c) $x^2-3x+2 \geq 0$ ✓

$(x-1)(x-2) \geq 0$ ✓



$x \leq 1,\ x \geq 2$ ✓

Factorise ✓
sketch ✓

d) $-2 = 3x-1$ | $3x-1 = 2$

$-1 = 3x$ | $3x = 3$

$x = -1/3$ | $x = 1$ 3

check. ✓ ✓

e) $\dfrac{1}{x^2-1}$ $x^2-1=0$ ✓

$x = \pm 1$ ✓

$\therefore\ x \neq \pm 1$ Domain 2

f) $2x^2+5x-3=0$

i) $\alpha+\beta = \dfrac{-5}{2}$ ✓

$\alpha\beta = -\dfrac{3}{2}$ 1

ii) $\dfrac{1}{\alpha}+\dfrac{1}{\beta} = \dfrac{\alpha+\beta}{\alpha\beta}$ 1

$= -5/2 \,/ -3/2 = \dfrac{5}{3}$

iii) $\alpha^2+\beta^2$

$= (\alpha+\beta)^2 - 2\alpha\beta$ ✓

$= \dfrac{25}{4} - 2\left(-\dfrac{3}{2}\right)$ 2

$= 9\dfrac{1}{4}$ ✓

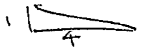
other side

Q2

a) i diagram

~~⇒~~ $3x+2y-12=0$

$R(4,0)$ ✓



$\sqrt{1+16}$

$= \sqrt{17}$ ✓ 2

ii) $Q(5,6)$ $\dfrac{6-1}{5-0} = \dfrac{5}{5} = 1$ ✓ 1

$P(0,1)$

iii) $y-1 = 1(x-0)$ ✓

$y-1 = x$ 2

$x-y+1=0$ ✓

iv) $2(x-y+1)=0 \Rightarrow 2x-2y+2=0$

$3x+2y-12=0 \Rightarrow 3x+2y-12=0$

$5x-10=0$

$y=3$ $x=2$ 3

method (1 mark)

v) $d = \dfrac{|(3)(0)+(2)(1)+(-12)|}{\sqrt{(3)^2+(2)^2}}$ ✓

$= \dfrac{|-10|}{\sqrt{13}} = \dfrac{10}{\sqrt{13}}$ units ✓ 3

vi) $m_{PQ} = 1$

$\tan\theta = 1$ $\therefore 45^\circ$ ✓

↑ know

viii) below the line ✓ 1

b) $A_1 = \dfrac{3}{3}[8+4(4)+6]$

$= 30$

$A_2 = \dfrac{3}{3}[6+4(5.7)+8]$

$= 36.8$

$\therefore A_1 + A_2 = 66.8\ m^2$ ✓ 3

Rule ✓
Application ✓

$A = \dfrac{3}{3}\left[8 + 2(6) + 4(9.7) + 8\right]$

HY HSC '10

Q3 $A(1,4),\ B(-3,5)$

a) $\sqrt{(x-1)^2+(y-4)^2} = 2\sqrt{(x--3)^2+(y-5)^2}$

$x^2-2x+1+y^2-8y+16$

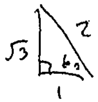
$= 4[x^2+6x+9+y^2-10y+25]$

$\Rightarrow 4x^2+24x+4y^2-40y+136$

$0 = 3x^2+26x+3y^2-32y+119$ ✓ 3

✓ shows $PA = 2PB$
✓ correct Expansion

b) i)



$\sin 60 = \dfrac{\sqrt{3}}{2}$

ii) $\sin 60 = \dfrac{30}{AC}$

$AC = \dfrac{30}{\sin 60}$ ← $\dfrac{\sqrt{3}}{2}$

$= \dfrac{60}{\sqrt{3}}$ cm 2

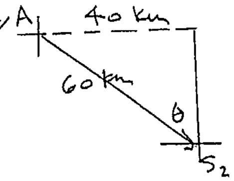
c) $\sin x = -\dfrac{1}{2}$

calc $x = -30$ ← steps ✓

But $x = 330^\circ$ ✓

$x = 210^\circ$ ✓ 3

d)



