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BRIGIDINE COLLEGE RANDWICK

Ave.
62%

MATHEMATICS

YEAR 12

56·1 / 90.

HALF YEARLY

2004

(TIME - 2 HOUR)

Directions to candidates

* *Put your* ***name at the top*** *of this paper and on each of the* ***6 sections*** *that are to be collected.*
* *All 6 questions are to be attempted.*
* *All 6 questions are of equal value.*
* *All questions are to be answered on separate pages and will be collected in separate bundles at the end of this exam.*
* *All necessary working should be shown in every question.*
* *Full marks may not be awarded for careless or badly arranged work.*

Question 1 *(Start a new page)*

a. Determine the value of $\dfrac{32.5 - 24.1^4}{6.1 \times 4.3}$ correct to 3 significant figures. 2

b. Show that $0.\overline{9}$ (i.e. 0.99999...) is equivalent to the number 1. 2

c. Solve the inequation: $2x^2 - x \geq 10$ 3

d. Determine $\lim_{x \to 3} \dfrac{x^2 - 9}{x - 3}$ 2

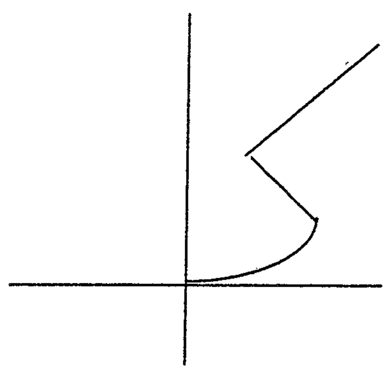
e. Show that $\dfrac{\sqrt{32} - \sqrt{8}}{\sqrt{2}}$ represents a Rational Number and state its value. 2

f. Completely factorise $x^3 - 27$ 1

g. Copy this figure to the right onto your exam page.

i. Using a dotted line, complete f(x) if f(x) is an even function. 1

ii. Using a solid line, complete f(x) if f(x) is a n odd function. 2



Question 2 *(Start a new page)*

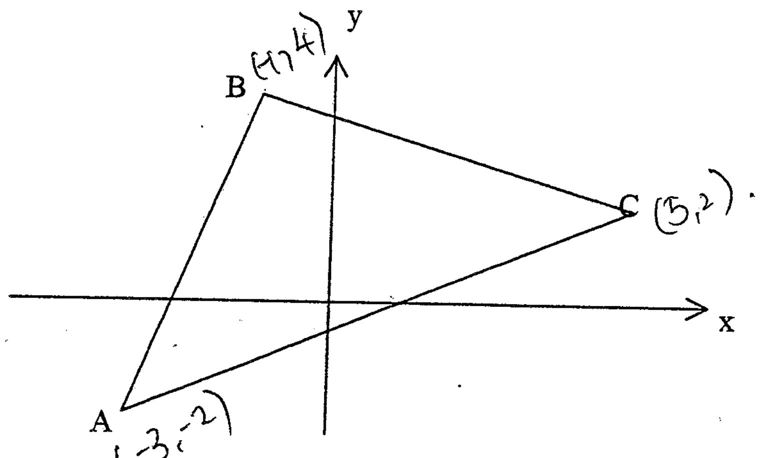
a. Differentiate the following, leaving answers simplified and with positive indices.

i. $\sqrt{x+2}$ 1

ii. $(5x^3 - 2)^3$ 2

b. Solve the following $|3x - 2| = 4x - 7$ 3

c. The diagram below shows points A(-3,-2) , B(-1,4) , C(5,2)
Copy this diagram onto your exam page.



i. Find the gradient of AC. 1

ii. P is the midpoint of AC. Show that the coordinate of P is (1,0). Mark P on the diagram above. 1

iii Show that the equation of the line perpendicular to AC and passing through P is $2x + y - 2 = 0$ 2

iv. Show that B lies on the line $2x + y - 2 = 0$ 1

v. Show that the length of BP is $2\sqrt{5}$ 2

vi. Find the area of Δ ABC . 2

Question 3 *(Start a new page)*

a. If α and β are the roots to the equation $x^2 - 2x - 5 = 0$.
Find the value of

