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BRIGIDINE COLLEGE RANDWICK

MATHEMATICS

YEAR 12

HALF-YEARLY
2003

(TIME - 2 HOUR)

DIRECTIONS TO CANDIDATES

- *Put your **name** at the top of this paper and on **each of the 6 sections** to be collected.*
- *All 6 **questions** may be attempted.*
- *All 6 **questions** are to be answered on **separate pages** and will be collected in **separate bundles** at the end of this exam.*
- *All questions are of equal value.*
- *All necessary working should be shown in every question.*
- *Full marks may not be awarded for careless or badly arranged work.*

Question 1 (start a new page)

a. Completely factorise $8x^3 - 27$. 1 m

b. $\dfrac{\sqrt{41.6 + 39.5}}{0.52 + 321}$ 2 m

(correct to 3 significant figures)

c. Find $\dfrac{d(x^3 - 2x + \sqrt{x})}{dx}$ 2 m

d. A diamond ring appreciates in value by 1.5% per year. If the present value of the ring is \$2350, what would be the value of the ring in 12 years time? 2 m

e. Find the primitive function of $\dfrac{1}{(1-4x)^4}$ 2 m

f. Find the locus of all the points P (x,y) whose distance from A (1,4) is twice its distance from B (-3,5). 3 m

g. Write down the equation of the tangent to 3 m

the curve $y = \sqrt{x} + 1$ when $x = 4$.

Question 2 (start a new page)

a. Show that $\dfrac{2\sqrt{3}}{\sqrt{3} + \sqrt{2}}$ may be expressed in the form **2 m**

$a + b\sqrt{6}$ and state the values of a and b.

b. Solve the inequalities given by

i. $\left|\dfrac{3x - 2}{2}\right| < 4$ **3 m**

ii. $2x^2 + x \geq 6$ **3 m**

c. If α and β are the roots to the equation $2x^2 + 3x - 4 = 0$,

find the value of: **4 m**

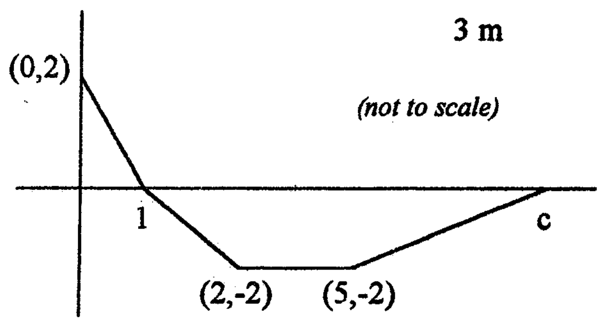
i. $\alpha + \beta$

ii. $\alpha^2\beta^2$

iii. $\alpha^2 + \beta^2$

d. This figure to the right is made up of a relation r (x) which traces out a triangle and a trapezium. **3 m**

The triangle has vertices (0,0), (1,0) and (0,2). The trapezium has vertices (1,0), (2,-2), (5,-2) and (c,0).



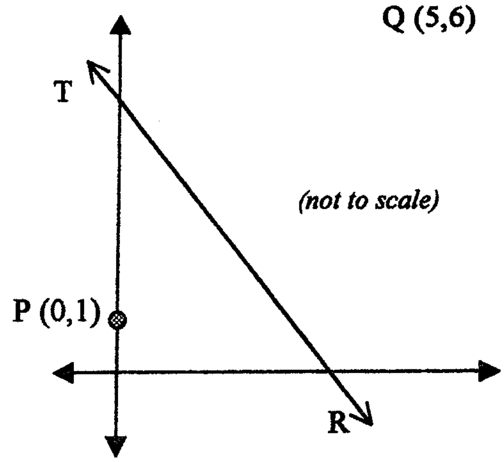
It is given that $\int_0^c r(x)\, dx = -8.5$, determine the value of c.

Question 3 (start a new page)

In the diagram to the right, P and Q have coordinates (0,1) and (5,6) respectively.

The line through T and R has equation $3x + 2y - 12 = 0$.