N θ W bearing

$\sin\theta = \dfrac{40}{60}$

$\theta = 41°49'$ 4

∴ 41° W ✓

diagram ✓ | bearing ✓
Ans ✓

e) $\dfrac{dN}{dt} > 0,\ \dfrac{d^2N}{dt^2} < 0$

SALES ARE INCREASING but slowing down with time

Q4

a) $\log_a 54$

$\log_a(2.3.3.3)$ ✓

$\log_a 2 + \log_a 3^3$

$\log_a 2 + 3(\log_a 3)$

$0.23 + 3(0.33)$ ✓

$= 1.22$ 3

Concept of

b) $y = \sqrt{x}+1$ $x=4$

$y=3$ ✓

$y' = \dfrac{1}{2}x^{-\frac{1}{2}}$

$= \dfrac{1}{2\sqrt{x}}$

$= \dfrac{1}{2\sqrt{4}} \Rightarrow \dfrac{1}{4}$ ✓

$y-3 = \dfrac{1}{4}(x-4)$

$4y-12 = x-4$ ✓

$x-4y+8=0$ 2

c) $\dfrac{dy}{dx} = 3x^2-4x+7$

$y = x^3-2x^2+7x+c$ ✓

$-1 = 1-2+7+c$ ✓

$-7 = c$

$y = x^3-2x^2+7x-7$ ✓ 3

d) $y = 2x^2 + ax^{-2}$ $x=3$ ✓

$y' = 4x - 2ax^{-3}$ ✓ $y'=0$

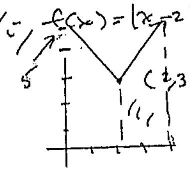
$0 = 12 - \dfrac{2a}{27}$

$\dfrac{2a}{27} = 12$ ✓

$a = 162$ 4

use

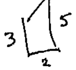
e) i) $f(x) = |x-2|+3$



1 mark corner
1 mark point (another)