i. $\alpha + \beta$ and $\alpha\beta$ 1

ii. $\dfrac{1}{\alpha} + \dfrac{1}{\beta}$ 1

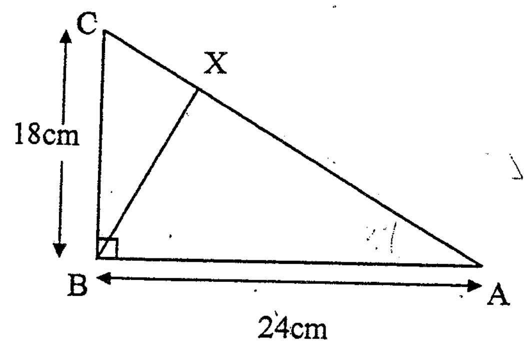
iii. $\alpha^2 + \beta^2$ 2

b. Solve for α where $0 \le \alpha \le 360°$, if $3 \sec \alpha = 4 \cos \alpha$. 4

c. In this figure to the right,
$\angle ABC = 90°$, AB = 24cm,
BC = 18cm.

i. Find $\angle CAB$ (nearest degree) 1

ii. The point X is on AC such that BX is perpendicular to AC.
Find the lengths of AX and XC.
(nearest tenth) 3



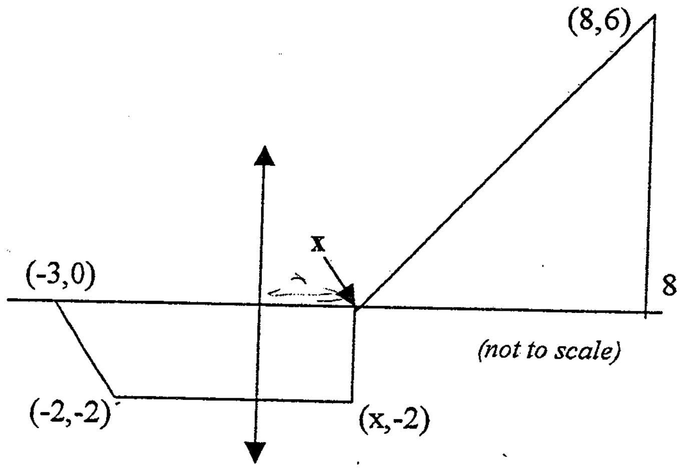
d. 3

Consider this diagram to the right.

The trapezium and triangle are traced out by the curve f(x).

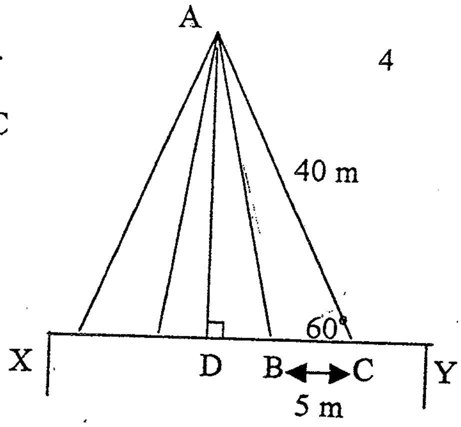
If $\int_{-3}^{8} f(x)\, dx = 0$,

determine the value of **x**,
leaving answer in exact form.



Question 4 *(Start a new page)*

a. A horizontal bridge was built between point X and Y. Cables were used to support the bridge as shown to the right. The distance between the cables AB and AC was 5 metres. Cable AC was 40 metres long and $\angle ACB = 60°$. 4



i. Show that the height of A above the horizontal bridge is $20\sqrt{3}$ metres.

ii. Determine the exact length of cable AB.

b. Consider the curve given by $y = 3x^2 - x^3$.