Copy the diagram onto your answer page.



a. Find the length of PR. 1 m

b. Show that the gradient of PQ is 1. 1 m

c. Show that the equation of the line through P and Q is $x - y + 1 = 0$ 2 m

d. Find the coordinates of the point at which the lines $x - y + 1 = 0$ and $3x + 2y - 12 = 0$ intersect. 2 m

e. Find the perpendicular distance of the point P from the line $3x + 2y - 12 = 0$. 2 m

f. Find the angle which the line PQ makes with the positive direction of the x – axis. 2 m

g. On your diagram shade the region satisfying the inequalities $3x + 2y - 12 \geq 0$ and $x - y + 1 \geq 0$. 2 m

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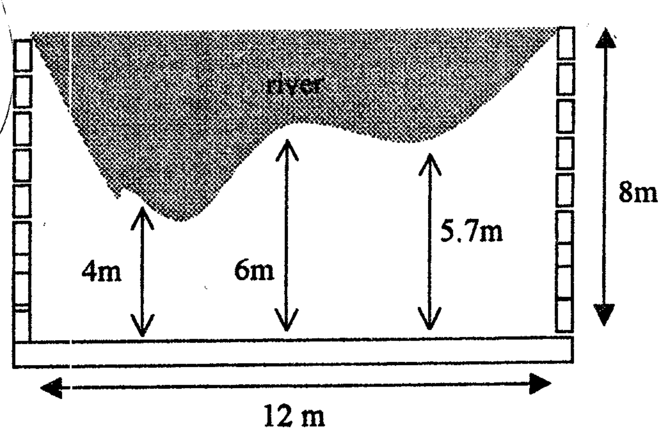
3 m

h.



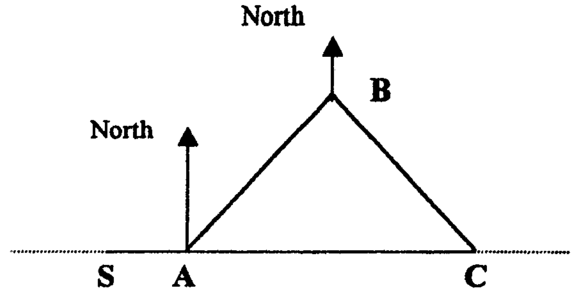
A market garden is bounded on one side by a river and the other 3 sides by stones.

Its dimensions are shown in this diagram to the left.

By using Simpson's Rule with two applications, determine the area of this market garden.

Question 4 (start a new page)

a. From the start **S**, Sharon rode 3 km due east to **A**. At **A**, she proceeded on a bearing of 055° for 10 km to **B**. At **B**, she changed course to a bearing of 130° and continued in this direction until she reached the finish at **C**. (**C** is due east of the start **S** and **A**)



i. Copy this diagram onto your answer sheet and display this information. 1 m

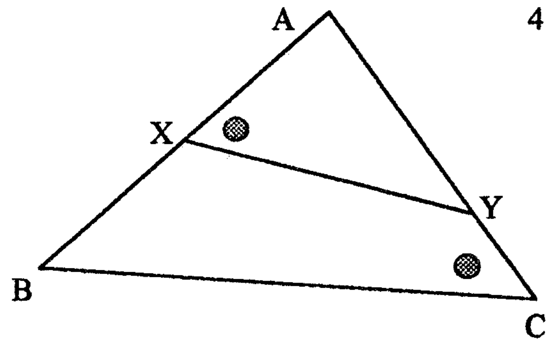
ii. Show that $\angle ACB = 40°$. 1 m

iii. Use the sine rule to find the distance from **B** to **C** (nearest km). 2 m

iv. It took Sharon 24 minutes to travel from the start to the finish. What was her average speed in km / hr? 2 m

b. In this figure to the right, $\angle AXY = \angle ACB$. 4 m

Redraw this figure onto your exam page.