ii) $\int_2^4 f(x)\,dx$

Area



2

$2\left(\dfrac{3+5}{2}\right)$

$= 8$

HSC HY 10

Q5

a/ i/ $x \ln x$

$x \frac{1}{x} + \ln x$

$1 + \ln x$ ✓ 1

ii/ $e^{\frac{1}{2}x}$

$\frac{1}{2} e^{\boxed{\frac{1}{2}x}} \frac{1}{2} x^{-\frac{1}{2}}$ ✓

$\frac{1}{4} \frac{e^{\sqrt{x}}}{\sqrt{x}} = \frac{e^{\sqrt{x}}}{4\sqrt{x}}$ ✓ 2

b/ i/ $\frac{1}{3} \int_0^1 e^{x^3} \boxed{3} x^2 \, dx$

$\frac{1}{3} e^{x^3} \Big|_0^1$ ✓

$= \frac{1}{3} \left[e^1 - e^0 \right]$ ✓ 2

$= \frac{1}{3} [e - 1]$

ii/ $\int_1^2 \frac{1}{x+1} \, dx$ ✓

$\ln (x+1) \Big|_1^2$ ✓ 2

$\ln (3) - \ln 2 = \ln \frac{3}{2}$

c/ i/ $33 = \frac{1}{4} 2\pi (x) + y + x$

$33 - \frac{1}{2}\pi x - x = y$

$y = 33 - x\left[\frac{\pi}{2} + 1\right]$ ✓ 1

ii/ $A = \frac{1}{4}\pi x^2 + xy$

$= \frac{1}{4}\pi x^2 + x\left[33 - \frac{\pi x}{2} - x\right]$

$= \frac{1}{4}\pi x^2 + 33x - \frac{\pi x^2}{2} - x^2$ 2

$= 33x - \frac{\pi}{4}x^2 - x^2$

$= 33x - \left(\frac{\pi}{4} + 1\right)x^2$

iii/ $A' = 33 - 2\left(\frac{\pi}{4} + 1\right)x$

$\left(\frac{2\pi}{4} + 2\right)x = 33$ ✓ 3

$\left(\frac{\pi}{2} + \frac{4\pi}{2}\right)x = 33$ ✓

$x = \frac{33}{5\pi} = \frac{66}{5\pi}$

Since ∴ max proved

Q6 a/ $\int_0^2 \left[x^2 + 4 - x^3\right] dx$ ✓

$= \frac{x^3}{3} + 4x - \frac{x^4}{4} \Big|_0^2$ ✓ 3

$= \left[\frac{8}{3} + 8 - \frac{16}{4}\right] - 0$ ✓

$= 6\frac{2}{3} \; u^2$

b/ $f(x) = \frac{1}{2}x^4 - 4x^2$ ∴ $f(x) = f(-x)$ 1

i/ $f(-x) = \frac{1}{2}(-x)^4 - 4(-x)^2$

$= \frac{1}{2}x^4 - 4x^2$

ii/ $0 = \frac{1}{2}x^4 - 4x^2$ 2

$0 = x^4 - 8x^2$

$0 = x^2(x^2 - 8) \longleftarrow x^2 = \pm\sqrt{8}$

intercepts

$x = 0, \quad x = \pm 2\sqrt{2}$

iii/ $f'(x) = 2x^3 - 8x$ ✓ 2

$f''(x) = 6x^2 - 8$ ✓

iv/ $f'(x) = 0 \; \Big| \; 0 = 2x(x^2 - 4)$

$x = -2$	$x = 0$	$x = 2$
$y'' = \text{pos}$	$y'' = \text{neg}$	$y'' = \text{pos}$
∪ min	∩ max	∪ min
$(-2, -8)$	$(0, 0)$	$(2, -8)$

3

v/ $f''(x) = 0 \; \Big| \; 0 = 6x^2 - 8$

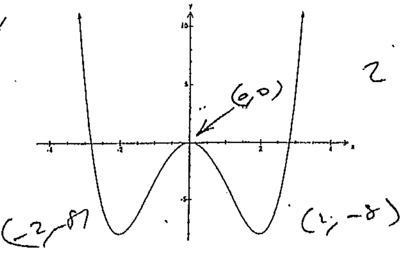
$x^2 = \frac{4}{3}$

$x = \pm \frac{2}{\sqrt{3}}$ ✓ 2

x	←	$\frac{2}{\sqrt{3}}$	→
y''	neg	0	pos

NB ✓
Even

vi/



2 2

½ Yearly 2010

a) $\sqrt{\frac{68.46}{5.858\ldots}} = \sqrt{11.6857\ldots}$ ✓

$= 3.42$ ✓ 2

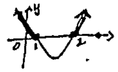
) $8x^3 - 27 = (2x)^3 - 3^3$

$= (2x-3)(4x^2+6x+9)$ ✓ 1

) $x^2 \geq 3x - 2$

$x^2 - 3x + 2 \geq 0$

$(x-2)(x-1) \geq 0$ ✓



✓ 3

$x \leq 1$ or $x \geq 2$ ✓

) $|3x - 1| = 2$

$3x - 1 = \pm 2$ ✓ 3

$3x - 1 = -2$ or $3x - 1 = 2$

$3x = -1 \qquad 3x = 3$

$x = -\frac{1}{3}$ ✓ or $x = 1$ ✓

check solutions.

) $\frac{1}{x^2 - 1}$ Domain

$x^2 - 1 \neq 0$ 2

$x \neq \pm 1$ ✓ ✓

Domain all real x except $\{x \neq -1 \text{ or } x \neq 1\}$.

e) Sales N are increasing but slow down with time ∴ $\frac{dN}{dt} > 0$

$\frac{d^2N}{dt^2} < 0$. 2

$a = 2, \; b = 5, \; c = -3$

i) $\alpha + \beta = -\frac{b}{a}$

$= -\frac{5}{2}$ ✓ 1

$\alpha\beta = \frac{c}{a} = -\frac{3}{2}$ ✓

ii) $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta + \alpha}{\alpha\beta}$

$= \frac{-5/2}{-3/2}$

$= \frac{5}{3}$ ✓ 1

iii) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$ ✓

$= \left(-\frac{5}{2}\right)^2 - 2 \times -\frac{3}{2}$

$= \frac{25}{4} + 3$

$= 9\frac{1}{4}$ ✓ 2.

a) $PA = 2PB$ ✓

$PB = \sqrt{(x--3)^2 + (y-5)^2}$

$= \sqrt{(x+3)^2 + (y-5)^2}$

$PA = \sqrt{(x-1)^2 + (y-4)^2}$ 3

$2\sqrt{(x+3)^2 + (y-5)^2} = \sqrt{(x-1)^2 + (y-4)^2}$ ✓

Square both sides

$4\left\{(x+3)^2 + (y-5)^2\right\} = (x-1)^2 + (y-4)^2$

$4\left[x^2 + 6x + 9 + y^2 - 10y + 25\right]$

$= \left[x^2 - 2x + 1 + y^2 - 8y + 16\right]$ ✓

$4x^2 + 24x + 4y^2 - 40y + 136$

$= x^2 - 2x + y^2 - 8y + 17$

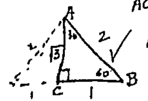
$3x^2 + 26x + 3y^2 - 32y + 119 = 0$

$\left(x + 1\frac{3}{3}\right)^2 + \left(y - \frac{16}{3}\right)^2 = \frac{68}{9}$ ✓

