7/90 (7′) 79%

a solid result Ashleigh concentrate on this.

Student Ashleigh Sparrow

Teacher Mr Eames

BRIGIDINE COLLEGE RANDWICK

MATHEMATICS

HSC

HALF YEARLY

2005

(TIME - 2 HOUR)

Directions to candidates

* *Put your **name at the top** of this paper and on each of the **6 sections** that are to be collected.*
* *All 6 questions are to be attempted.*
* *All 6 questions are of equal value.*
* *All questions are to be answered on separate pages and will be collected **in separate** bundles at the end of this exam.*
* *All necessary working should be shown in every question.*
* *Full marks may not be awarded for careless or badly arranged work.*

Question 1 *(Start a new page)*

a. Calculate $\dfrac{\sqrt{41.6 - 39.5}}{0.52 + 321}$ (to 3 significant figures) *2 m*

b. Having received a 12 % discount, Amanda paid \$ 385 for a VCR. What was the original price on this VCR? *2 m*

c. Express $\sqrt{32} - \sqrt{8}$ in the form $\sqrt{x}$ and state the value of x. *2 m*

d. Solve the inequation $x^2 + 2 \geq 3x$ *3 m*

e. Neatly sketch the curve $y = |x - 2|$ *2 m*

f. Differentiate

i. $(x^3 + 3x^2)^3$ *2 m*

ii. $6x^2 + \dfrac{1}{x}$ *2 m*

Question 2 *(Start a new page)*

a. Completely factorise $27x^3 - 8$. *1 m*

b. Solve for x if $|3x - 18| = 2 - x$ *3 m*

c. If α and β are the roots to the equation $x^2 = 5 - 2x$.

i. State the value of $\alpha + \beta$ and $\alpha\beta$ *1 m*

and hence find

ii. $\dfrac{1}{\alpha} + \dfrac{1}{\beta}$ *1 m*

iii. $\alpha^2 + \beta^2$ *2 m*

d. Evaluate $\sum_{n=3}^{6} (n^3 - 1)$ *2 m*

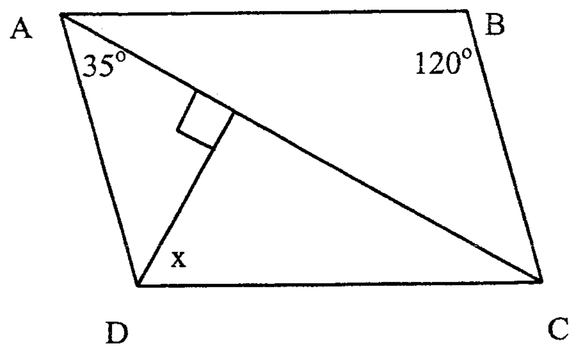
e. The gradient function of a curve f (x) is given by $\frac{1}{3}x^2 - 4x + 3$.
The curve f (x) passes through the point (3,-2).

Find the equation of f (x). *3 m*

f. For triangle ABC, sides AB and BC measure 12m and 15m respectively. *2 m*
<ABC measures 60°. Determine the exact area of this triangle ABC.

Question 3 *(Start a new page)*

a.

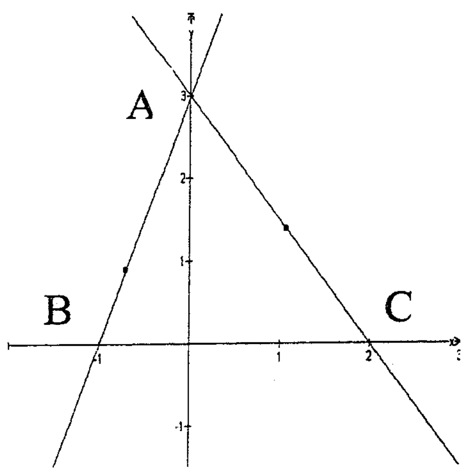


ABCD is a parallelogram.