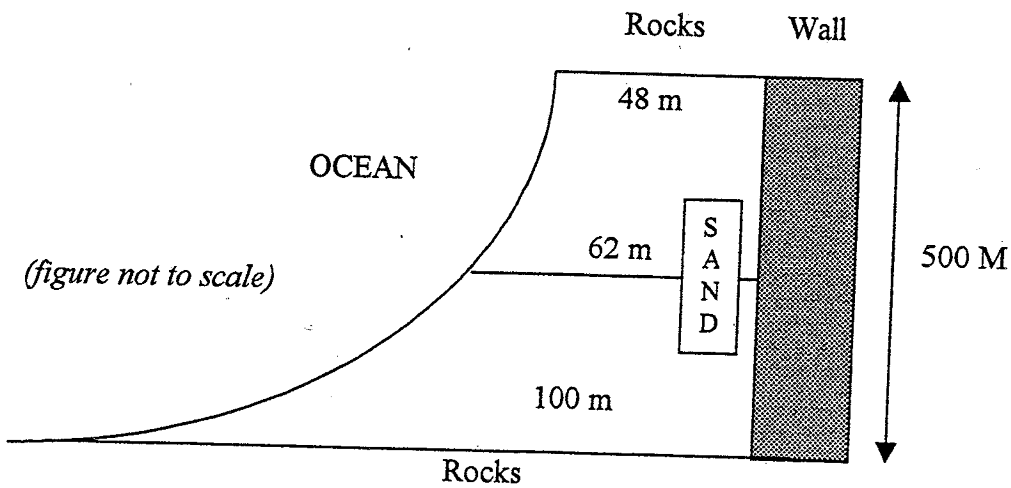
i. Show that when $x = 0$ and $x = 2$ there are two stationary values and determine their nature. 3

ii. Determine any possible points of inflection. 1

iii. Sketch the curve, showing the above features and indicating where it crosses the x axis. 2

iv. Find the equation of the tangent to the curve at the point $(-1,4)$. 2

c. A local council wanted to calculate the area of sand on one of its beaches. The beach was surveyed and the diagram below shows a notepad diagram of the beach bounded by a wall, two sets of rocks and the ocean. Measurements were taken at either end of the beach and from the middle of the wall. 3



By using, *The Trapezoidal Rule* and three function values, calculate the approximate area of sand.

Question 5 *(Start a new page)*

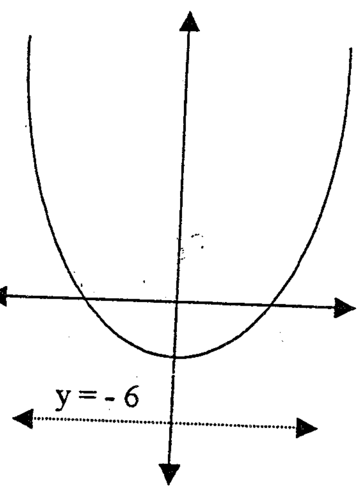
a. The locus of a point P moves in a plane so that its distance from a point A (2,3) is twice its distance from a point B (-1,-3). 3

b. In a nearby room, Caroline notices that water is dripping from a tap into a 42 cm high bucket. After 1 minute the depth of the water is 2 mm, after 2 minutes the depth is 6 mm, after 3 minutes it is 12 mm, after 4 minutes 20 mm and so on.

i. Show that this represents an Arithmetic progression and determine the increase in depth in the 10^{th} minute? 2

ii. When will this bucket be full? 2

c. This parabola to the right has as its directrix $y = -6$ and its vertex at $(0, -2)$.



i. Determine the focal length of this parabola. 1

ii. Show that the equation of this parabola may be given by $x^2 = 8y + 16$ 1

iii. Determine the x intercepts of this curve. 1

iv. Determine the area bounded by this parabola ($x^2 = 8y + 16$) and the x axis. 3

v. A mold for a vase is designed by the rotation of this parabola ($x^2 = 8y + 16$) between the lines $y = 0$ and $y = 8$ about the y-axis. 2

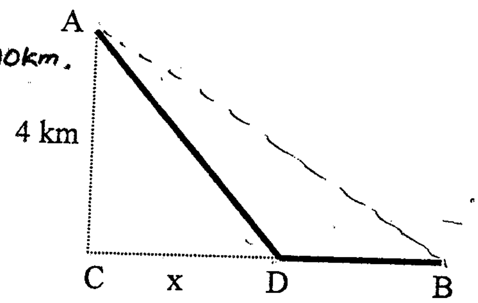
Determine the volume of this vase.

Question 6 *(Start a new page)*

a. i. Neatly sketch the curve given by $y = \dfrac{1}{x - 2} + 1$ identifying all necessary features that assisted your sketch. 2

ii. The area bounded by this curve and the lines $x = 3$ and $x = 5$ is to be rotated about the x axis. Establish a suitable Integral that would calculate this volume. **(Do Not Solve)** 1

b A bushwalker is going from A to B, along the trails AD and DB. She can travel at 6 km / hr when walking AD and 10 km / hr when walking from D to B. The distance of AC is 4 km, and CB is 10km.



