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Teacher: MR LISI 4/4/06.

BRIGIDINE COLLEGE RANDWICK

MATHEMATICS

HSC

HALF-YEARLY

2006

(TIME: 2 HOURS)

Directions to candidates:

- Write your **name** at the top of this question paper and each of the **6 sections** to be handed in.
- All 6 questions are to be attempted.
- All 6 questions are of equal value.
- All questions are to be answered on **separate pages** and will be collected **separately at the conclusion of this exam.**
- All necessary working should be shown for every question.
- Full marks may not be awarded for careless or badly arranged work.

QUESTION 1 *(Start a new page)*

(a) Find the value of $11^{-2\cdot1}$ correct to two significant figures. 2

(b) A merchant buys flour from a wholesaler and then sells it at a profit of 40%. If the merchant sells a packet of flour for \$4·76, what price does he pay the wholesaler per packet of flour? 2

(c) Find integers a and b such that $(7 + \sqrt{3})^2 = a + b\sqrt{3}$. 2

(d) Solve the inequation $5 - 3x \geq 11$. 2

(e) Express the recurring decimal $0.\dot{5}\dot{7}$ (i.e. 0·575757 ...) as a simple fraction 2

(f) Differentiate $(5x^3 + 7)^4$. 2

QUESTION 2 *(Start a new page)*

(a) Completely factorise $2x^2 - 7x - 15$. 2

(b) Solve $|x - 1| = 4$. 2

(c) The graph of $y = f(x)$ passes through the point $(3, 1)$ and $f'(x) = 4x + 7$. Find $f(x)$. 2

(d) Find the values of p for which the equation $9x^2 - px + 16 = 0$ has:

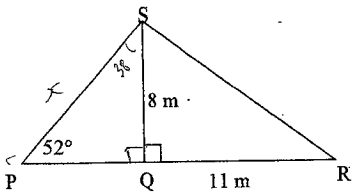
(*i*) exactly one real root. 2

(*ii*) real roots. 2

(e) For all x in the domain $0 \leq x \leq 5$ a function $h(x)$ satisfies $h'(x)<0$ and $h''(x)<0$. Sketch a possible graph of $h(x)$ in this domain. 2

QUESTION 3 *(Start a new page)*

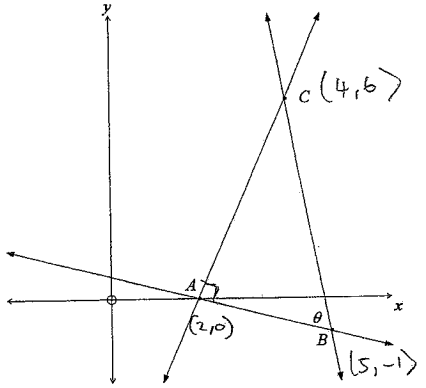
(a)



RQ = 11 m
SQ = 8 m
∠SPQ = 52°

Find the length of SP to the nearest centimetre 2

(b)



The diagram shows the points $A(2, 0)$, $B(5, -1)$ and $C(4, 6)$. Angle ABC is θ.

(*i*) Show the equation of the line AC is $3x - y = 6$. 2

(*ii*) Show the gradient of AB is $-\frac{1}{3}$. 1

(*iii*) Show the length of AB is $\sqrt{10}$ units. 1

(*iv*) Show AB and AC are perpendicular., 1

(*v*) Find $\tan\theta$. 2

(*vi*) Find the equation of the circle with centre A that passes through B. 2

(*vii*) Copy the diagram from the question onto your answer page and shade the region satisfying the inequality $3x - y \leq 6$ 1

QUESTION 4 *(Start a new page)*

(a) The following table lists the values of a function for three values of x.

x	2·0	3·0	4·0
$f(x)$	2·6	7·8	1·4

Use three function values to estimate $\int_2^4 f(x)dx$ by:

(*i*) Simpson's rule 2

(*ii*) the trapezoidal rule 2

(b) The fourth term of an arithmetic series is 22 and the seventh term is 10.

(*i*) Find the common difference. 1

(*ii*) Find the sum of the first 12 terms. 2

(c) The first term of a geometric series is 81 and the fourth term is 3.

(*i*) Find the common ratio. 1

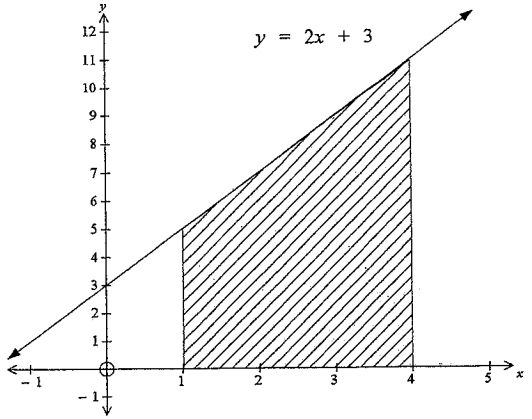
(*ii*) Find the limiting sum of the series. 1

(d) A parabola has equation $x^2 = 12(y + 2)$.