Redraw this firgure on to your exam page and determine the value for x, giving reasons

3 m

b. The points A (0,3), B (-1,0) and C (2,0) are the vertices of a triangle.



i. Copy this diagram on to your exam page and find the gradient of the line AC. *1 m*

ii. Show that the equation of AC is *2 m*

$$3x + 2y - 6 = 0.$$

iii. BE is the altitude from B to AC. Show that BE has equation *2 m*

$$2x - 3y + 2 = 0$$

(BE is perpendicular to AC)

iv. Calculate the length of the line segment BE. *2 m*

v. Determine the area of triangle ABC. *2 m*

c. Solve the equation $2 \sin x = -\sqrt{3}$ for $0^\circ \leq x \leq 360^\circ$. *3 m*

Question 4 *(Start a new page)*

a. Consider the curve given by $f(x) = x^2 (x - 3)$

i. State the first and second derivative of $f(x)$. *2 m*

ii. Show that there are two stationary values when $x = 0$ and when $x = 2$ and determine their nature. *2 m*

iii. Show that there exists a point of inflection and name this point. *1 m*

iii. Using the above information, neatly sketch this curve, indicating where it crossed the coordinate axes. *2 m*

iv. Determine the equation of the normal to this curve $f(x)$ when $x = 1$. *2 m*

iv. Determine the area of this curve trapped by the x axis and the ordinates $x = 0$ and $x = 3$. *3m*

b. The points A and B in a plane are given by A (2,1) and B (-2,1). Find the equation of the locus of P (x,y), if angle APB is a right angle. *3 m*

Question 5 *(Start a new page)*

a. Use Simpson's Rule with 3 function values (2 subintervals) to approximate the area enclosed between the curve $y = \dfrac{1}{(x + 1)^2}$ and the lines $x = 0$ and $x = 4$ correct to 2 significant figures. *2 m*

b. Evaluate

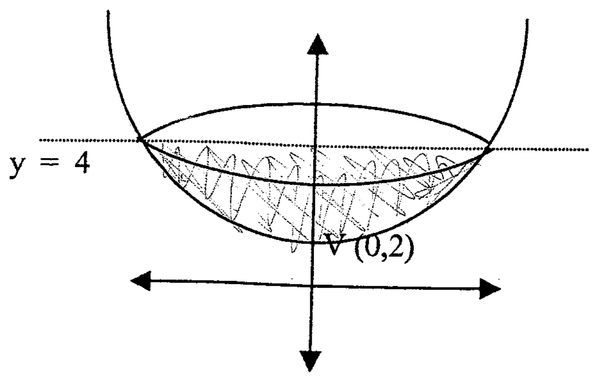
i. $\displaystyle\int_{-1}^{1} (3x + 8)\, dx$ *2 m*

ii. $\displaystyle\int_{0}^{1} \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right) dx$ *2 m*

c. i. Neatly sketch the two curves $f(x) = x$ and $g(x) = x^2 - 3x$ and show that they intersect at the points (0,0) and (4,4). *2 m*

ii. Determine the area bounded by these two curves f(x) and g(x). *3 m*

d. This parabola to the right has its vertex at the point (0,2) and its focus at the point (0,4).



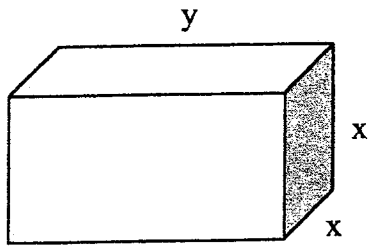
i. Show that the equation of this parabola is given by $y = \frac{1}{8}x^2 + 2$ *1 m*

ii. The area trapped by this parabola and the focal chord y = 4 is rotated about the y axis. Determine the resultant volume. *3 m*

Question 6 *(Start a new page)*

a. The population of a small country town is decreasing at an increasing rate. On your exam page roughly sketch this information, given that N is the population of the town at a given time t, what statement can be made about dN/dt and d^2N/dt^2. *2 m*

b. A metal box, open at the top is made to hold 36 m^3. Each end is a square of side x m.