circle ctr $\left(-\frac{13}{3}, \frac{16}{3}\right)$ radius $\frac{\sqrt{68}}{3}$

b)

i) $\sin 60°$



$AC^2 = 2^2 - 1^2$

$= 3$

$AC = \sqrt{3}$ by pythagoras 1

$\sin 60 = \frac{\sqrt{3}}{2}$

ii) $\sin 60° = \frac{30}{AC}$ ✓ 2

$\frac{\sqrt{3}}{2} = \frac{30}{AC}$

AC = ~~60~~ ∴ AC = 20√3 cm ✓

$\sin x = -\frac{1}{2}$

x is in the 3rd & 4th quadrant

S	A
T	C

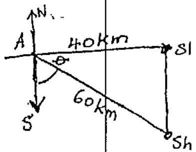
Acute angle $\sin x = \frac{1}{2}$ acute $x = 30°$ ✓

$x = 180 + 30$ and $x = $

$= 210°$

$x = 210°$ ✓, $330°$ ✓

d)



$\cos\theta = \frac{40}{60}$ ✓

$\theta = 48°11'$ ✓

Bearing

$90 + 48°11 = 1$

OR $S41°49'E$ ✓

5a

i) $\frac{d}{dx}(x\log x) = x \cdot \frac{1}{x} + \log x$

$= 1 + \log x$ ✓

ii) $e^{\sqrt{x}} = e^{x^{1/2}}$ ✓

$= e^{x^{1/2}} \cdot \frac{1}{2}x^{-1/2}$ ✓

$= \boxed{= \frac{e^{\sqrt{x}}}{2\sqrt{x}}}$

b) (i) $\int_0^1 x^2 e^{x^3} dx$

$\frac{d}{dx} e^{x^3} = 3x^2 e^{x^3}$.

∴ $\frac{1}{3}\int_0^1 3x^2 e^{x^3} dx = \left[\frac{1}{3} e^{x^3}\right]_0^1$ ✓

$= \frac{1}{3}\left\{e^1 - e^0\right\}$

$= \frac{1}{3}(e - 1)$ ✓

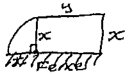
ii) $\int_1^2 \frac{1}{x+1} dx = \left[\ln(x+1)\right]_1^2$ ✓

$= \ln 3 - \ln 2$ ✓

$= \ln \frac{3}{2}$

c)

c)



i) Perimeter to be roped $= x + y + \frac{1}{4} \times 2\pi \times x$

$= x + y + \frac{\pi x}{2}$

33m of rope

∴ $x + y + \frac{\pi x}{2} = 33$

$y = 33 - \frac{\pi x}{2} - x$

$y = 33 - \left(\frac{\pi}{2} + 1\right)x$ ✓

ii) $A = xy + \frac{\pi \times x^2}{4}$ Area of circle $= \pi r^2$

$= x\left(33 - \left(\frac{\pi}{2} + 1\right)x\right) + \frac{\pi x^2}{4}$ ✓

$= 33x - \left(\frac{\pi}{2} + 1\right)x^2 + \frac{\pi x^2}{4}$ Show

$= 33x - \frac{\pi}{2}x^2 + \frac{\pi}{4}x^2 - x^2$

$= 33x - \frac{\pi}{4}x^2 - x^2$ ✓

$A = 33x - \left(\frac{\pi}{4} + 1\right)x^2$.

iii) $\frac{dA}{dx} = 33 - 2\left(\frac{\pi}{4} + 1\right)x$ ✓

$33 - 2\left(\frac{\pi}{4} + 1\right)x = 0$ ✓

$2\left(\frac{\pi}{4} + 1\right)x = 33$

$\left(\frac{\pi}{4} + 1\right)x = \frac{33}{2}$

d) $V = \pi \int_1^2 y^2 dx$

$= \pi \int_1^2 \frac{1}{x} dx$

$= \pi \left[\ln x\right]_1^2$

$= \pi\left\{\ln 2 - \ln 1\right\}$

$= \pi \ln 2$

$= 2.1776$

$A'' = -2(\pi$

< 0

∴ $x = $