If CD = x km, show that the time 'T' taken to travel from a A to B, by this route is given by

i. $T = \dfrac{\sqrt{x^2 + 16}}{6} + \dfrac{10 - x}{10}$ 1

ii. Determine the shortest time to go from A to B in this way. 5

c. i. On the same number plane, sketch the graphs of 2

$$y = |x - 3| + 2 \quad \text{and} \quad y = -x^2 + 8x - 15$$

ii. Determine the area bounded between these two curves and the lines $x = 3$ and $x = 5$. 4

- end of exam -

mathematics yr 12 2 yrly

Q1 a) -12900

$-12859.61\ldots$ (1 mark)

b) $x = 0.999\ldots\ldots$ ①

$10x = 9.999\ldots\ldots$ ②

$\cancel{100x = 99.999}\ldots\ldots$ ③

③ − ②

$9x = 9$

$\underline{x = 1}$

c) $2x^2 - x \geq 10$

$2x^2 - x - 10 \geq 0$

$(2x-5)(x+2) \geq 0$

$x = 2\frac{1}{2}$ $x = -2$

$x \leq -2$ $x \geq 2\frac{1}{2}$

d) $\lim_{x\to 3} \frac{x^2-9}{x-3}$

$\lim_{x\to 3} \frac{(x-3)(x+3)}{x-3}$

$\lim_{x\to 3} x+3 = 3+3 = \underline{6}$

e) $\frac{\sqrt{32}-\sqrt{8}}{\sqrt{2}}$

$= \frac{\sqrt{16}\times\sqrt{2} - \sqrt{4}\times\sqrt{2}}{\sqrt{2}}$

$= \frac{4\sqrt{2} - 2\sqrt{2}}{\sqrt{2}}$

$= \frac{2\sqrt{2}}{\sqrt{2}}$

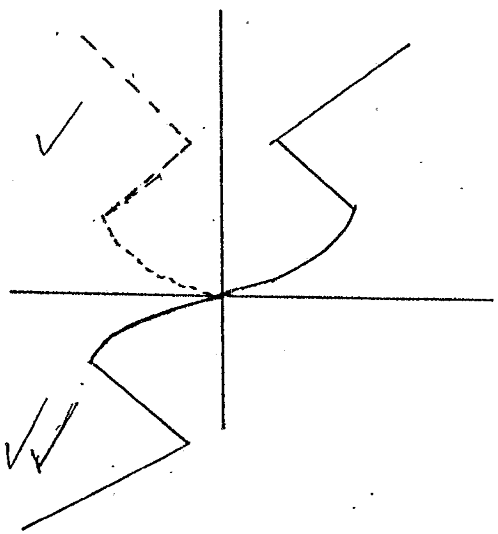
$= 2$

f) $x^3 - 27$

$x^3 - 3^3$

$(x-3)(x^2+3x+9)$

g)



Q2 i) $\frac{d}{dx}(x+2)^{\frac{1}{2}}$

$= \frac{1}{2}(x+2)^{-\frac{1}{2}}.1$

$= \frac{1}{2\sqrt{x+2}}$

ii) $\frac{d}{dx}(5x^3-2)^3$

$= 3(5x^3-2)^2 . 15x^2$

$= 45x^2(5x^3-2)^2$

b) $|3x-2| = 4x-7$

$3x-2 = 4x-7$ or $-(3x-2) = 4x-7$

$x = 5$ $-3x+2 = 4x-7$

$9 = 7x$

$x = \frac{9}{7}$

Test $x = 5 \to |3\times5-2| = 4\times5-7$

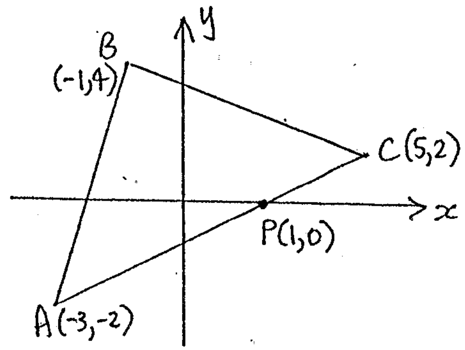
$13 = 13$ TRUE

$x = \frac{9}{7} \to |3\times\frac{9}{7}-2| = 4\times\frac{9}{7}-7$

$1\frac{6}{7} = -1\frac{6}{7}$ ✗ REJECT

Ans = $\underline{x=5}$

c)