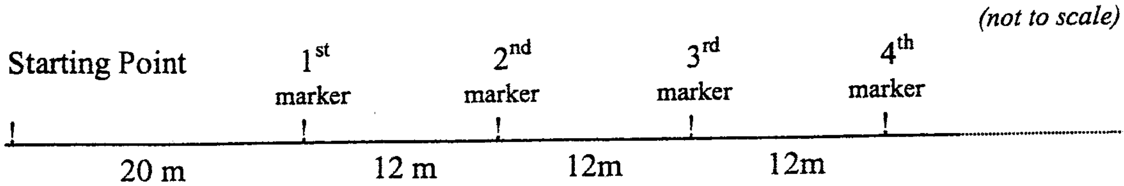
i. Show that the total surface area is given by

$$A = 2x^2 + \frac{108}{x}\ m^2.$$ *1 m*

ii. Find the least possible area of sheet metal that could be used to make it. *3 m*

c. In a training drill at an athletics club an athlete must run from a starting point to the first marker and back to the starting point then out to the second marker and back to the starting point, then out to the third marker and back to the starting point and so on. The markers and the starting point are in a straight line with the first marker 20 m from the starting point and each marker 12 m apart thereafter as indicated in the diagram below.

(not to scale)



Tonight the coach has had 18 markers laid out.

i. How far from the starting point would the 18^{th} marker have been placed? *2 m*

ii. How far in total would an athlete run in completing the drill tonight? *2 m*

½ yrly 2005 yr12 2U maths.

1a 4.51×10^{-3} or 0.00451 ✓✓

4.51 ✓ or any number written out fully then rounded correctly ✓

b)

88%	12%
\$385	

$88\% = 385$ ✓

$1\% = 4.375$

$100\% = \$437.50$ ✓

c) $\sqrt{32}-\sqrt{8}$

$\sqrt{16}\times\sqrt{2}-\sqrt{4}\times\sqrt{2}$

$4\sqrt{2}-2\sqrt{2}$ ✓

$2\sqrt{2}$

$=\sqrt{4}\times\sqrt{2}$

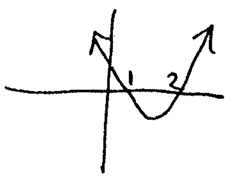
$=\sqrt{8}$

$x=8$ ✓

d) $x^2+2\geq 3x$

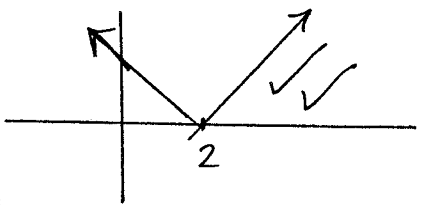
$x^2-3x+2\geq 0$

$(x-2)(x-1)\geq 0$ ✓



$x\leq 1,\ x\geq 2$ ✓ ✓

e) $y=|x-2|$



✓✓

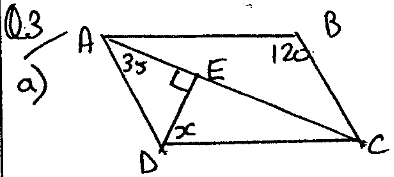
f) i) $\underbrace{3(x^3+3x^2)^2}_{✓}\times(3x^2+6x)$ ✓

ii) $\frac{d}{dx}(6x^2+x^{-1})$

$12x-x^{-2}$ or $12x-\frac{1}{x^2}$

✓ ✓

Q3 a)



$\angle ADE+90+35=180°$ ($\angle$ sum Δ) ✓

$\angle ADE=55$ ✓

$120=x+55$ (opp $\angle$'s of parallelogram)

$x=65°$ ✓

b) i) $m=\frac{rise}{run}=-\frac{3}{2}$ ✓

ii) $A(0,3)$ $C=(2,0)$

$m=\frac{rise}{run}=\frac{-3}{2}$ use say $(2,0)$

$y-0=\frac{-3}{2}(x-2)$ ✓

$y=-\frac{3}{2}x+3$

$2y=-3x+6$

$3x+2y-6=0$ ✓

iii) m of BE $=\frac{2}{3}$ ✓ $B(-1,0)$

$y-0=\frac{2}{3}(x+1)$

$y=\frac{2}{3}x+\frac{2}{3}$ ✓

$3y=2x+2$

$0=2x-3y+2$

iv) $d=\frac{|3\times -1+2\times 0-6|}{\sqrt{2^2+3^2}}$ ✓

$d=\frac{9}{\sqrt{13}}$ ✓

or $d=\frac{9}{\sqrt{13}}\times\frac{\sqrt{13}}{\sqrt{13}}=\frac{9\sqrt{13}}{13}$