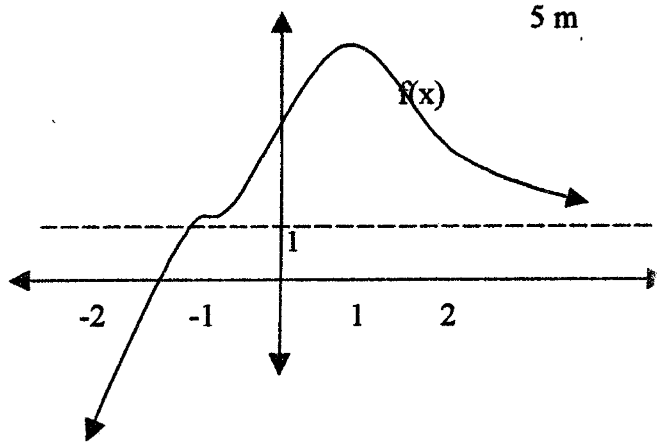
i. Prove that $\Delta AXY \ /// \ \Delta ACB$.

ii. Hence, or otherwise, show that

$AB \cdot AX = AC \cdot AY$

5 m

c. The following questions refer to the graph of f(x) to the right:



i. For what values of x is f(x) increasing?

ii. For what values of x is f(x) concave up?

iii. State what is happening to the curve when $x = -1$ and when $x = 1$.

iv. Find $\lim_{x \to \infty} f(x)$.

Question 5 (start a new page)

a. Consider the curve given by $f(x) = x(x - 3)^2$. 6 m

i. Determine its x and y intercepts.

ii. Show that there exists stationary values at $x = 1$ and $x = 3$ and determine their nature.

iii. Show the existence of a point of inflection.

iv. Draw a sketch of $f(x)$ in domain $-1 \leq x \leq 4$.

b. The table below lists the number of sales for a new product N over a period of time t (months). 2 m

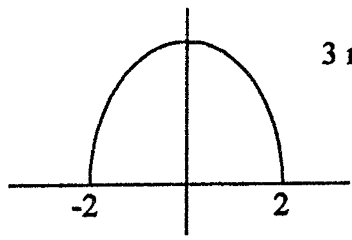
t	1	2	3	4	5	6
N	5000	7500	8750	9375	9687	9849

2500 1250 625

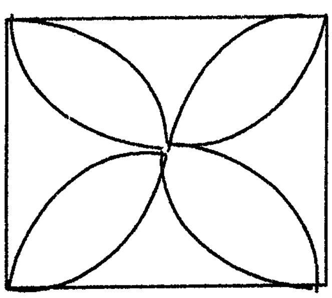
Comment on these sales in terms of $\frac{dN}{dt}$ and $\frac{d^2N}{dt^2}$ justifying your answer in words.

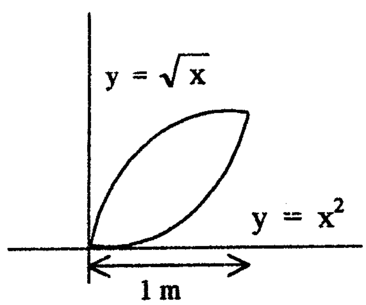
c. To the right is a sketch the curve $y = -x^2 + 4$ in the domain $-2 \leq x \leq 2$. 3 m

Determine the volume formed when the area bounded by this curve and the x axis is rotated about the x - axis.



d. The design for a tile is shown below and the upper section of the tile on a coordinate plane. It consists of a flower with four petals. The curves making up the upper right section petals are parabolas with equations $y = x^2$ and $y = \sqrt{x}$.





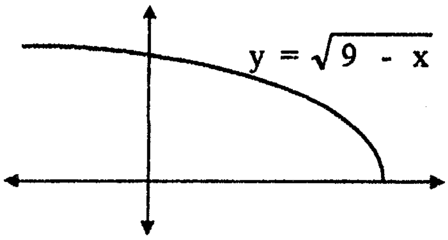
4 m

i. Determine the area bounded by the two curves $y = x^2$ and $y = \sqrt{x}$.

ii. If the flowers are to be painted yellow, what percentage of the tile will be yellow?