i) $m = \frac{2--2}{5--3} = \frac{1}{2}$

ii) $M = \left(\frac{5-3}{2}, \frac{2-2}{2}\right)$

$M = (1, 0)$

iii) m of ⊥ $= -2$

pt$(1,0)$

$y - 0 = -2(x-1)$

$y = -2x+2$

$y + 2x - 2 = 0$

iv) $B(-1,4)$

$4 + 2\times -1 - 2 = 0$

$4 - 2 - 2 = 0$

∴ TRUE

v) $BP = \sqrt{(1+1)^2 + (0-4)^2}$

$= \sqrt{4+16}$

$= \sqrt{20}$

vi) $AC = \sqrt{(5+3)^2 + (2+2)^2}$

$= \sqrt{64+16}$

$= \sqrt{80}$

∴ Area $= \frac{1}{2}\times\sqrt{80}\times 2\sqrt{5}$

$= 20$

3 a) i) $\alpha + \beta = 2$

$\alpha\beta = -5$

ii) $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = -\frac{2}{5}$

iii) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= (2)^2 - 2\times -5$

$= 14$

b) $3\sec\alpha = 4\cos\alpha$

$\frac{3}{\cos\alpha} = 4\cos\alpha$

$3 = 4\cos^2\alpha$

$\cos^2\alpha = \frac{3}{4}$

$\cos\alpha = \pm\frac{\sqrt{3}}{2}$

$\alpha = 30°, 150°, 210°, 330°$

c) i) $\tan\theta = \frac{18}{24}$

$\theta = 36°52' = 37°$

ii) $\cos 37° = \frac{AX}{24}$

$AX = 19.2$ cm

$AC^2 = 24^2 + 18^2$

$AC^2 = 900$

$AC = 30$

∴ $XC = 30 - 19.2 = 10.8$

equal area of trapezium

$\Delta = \frac{1}{2}x(8-x)\times 6$

Trapezium $= \frac{1}{2}\times 2[(x+2)+(x+3)]$

$\frac{1}{2}\times(8-x)\times 6 = \frac{1}{2}\times 2[(x+2)+(x+3)]$

$3(8-x) = 2x+5$

$24-3x = 2x+5$

$19 = 5x$

$x = \frac{19}{5} = 3\frac{4}{5}$

Q4

a) i) $\sin 60 = \frac{AD}{40}$

$AD = 40\sin 60$

$= 40\times\frac{\sqrt{3}}{2}$

$= 20\sqrt{3}$

ii) $40^2 = (20\sqrt{3})^2 + DC^2$

$DC = 20$

$\therefore DB = 20-5 = 15$

$\therefore AB^2 = (20\sqrt{3})^2 + (15)^2$

$AB = \sqrt{1425}$

$AB = \sqrt{25}\times\sqrt{57}$

$AB = 5\sqrt{57}$

b) i) $y = 3x^2 - x^3$

$y' = 6x - 3x^2$

$y'' = 6 - 6x$

$6x - 3x^2 = 0$

$3x(2-x) = 0$

$x=0$, $y=0$ $x=2$, $y=4$

$y''>0$ $\therefore$ MIN $y''<0$ $\therefore$ MAX

ii) $y''=0$

$6-6x=0$

$x=1$

$y=2$

Test:

x	←	1	→
y''	+	0	−

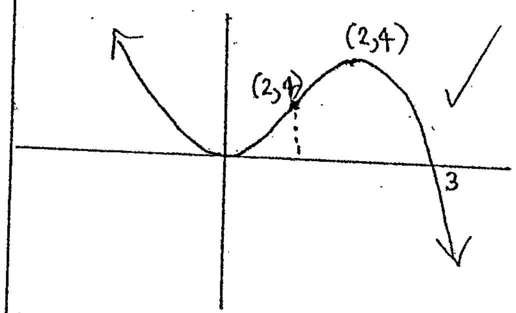
$\therefore$ True $(1,2)$ is a pt of inflexion