v) $AC=\sqrt{3^2+2^2}$

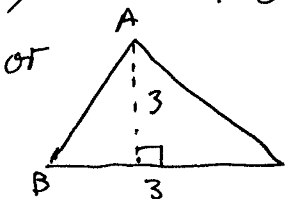
$AC=\sqrt{13}$ ✓

$\therefore A=\frac{1}{2}\times b\times h$

$=\frac{1}{2}\times\sqrt{13}\times\frac{9}{\sqrt{13}}$ ✓

$=4.5$ units^2

or



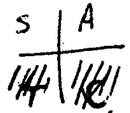
✓ ✓

$A=\frac{1}{2}\times 3\times 3$

$=4.5$

c) $2\sin x=-\sqrt{3}$

$\sin x=-\frac{\sqrt{3}}{2}$



$x=180+60,\ 360-60$

$x=240,\ 300$ ✓ ✓

recognition of 60° ✓

Q5

a)

x	0	2	4
$f(x)$	1	$\frac{1}{9}$	$\frac{1}{25}$

$h=2$ ✓

$A\doteqdot\frac{2}{3}\left[1+4\times\frac{1}{9}+\frac{1}{25}\right]$ ✓

$\doteqdot\frac{668}{675}\doteqdot 0.99$ ✓

b) i) $\left[\frac{3x^2}{2}+8x\right]_{-1}^{1}$ ✓

$=(\frac{3}{2}+8)-(\frac{3}{2}-8)=16$ ✓

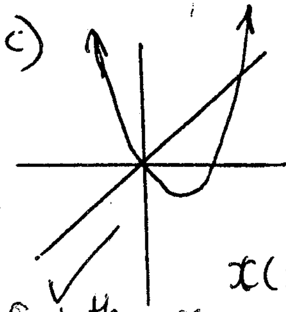
ii) $\int_0^1 x^{\frac{1}{2}}+x^{-\frac{1}{2}}\,dx$

$\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}+\frac{x^{\frac{1}{2}}}{\frac{1}{2}}\right]_0^1$ ✓

$\left[\frac{2x^{\frac{3}{2}}}{3}+2x^{\frac{1}{2}}\right]_0^1$

$=(\frac{2}{3}+2)-0=2\frac{2}{3}$ or $\frac{8}{3}$ ✓

c)



✓ for both sketches

$f(x)=x$

$g(x)=x^2-3x$

$f(x)=g(x)$

$x^2-3x=x$

$x^2-4x=0$

$x(x-4)=0$ ✓

$x=0$ $x=4$

$y=0$ $y=4$

Q5 c) ii)

$A = \int_0^4 x-(x^2-3x)\,dx$ ✓

$= \int_0^4 x - x^2+3x\,dx$

$= \int_0^4 4x - x^2\,dx$

$\left[\frac{4x^2}{2} - \frac{x^3}{3}\right]_0^4$ ✓

$\left(2\times4^2 - \frac{4^3}{3}\right) - (0)$

$= 10\frac{2}{3}$ units^2 ✓

d) i) $(x-h)^2 = 4a(y-k)$

vertex $=(h,k) = (0,2)$

$a=$ focal length $a=2$

$(x-0)^2 = 4\times2(y-2)$

$x^2 = 8y-16$

$8y = x^2+16$

$y = \frac{1}{8}x^2+2$ ✓

ii) $V = \pi\int_2^4 x^2\,dy$

$x^2 = 8y-16$ (see (i))

$V = \pi\int_2^4 8y-16\,dy$ ✓

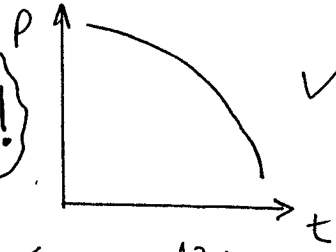
$V = \pi\left[\frac{8y^2}{2} - 16y\right]_2^4$

$V = \pi\left[\left(\frac{8\times4^2}{2} - 16\times4\right) - \left(\frac{8\times2^2}{2} - 16\times2\right)\right]$

$V = 16\pi$ units^3 ✓ ✓

Q6 a)

Label Axes!!!