Question 6 (start a new page)

a. The area trapped between the curve $y = \sqrt{9 - x}$ and the coordinate axes is rotated about the y-axis, determine the resultant volume. **4 m**



b. i. Sketch these curves $y = x^3$ and $y = x^2 + 4$ on the same number plane. **5 m**

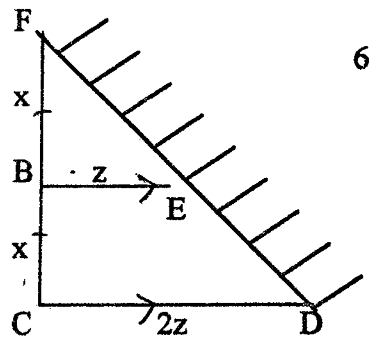
ii. Verify that the curves $y = x^3$ and $y = x^2 + 4$ intersect at the point (2, 8).

iii. Find the area of the region bounded by the curves $y = x^3$ and $y = x^2 + 4$ in the first quadrant.

c. A farmer wishing to keep her animals separate, sets up a field (right) so that fences will exist at FC, CD and BE. **6 m**

The side FD is a brick wall.

B is in the middle of FC and CD is parallel and twice the length of BE.



If FB = x metres and BE = z metres, write down expressions in terms of x and z for:

i. The area, A, of the field FCD.

ii. The amount of fencing, L, that the farmer would need.

iii. If the area of the field is 1200 m^2, show that the length of fencing required is given by

$$L = 2x + \frac{1800}{x} \text{ metres}$$

iv. Hence find the values of x and z so that the farmer uses the minimum amount of fencing.

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Q2 a) $\frac{2\sqrt{3}}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}$ ✓

$\frac{6-2\sqrt{6}}{3-2}$

$= 6-2\sqrt{6}$ ✓

$\therefore a=6,\ b=-2$ ✓

b) i) $\left|\frac{3x-2}{2}\right| < 4$

$-4 < \frac{3x-2}{2} < 4$ ✓

$-8 < 3x-2 < 8$ ✓

$-6 < 3x < 10$

$-2 < x < \frac{10}{3}$ ✓

ii) $2x^2+x \geq 6$

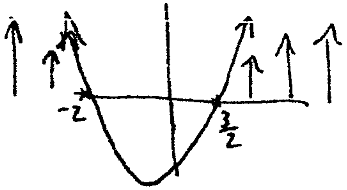
$2x^2+x-6 \geq 0$

$2x \quad -3$

$x \quad +2$

$(2x-3)(x+2) \geq 0$ ✓

$x=\frac{3}{2}$ $\quad x=-2$ ✓



$x \leq -2 \qquad x \geq \frac{3}{2}$ ✓

c) $2x^2+3x-4=0$

i) $\alpha+\beta = \frac{-3}{2}$ ✓

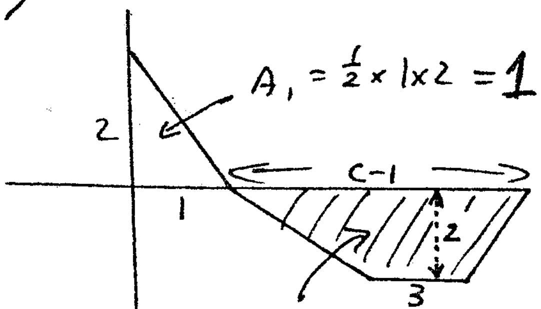
ii) $(\alpha\beta)^2 = \left(\frac{-4}{2}\right)^2 = (-2)^2 = 4$ ✓

iii) $\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$ ✓

$= \left(-\frac{3}{2}\right)^2 - 2\times -2$ ✓

$= 6\frac{1}{4}$ ✓

d)



$A_1 = \frac{1}{2}\times 1\times 2 = 1$ ✓

$A_2 = \frac{1}{2}\times h\times(a+b)$

$= \frac{1}{2}\times 2\times(3+c-1)$

$A_2 = 2+c$

but A_2 is below x-axis

$\therefore A_2$ is negative

$A_2 = -(2+c) = -2-c$ ✓

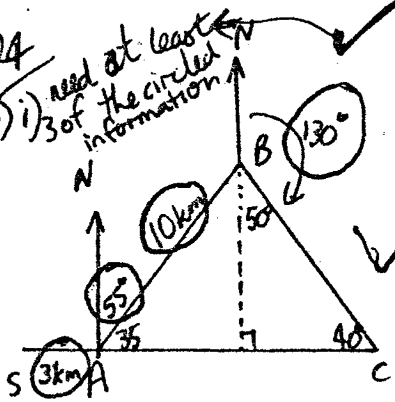
$\therefore A_2 + A_1 = \int_0^c r(x)\,dx = -8.5$