iii) cuts x-axis when $y=0$

$0 = 3x^2 - x^3$

$0 = x^2(3-x)$

$x=0$ $x=3$



iv) $y' = 6x-3x^2$ at $(-1,4)$

$m = 6\times -1 - 3\times(-1)^2$

$m = -9$

$y-4 = -9(x+1)$

$y = -9x-5$

c) $h = \frac{500}{2} = 250$

$A \doteqdot \frac{250}{2}[48 + 2\times 62 + 100]$

$\doteqdot 34000$

Q5

a) $A(2,3)$ $P(x,y)$ $B(-1,-3)$

$PA = 2PB$

$\sqrt{(x-2)^2+(y-3)^2} = 2\sqrt{(x+1)^2+(y+3)^2}$

$x^2-4x+4+y^2-6y+9 = 4[x^2+2x+1+y^2+6y+9]$

$x^2+y^2-4x-6y+13 = 4[x^2+y^2+2x+6y+10]$

$x^2+y^2-4x-6y+13 = 4x^2+4y^2+8x+24y+40$

$0 = 3x^2+3y^2+12x+30y+27$

$0 = x^2+y^2+4x+10y+9$

b) i) 1min   2min   3   4 ....

2 deep   6 deep   12 deep   20 deep

$T_1=2$ $T_2=4$ $T_3=6$ $T_4=8$

increase in depth.

$\therefore 4-2 = 6-4$ $\therefore$ AP.

$T_{10} = 2+(10-1)\times 2 = 20$

ii) 4.2cm → 420mm $= S_n$

$n=?$

$a=2$

$d=2$

$S_n = \frac{n}{2}(2a+(n-1)d)$

$420 = \frac{n}{2}(4+(n-1)2)$

$840 = n(4+2n-2)$

$840 = n(2+2n)$

$0 = 2n^2+2n-840$

$0 = n^2+n-420$

$(n+21)(n-20)=0$

$n=-21$ reject $n=20$ accept.

c) i) $a=4$ -2 ↕ -6

ii) $\cancel{(x+2)^2 = 4\times 4(y-0)}$

ii) $(x-0)^2 = 4\times 4(y+2)$

$x^2 = 16y+32$

Note: The mark for this question was given here

Not $x^2 = 8y+16$

iii) $x^2 = 16y+32$ OR $\therefore x^2 = 8y+16$

when $y=0$:

$x^2 = 32$ $x^2 = 16$

$x = \pm\sqrt{32}$ $x = \pm 4$

iv) If $x^2 = 16y+32$ If $x^2 = 8y+16$

$y = \frac{x^2}{16} - 2$ $y = \frac{x^2}{8} - 2$

$A = \left|2\int_0^{\sqrt{32}} \frac{x^2}{16} - 2\right|$ $A = \left|2\int_0^{4} \frac{x^2}{8} - 2\right|$

$= 2\left[\frac{x^3}{48} - 2x\right]_0^{\sqrt{32}}$ OR $2\left[\frac{x^3}{24} - 2x\right]_0^{4}$

$= 2\times\left[\frac{32\sqrt{32}}{48} - 2\sqrt{32}\right]$ OR $2\left[\frac{64}{24} - 8\right]$

$= |-15.08\ldots|$ $= |-10\frac{2}{3}|$

$= 15.08\ u^2$ $= 10\frac{2}{3}\ u^2$

v) $V = \pi\int_0^8 16y+32$ OR $V = \pi\int_0^8 8y+16$

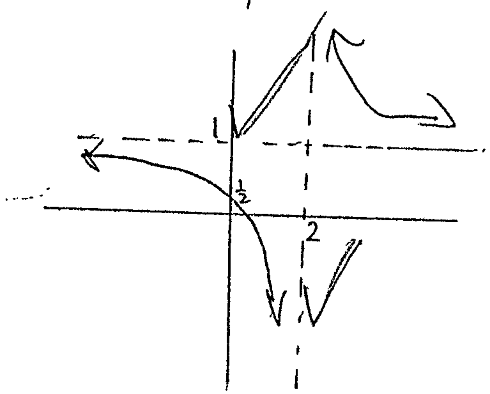
$V = \pi[8y^2+32y]_0^8$ $V = \pi[4y^2+16y]_0^8$