✓

$\frac{dN}{dt} < 0 \qquad \frac{d^2N}{dt^2} < 0$

✓ (from your graph)

b) i) $S.A = 2x^2 + xy + \cancel{2x}y + xy$

$= 2x^2 + 3xy$

$V = x^2y = 36$

$y = \frac{36}{x^2}$ sub in S.A

$A = 2x^2 + 3x\times\frac{36}{x^2}$

$A = 2x^2 + \frac{108}{x}$ ✓

ii) $A = 2x^2 + 108x^{-1}$

$A' = 4x - 108x^{-2}$

$A'' = 4 + 216x^{-3}$

$A'=0 \qquad 4x - \frac{108}{x^2} = 0$

$4x^3 - 108 = 0$

$x^3 = 27$

$x = 3$ ✓

$A'' = 4 + 216\times3^{-3} > 0$ ✓

$\therefore$ MIN when $x=3$

$A = 2\times3^2 + \frac{108}{3} = 54\text{ m}^2$ ✓

c)

$$\begin{array}{ccccc} M_1 & M_2 & M_3 & \cdots & M_{18} \\ \downarrow & \downarrow & \downarrow & & \downarrow \\ 20 & 12 & 12 & & 12 \\ & \uparrow & \uparrow & & \uparrow \\ & T_1 & T_2 & \cdots & T_{17} \end{array}$$

i) $17\times12 + 20 = 224$ m ✓ ✓

ii) $40+64+88+112+\ldots\ldots$ ✓

$T_1 \quad T_2 \quad T_3 \quad T_4 \quad \ldots\ T_{18}$

$a=40$

$d=24$

$n=18$

$S_n=?$

$S_n = \frac{n}{2}(2a+(n-1)d)$

$= \frac{18}{2}[2\times40+(18-1)\times24]$

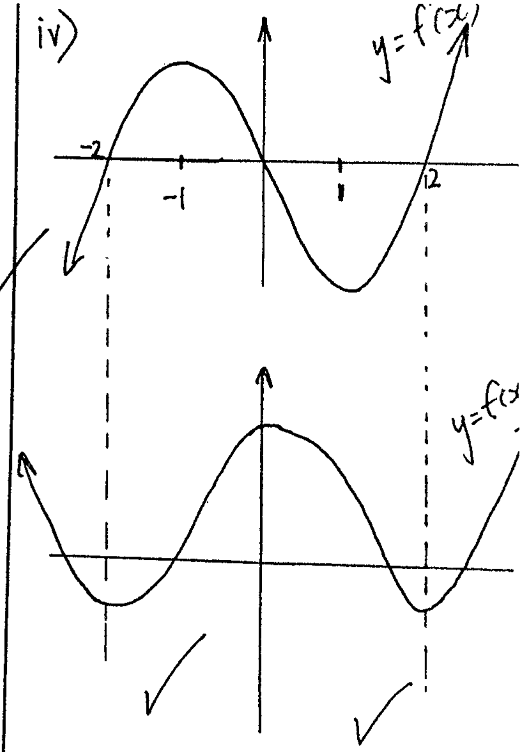
$= 4392$ ✓

d) i) $x=-2, 0, 2$ ✓

ii) $x=0$ ✓

iii) $x=-1, 1$ ✓

iv)



✓ ✓

(3 turning pts in the correct spots = 1 mark).

Question was made very difficult for those students (nearly all) who didn't set up a number pattern first before starting on the formula

* Note $S = 2196 \Rightarrow$ 1 mark (students forgot to double).