$-2-c+1 = -8.5$

$-1-c = -8.5$

$c = 7.5$ ✓

Q4 a) i) need at least 3 of the circled information ✓



ii) see diagram ✓

iii) $\frac{BC}{\sin 35} = \frac{10}{\sin 40}$ ✓

$BC = 8.9\text{km} \doteqdot 9\text{km}$ ✓

iv) $3+10+9\text{km} = 22\text{km}$ ✓

$s = \frac{d}{t} = \frac{22}{\left(\frac{24}{60}\right)}$ ✓ 55km/h

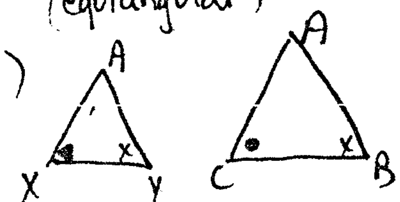
b) i) $\angle A$ common ✓

$\angle AXY = \angle ACB$ (given) ✓

$\therefore \Delta AXY \,|||\, \Delta ACB$

(equiangular)

ii)



$\frac{AB}{AC} = \frac{AY}{AX}$ ✓

$\therefore AB.AX = AC.AY$ ✓

c) i) $x<1,\ x\neq -1$ ✓

ii) ~~$x<-1$~~, ~~$0<x<2$~~

$-1<x<0,\ x>2$ ✓

iii) * horizontal point of inflexion at $x=-1$ ✓

* max at $x=1$ ✓

iv) $f(x)=1$ ✓

Q6 $y=\sqrt{9-x}$

when $x=0,\ y=3$ ✓

$\therefore V = \pi\int_0^3 x^2\,dy$ $\quad \begin{cases} y=\sqrt{9-x} \\ y^2 = 9-x \\ x = 9-y^2 \\ x^2 = (9-y^2)^2 \end{cases}$ ✓

$V = \pi\int_0^3 (9-y^2)^2\,dy$

$V = \pi\int_0^3 81-18y^2+y^4\,dy$ ✓

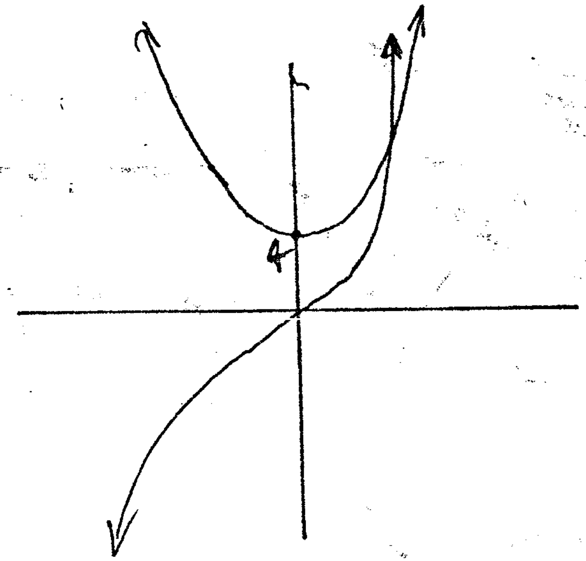
$= \pi\left[81y - 6y^3 + \frac{y^5}{5}\right]_0^3$