$= 256\pi$ $= 384\pi$

Q6

a) i) $y=\frac{1}{x-2}+1$

$x \neq 2$



cuts y-axis when $x=0$

$y=\frac{1}{2}$

ii) $V=\pi\int_3^5 y^2\,dx$

$y=\frac{1}{x-2}+1$

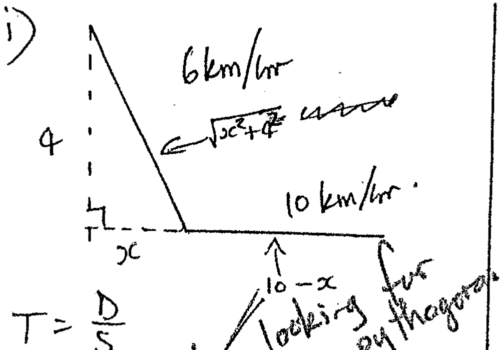
$y=\frac{1+x-2}{x-2}$

$y=\frac{x-1}{x-2}$

$V=\pi\int_3^5\left(\frac{x-1}{x-2}\right)^2dx$

or $V=\pi\int_3^5\left(\frac{1}{x-2}+1\right)^2dx$

b) i)



$\therefore T=\frac{D}{S}$

$=\frac{\sqrt{x^2+16}}{6}+\frac{10-x}{10}$

ii) $\cancel{\frac{dT}{dx}=\frac{1}{6}\times\frac{1}{2}(x^2+4)^{-\frac{1}{2}}+\frac{1}{10}}$

ii) $\frac{dT}{dx}=\frac{1}{6}\times\frac{1}{2}(x^2+16)^{-\frac{1}{2}}\times 2x-\frac{1}{10}$

$\frac{dT}{dx}=\frac{x}{6\sqrt{x^2+16}}-\frac{1}{10}$

$\frac{dT}{dx}=0$

$\frac{x}{6\sqrt{x^2+16}}-\frac{1}{10}=0$

$\frac{x}{6\sqrt{x^2+16}}=\frac{1}{10}$

$10x=6\sqrt{x^2+16}$

$\frac{10x}{6}=\sqrt{x^2+16}$

$\frac{100x^2}{36}=x^2+16$

$100x^2=36x^2+576$

$64x^2=576$

$x^2=9$

$x=\pm 3$

$x=3$

x	$\leftarrow$	3	$\rightarrow$
y'	$-$	0	$+$

$\therefore$ MIN when $x=3$

$\therefore T=\frac{\sqrt{(3)^2+16}}{6}+\frac{10-3}{10}$

$\cancel{=1\frac{4}{15}\text{ hrs or }1.26\text{ hrs}}$

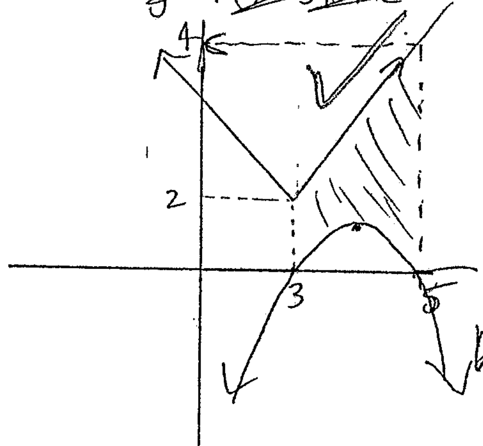
$\cancel{=1\text{ hr }16\text{ mins}}$

$\cancel{=1\frac{2}{15}\text{ hrs or } 8\text{ mins}}$

$=1\frac{8}{15}$ hrs

$=1$ hr 32 mins

c) i) $\cancel{y=|x-3|+2}$



$y=-x^2+8x-15$

$0=-x^2+8x-15$

$x^2-8x+15=0$

$(x-3)(x+5)=0$

$x=3,5$

$x=\frac{-8}{2\times -1}=4$

$y=1$

$(4,1)$

ii) Trapezium:

$A=\frac{1}{2}\times 2\times(2+4)=6$

$A=\int_3^5 -x^2+8x-15\,dx$

$=\left[-\frac{x^3}{3}+4x^2-15x\right]_3^5$

$=\left(\frac{-5^3}{3}+4\times 25-15\times 5\right)-\left(-\frac{3^3}{3}+4\times 9-15\times 3\right)$

$=-16\frac{2}{3}--18$

$=1\frac{1}{3}\ u^2$

$\therefore A=6-1\frac{1}{3}=4\frac{2}{3}$