(*i*) Find the coordinates of the vertex of the parabola. 1

(*ii*) Find the equation of the directrix of the parabola. 2

QUESTION 5 *(Start a new page)*

(a) 

4

The region which lies between the x-axis and the line $y = 2x + 3$ from $x = 1$ to $x = 4$ is rotated about the x-axis to from a solid. Find the volume of the solid.

(b) On 1 July 1995 Stephanie invested \$15 000 in a bank account that paid interest at a fixed rate of 8% per annum, compounded annually.

(*i*) Show the amount in the account, after the payment of interest on 1 July 2005 if no additional deposits were made, was \$32 383·87. 2

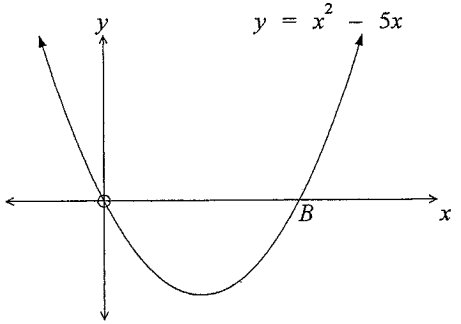
(*ii*) In fact Stephanie added \$1000 to her account on 1 July each year, beginning on 1 July 1996. 3

How much was in her account on 1 July 2005 after payment of interest into her account?

(*iii*) Stephanie's sister, Gemma, invested \$15 000 in an account at another bank on 1 July 1995 and made no further deposits. On 1 July 2005, the balance in Gemma's account was \$53 217. 3

What was the annual rate of compound interest paid on Gemma's account? (Express your answer as a percentage correct to one decimal point.)

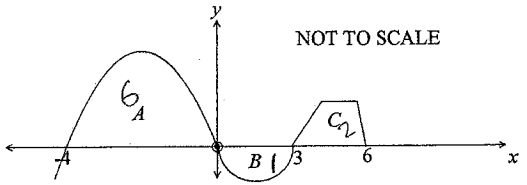
QUESTION 6 *(Start a new page)*

(a) 

The diagram shows the graph of the function $y = x^2 - 5x$.

(*i*) Find the x-coordinate of the point B where the curve crosses the positive x-axis. 1

(*ii*) Find the area of the region bounded by the curve $y = x^2 - 5x$ and the x-axis. 2

(b) 

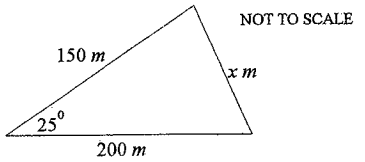
2

The graph of the function f is shown in the diagram. The area A, below f and above the x-axis from $x = -4$ to $x = 0$, is 6 square units. The area B, below the x-axis and above f from $x = 0$ to $x = 3$, is 1 square unit. The area C, between f and the x-axis from $x = 3$ to $x = 6$, is 2 square units.

Evaluate $\int_{-4}^{6} f(x)dx$

(c) Prove $(1 - \sin\theta)(1 + \sin\theta) = \cos^2\theta$ 2

(d) Solve for $0° \le \theta \le 360°$ to the nearest minute where necessary $\tan^2\theta - \tan\theta - 2 = 0$ 3

(e) 

2

Find the value of x (correct to one decimal place).

Yr 12 Half-Yearly 2006 : Mathematics I

Question 1

(a) $11^{-2.1} = 0.0065024...$ ✓
$= 0.0065$ (2 sig figs) ✓

(b) \$4.76 = 140% of cost price
1% of cost price = \$4.76 ÷ 140 ✓
Cost price = \$4.76 ÷ 140 × 100
= \$3.40 ✓

(c) $(7+\sqrt{3})^2 = a + b\sqrt{3}$
$(7+\sqrt{3})^2 = 49 + 14\sqrt{3} + 3$
$= 52 + 14\sqrt{3}$
$\therefore a = 52$ ✓
$b = 14$ ✓

(d) $5 - 3x \geq 11$
$-3x \geq 6$ ✓
$x \leq -2$ ✓

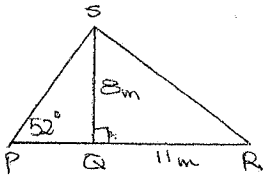
(e) $0.\dot{5}\dot{7} = 0.575757...$
Let $x = 0.575757...$ (1)
$100x = 57.575757...$ (2)
(2) − (1) $99x = 57$ ✓
$x = \frac{57}{99} = \frac{19}{33}$ ✓