MATHS HY 2005

Q2

a) $(3x-2)(9x^2+6x+4)$ ✓

b) $3x-18=2-x$ | $3x-18=x-2$

$4x = 20$ | $2x = 16$

$x = 5$ ✓ | $x = 8$ ✓ (2)

$|15-18| = 2-5$ | $|24-18| = 2-8$

FALSE | FALSE

$\therefore$ no sol^n ✓ (1)

c) $x^2 = 5-2x$

$x^2+2x-5=0$

i) $\alpha+\beta = -2$ ✓

$\alpha\beta = -5$ (1)

ii) $\frac{1}{\alpha}+\frac{1}{\beta}$

$= \frac{\alpha+\beta}{\alpha\beta} = \frac{-2}{-5} = \frac{2}{5}$ ✓ (1)

iii) $\alpha^2+\beta^2$

$= (\alpha+\beta)^2 - 2\alpha\beta$ ✓ (2)

$= (-2)^2 - 2(-5)$ ✓

$= 4+10 = 14$

d) $\sum_{n=3}^{6} (n^3-1)$

$(3^3-1)+(4^3-1)+(5^3-1)+(6^3-1)$ ✓ (2)

$= 26+63+124+215$ ✓

$= 428$

e) $f'(x) = \frac{1}{3}x^2 - 4x + 3$ ✓

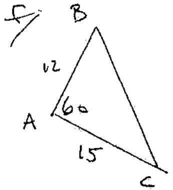
$y = \frac{x^3}{9} - 2x^2 + 3x + c$

$-2 = 27/9 - 18 + 9 + c$ ✓ (3)

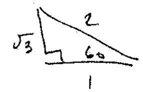
$-2 = -6 + c$

$4 = c$

$f(x) = \frac{x^3}{9} - 2x^2 + 3x + 4$ ✓

f) 

$A = \frac{1}{2}(12)(15)\sin 60$ ✓



$= \frac{1}{2}(12)(15)\frac{\sqrt{3}}{2}$ (2)

$= 45\sqrt{3}\ u^2$

Q4 $f(x) = x^2(x-3)$

i) $y = x^3 - 3x^2$

$y' = 3x^2 - 6x$ ✓ (2)

$y'' = 6x - 6$ ✓

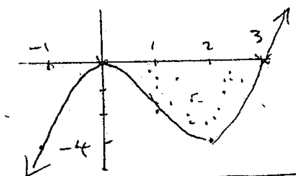
ii) $0 = 3x(x-2)$

$x=0$ $x=2$ ✓ (2)

$y''<0$ ∩ MAX $(0,0)$ $y''>0$ ∪ min $(2,-4)$ ✓

iii) $0 = 6(x-1)$ (1)

$x=1$ y'' neg 0 pos $(1,-2)$ ✓

iv) 

2 (2)

iv) $x=1,\ y=-2$ $m=-3$

$y--2 = \frac{1}{3}(x-1)$ 2 (2)

$3y+6 = x-1$

$0 = x-3y-7$

v) $A = \left|\int_0^3 (x^3-3x^2)\,dx\right|$

$= \left|\left[\frac{x^4}{4} - \frac{3x^3}{3}\right]_0^3\right|$

$= \left|20\frac{1}{4} - 27\right| - 0$ (3)

$= 6.75\ u^2$

Q4 $A(2,1)$, $B(-2,1)$ $\therefore$

b) $\frac{y-1}{x-2} \cdot \frac{y-1}{x--2} = -1$ ✓

$y^2-2y+1 = -(x-2)(x+2)$

$y^2-2y+1 = -x^2+4$ ✓ $\boxed{3}$

$x^2+y^2-2y-3=0$ ✓

nothing for $\sqrt{\ } = \sqrt{\ }$

1 ~~for~~ gradient statement

Notes

Q2

b. CORRECT $x=5$ ✓, $x=8$ ✓ But both rejected. If a student showed work & forgot to reject 1 answer a mark was still awarded
es $|-3| = -3$
Some rejected one but no work shown $\therefore$ no mark awarded

d. FEW RECEIVED 2 marks. $\therefore$ 1 mark for use of formula - considering it a G.P. or A.P.

Q4 b. most lost 3 marks / i.e. no marks awarded for $\sqrt{\ } = \sqrt{\ }$ / 1 mark if attempt at m