$= \pi\left[\left(81\times 3 - 6\times 3^3 + \frac{3^5}{5}\right) - (0)\right]$

$= 129.6\pi\ u^3$ ✓

$(407.15\,u^3)$

Q6 b. i)



ii) $y = x^3$ $\qquad$ $y = x^2 + 4$

sub (2,8) $\downarrow$

$8 = 2^3$ $\qquad$ $8 = 2^2 + 4$

$\therefore$ True intersect at (2, 8)

iii) $A = \int_0^2 x^2 + 4 - x^3 \, dx$

$A = \left[\frac{x^3}{3} + 4x - \frac{x^4}{4} \right]_0^2$

$= \left[\left(\frac{8}{3} + 8 - \frac{16}{4} \right) - (0) \right]$

$= \underline{6 \frac{2}{3}} \ u^2$

c) i) $A = \frac{1}{2} \times 2x \times 2z$

$A = 2xz$

ii) $L = 2x + 3z$

iii) $A = 2xz = 1200$

$z = \frac{1200}{2x} = \frac{600}{x}$ sub in part(ii)

$L = 2x + 3 \times \frac{600}{x}$

$L = 2x + \frac{1800}{x}$

iv) $L = 2x + 1800x^{-1}$

$L' = 2 - 1800x^{-2}$

$L'' = 3600x^{-3}$

st pt at $L' = 0$

$2 - 1800x^{-2} = 0$

$\frac{1800}{x^2} = 2$

$1800 = 2x^2$

$x^2 = 900$

$x = 30$

Nature: $L'' = \frac{3600}{x^3} \Rightarrow \frac{3600}{30^3} > 0$

$\therefore$ Min length when $\underline{x = 30}$

$z = \frac{600}{30} = 20$

Centre Number: .... Student Number: ....

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Q1(a) $8x^3-27=(2x)^3-3^3=(2x-3)(4x^2+6x+9)$ —— (1)

(b) $\sqrt{\dfrac{41.6+39.5}{0.52+321}} = 0.028009\ldots$ (1) ~ 0.0280 —— (2)

(c) $\dfrac{d}{dx}(x^3-2x+\sqrt{x}) = 3x^2-2+\frac{1}{2}x^{-1/2}$ or $3x^2-2+\dfrac{1}{2\sqrt{x}}$ —— (2)

(d) $A=2350(1.015)^{12}=\$2809.70$ —— (2)
(allow max 1 for evidence of concept)

(e) $\displaystyle\int\frac{dx}{(1-4x)^4} = \int(1-4x)^{-4}dx = \frac{1}{12}(1-4x)^{-3}+C$ —— (2)
(deduct 1 for error)

(f) $PA=2PB$ ie. $\sqrt{(x-1)^2+(y-4)^2}=2\sqrt{(x+3)^2+(y-5)^2}$ — (1)

$x^2-2x+1+y^2-8y+16=4[x^2+6x+9+y^2-10y+25]$ (1)

ie. $3x^2+3y^2+26x-32y+119=0$ —— (3)

(g) $y=\sqrt{x}+1$ $\quad y'=\frac{1}{2}x^{-1/2}$

$y'(4)=1/4$ (1)

When $x=4$, $y=3$ (1)

Tangent: $y-3=\frac{1}{4}(x-4)$ (1)

ie $y=\frac{1}{4}x+2$ OR $x-4y+8=0$ —— (3)

Centre Number: .... Student Number: ....

Q3 a) $PR=\sqrt{(4-0)^2+(0-1)^2}=\sqrt{17}$ —— (1)

b) $m(PQ)=\dfrac{6-1}{5-0}=1$ —— (1)

c) $y-1=1(x-0)$ ie $y=x+1$ —— (2)
OR $y-6=1(x-5)$ etc
OR Sub $Q(5,6)$ AND $P(0,1)$ into $x-y+1=0$

d) $x-y+1=0$ —(1)

$3x+2y-12=0$ —(2)

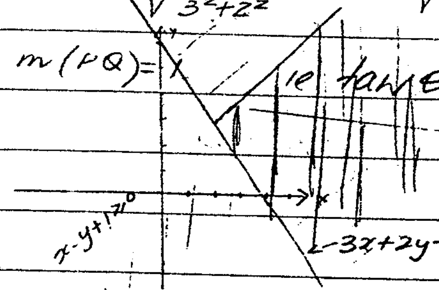
Sub. (1) $y=x+1$ into (2):
$3x+2(x+1)-12=0$
ie. $x=2$
$y=3$