(f) Let $u = 5x^3 + 7$
then $\frac{d(5x^3+7)^4}{dx}$
$= \frac{du^4}{du} \cdot \frac{du}{dx}$
$= 4u^3 \cdot \frac{d(5x^3+7)}{dx}$ ✓
$= 4(5x^3+7)^3 \times 15x^2$
$= 60x^2(5x^3+7)^3$ ✓

$\frac{du}{dx} = \frac{d(5x^3+7)}{dx}$
$= 15x^2$

Question 3

(a)

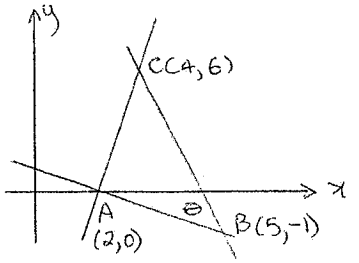


In $\triangle SPQ$, $\angle SQP = 90°$
and $\sin 52° = \frac{8}{SP}$

$SP = \frac{8}{\sin 52°}$ ✓

$= 10.15$m (to nearest cm) ✓

(b)



(i) $m_{AC} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6-0}{4-2} = 3$

3

4

c) i) $a = 81$

$T_4 = 3$

$3 = 81 \times r^3$

$r^3 = \frac{3}{81} = \frac{1}{27}$

$r = \frac{1}{3}$ ✓

ii) $S_\infty = \frac{a}{1-r}$

$= \frac{81}{1-\frac{1}{3}}$

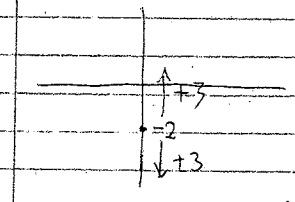
$= 121\frac{1}{2}$ ✓

d) $x^2 = 12(y+2)$

i) $(0, -2)$ ✓

ii) $4a = 12$

$a = 3$ ✓



$y = -5$ ✓

Q6a) i) $x^2 - 5x = 0$

$x(x-5) = 0$

$x = 0 \quad x = 5$ ✓

ii) $\int_0^5 x^2 - 5x \, dx$

$\left[\frac{x^3}{3} - \frac{5x^2}{2}\right]_0^5$ ✓

$= \left(\frac{5^3}{3} - \frac{5 \times 5^2}{2}\right) - (0)$

$= \left|-20\frac{5}{6}\right| = 20\frac{5}{6}$ or $\frac{125}{6}$

Nearly whole form got this wrong.

b) ✓

$6 + (-1) + 2 = 7$ ✓

Note $6 + 1 + 2 = 9$ (1 mark)

c) $(1 - \sin\theta)(1 + \sin\theta)$ No Fudging!

$= 1 - \sin^2\theta$ ✓

$= \cos^2\theta$ ✓

$1 - \sin\theta = \cos\theta$ shows incomplete understanding!

d) $\tan^2\theta - \tan\theta - 2 = 0$

$(\tan\theta - 2)(\tan\theta + 1) = 0$ ✓

$\tan\theta = 2$

$\theta = 63°26'$

and $180 + 63°26' = 243°26'$ ✓

$\tan\theta = -1$

$\theta = 180 - 45° = 135°$

and $360 - 45 = 315°$ ✓

(any 2 correct answers 1 mark)

e) $x^2 = 150^2 + 200^2 + 2 \times 150 \times 200 \times \cos 25$ ✓

$x^2 = 8121.53\ldots$

$x = 90.1$ ✓

(taking a square root of an incorrect formula 1 mark)

Q2

a) $2x^2 - 7x - 15$

$2x \times +3$

$x \times -5$

$(2x+3)(x-5)$ ✓ ✓

(note +, − swapped gets 1 mark)
* solving quadratic = 1 mark

b) $|x-1| = 4$

$x - 1 = 4$ $\quad x - 1 = -4$

$x = 5$ ✓ $\quad x = -3$ ✓

c) $f'(x) = 4x + 7$

$f(x) = 2x^2 + 7x + C$ ✓

at (3, 1)

$1 = 2 \times 3^2 + 7 \times 3 + C$

$C = -38$ ✓

$f(x) = 2x^2 + 7x - 38$

d) i) $\Delta = 0$

$b^2 - 4ac = 0$ ✓

$p^2 - 4 \times 9 \times 16 = 0$

$p^2 = 576$

$p = \pm 24$ ✓

ii) $\Delta \geq 0$

$b^2 - 4ac \geq 0$ ✓

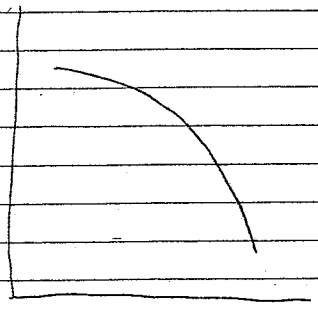
$p^2 - 4 \times 9 \times 16 \geq 0$

$p^2 - 576 \geq 0$

$(p-24)(p+24) \geq 0$

$p \leq -24, \; p \geq 24$ ✓

e)