OR $3\times(1)$: $3x-3y+3=0$
(2): $3x+2y-12=0$
$-5y+15=0$
ie $y=3$
$x=2$ —— (2)

e) $d=\dfrac{|3(0)+2(1)-12|}{\sqrt{3^2+2^2}}=\dfrac{10}{\sqrt{13}}$ or $\dfrac{10\sqrt{13}}{13}$ units —— (2)

(f) $m(PQ)=1$ ie $\tan\theta=1$, $\theta=45^\circ$ —— (2)

(g) —— (2)



(h) Approx area $= 1(8+4\times4+2\times6+4\times5.7+8)$

OR $1(8+4\times4+6)+1(6+4\times5.7+8)$

ie. $66.8\ m^2$ —— (3)

Single correct use of rule – max 2

Centre Number: .... Student Number: ....

Q5 a) $f(x)=x(x-3)^2$ (i) $x=0, y=0$; ie $(0,0)$

when $f(x)=0$, $x=0,3$

ie $(0,0), (3,0)$ — (2)

(ii) $f'(x)=1(x-3)^2+2x(x-3)$ — 1

$=(x-3)(x-3+2x)$ [OR $3x^2-12x+9$]

$=3(x-3)(x-1)$

when $f'(x)=0$

$x=1,3$ — 1

$y=4,0$

x	0	1	2
$f'(x)$	9	0	-3

ie local max at $x=1$

OR $f''(x)=3(x-1)+3(x-3)$

$=6x-12$

$f''(1)=-6<0$

ie concave down

∴ max at $x=1$.

$f''(3)=6>0$

∴ local min } — 1

x	2	3	4
$f'(x)$	-3	0	9

ie local min at $x=3$ (3)

(iii) at an inflexion $f''(x)=0$. ✓

ie $6x-12=0$.

∴ $x=2$, $y=2$

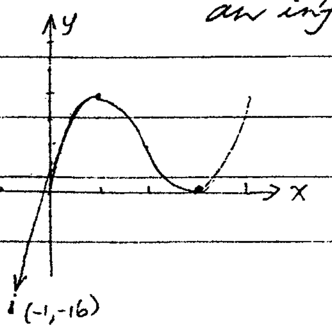
Here concavity changes since

x	1.5	2	2.5
$f''(x)$	-3	0	3

∴ inflexion (or since from above $x=2$ is NOT a st. pt then it must be an inflexion)

[Must indicate that $f''(x)=0$ is necc. but NOT sufficient] (1)

(iv)



✓ (correct from non-trivial above) (1)

(b) For $1\le t\le 6$ (2)

N is increasing ∴ $\frac{dN}{dt}>0$ — 1

$\frac{d^2N}{dt^2}=\frac{d\left(\frac{dN}{dt}\right)}{dt}$

but

N is increasing at a decreasing rate

ie. t	2	3	4	5	6
increase in N	2500	1250	625	312	162

ie $\frac{d^2N}{dt^2}<0$ — 1

(c) $V=\pi\int y^2dx$

$=\pi\int_{-2}^{2}(4-x^2)^2dx$ OR $2\pi\int_0^2(4-x^2)^2dx$ — 1

$=\pi\int_{-2}^{2}(16-8x^2+x^4)\,dx$

$=\pi\left[16x-\frac{8}{3}x^3+\frac{x^5}{5}\right]_{-2}^{2}$ — 1

$=\pi\left[\left(32-\frac{64}{3}+\frac{32}{5}\right)-\left(-32+\frac{64}{3}-\frac{32}{5}\right)\right]$

$=34\frac{2}{15}\pi$ OR $34.13\pi\ u^3$ — 1 (3)

$(107.2\ u^3)$

(d)(i) $A_{petal}=\left|\int_0^1(\sqrt{x}-x^2)\,dx\right|$ — 1 (3)

$=\left(\frac{2}{3}x^{3/2}-\frac{x^3}{3}\right)_0^1$

$=\frac{1}{3}\ u^2$ — 1

(ii) % of tile yellow $=\frac{4\times\frac{1}{3}}{2\times2}\times\frac{100}{1}$

$=33\frac{1}{3}\%$ — 1