1 mark for negative gradient
1 mark for negative concavity

Q4a)

i) $A = \frac{h}{3}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right]$

$A = \frac{1}{3}\left[2.6 + 4 \times 7.8 + 1.4\right]$ ✓ ✓

$A = 11.7\dot{3}$ or $11\frac{11}{15}$

ii) $A = \frac{h}{2}\left[f(a) + f(b)\right]$

$= \frac{1}{2}\left[2.6 + 2 \times 7.8 + 1.4\right]$ ✓ ✓

$= 9.8$

b) i) $A_4 = 22 \rightarrow 22 = a + 3d \; \ldots ①$

$A_7 = 10 \rightarrow 10 = a + 6d \; \ldots ②$

① − ② $\quad 12 = -3d$

$d = -4$ ✓

$a = 34$ ✓

ii) $S_{12} = \frac{12}{2}\left[2 \times 34 + 11 \times -4\right]$

$= 144$ ✓

The equation of AC is given by

$y - 0 = 3(x-2)$ ✓

$y = 3x - 6$

$3x - 6 = y$

$3x - y - 6 = 0$

$3x - y = 6$ ✓

(ii) $m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{-1-0}{5-2}$

$= -\frac{1}{3}$ ✓

(iii) Length of AB

$= \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$= \sqrt{(5-2)^2 + (-1-0)^2}$

$= \sqrt{3^2 + (-1)^2}$

$= \sqrt{10}$ ✓

(iv) If AB & AC are perpendicular then

$m_{AB} \times m_{AC} = -1$

Now $m_{AB} \times m_{AC} = -\frac{1}{3} \times 3$

$= -1$ ✓

(v) $\triangle ABC$ is right-angled at A as $AB \perp AC$

Thus $\tan\theta = \frac{AC}{AB}$

$AC = \sqrt{(4-2)^2 + 6^2}$

$= \sqrt{40}$

$= 2\sqrt{10}$ ✓

$\therefore \tan\theta = \frac{2\sqrt{10}}{\sqrt{10}}$

$= 2$ ✓

3(b)(vi)

Circle with centre (h,k) and radius r is given by

$(x-h)^2 + (y-k)^2 = r^2$

Centre: (2,0) Radius: $\sqrt{10}$

$(x-2)^2 + (y-0)^2 = (\sqrt{10})^2$ ✓

$(x-2)^2 + y^2 = 10$ ✓

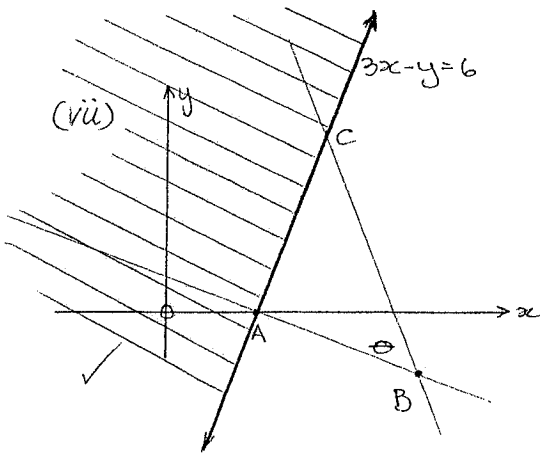
OR

$x^2 + y^2 - 4x + 4 = 10$

OR

$x^2 + y^2 - 4x = 6$

(vii)



✓

$3x - y = 6$

Test (0,0)

$3x - y = 3\times0 - 0$

$= 0 - 0$

$= 0 \leq 6$

Question 5

(a) $V = \pi \int y^2 dx$ ✓

$= \pi \int_1^4 (2x+3)^2 dx$

$= \pi \int_1^4 (4x^2+12x+9) dx$ ✓

$= \pi \left[\frac{4}{3}x^3+6x^2+9x\right]_1^4$

$= \pi \left[\frac{4}{3}\times 4^3+6\times 4^2+9\times 4-\left(\frac{4}{3}+6+9\right)\right]$ ✓

$= 201\pi$ unit^3 (Accept 631.46 unit^3) ✓

(b) (i) $P = \$15000$
$r = 8\%$ p.a. compounded annually
$n = 10$ years

$A = P(1+r)^n$
$= 15000(1+0.08)^{10}$ ✓
$= \$32383.87$ ✓

(ii) Amounts invested:

1995	\$15000 at 8%pa	for 10yrs	$\rightarrow \$15000\times 1.08^{10}$
1996	\$1000 at 8%pa	for 9yrs	$\rightarrow \$1000\times 1.08^{9}$
1997	\$1000 " "	" 8yrs	$\rightarrow \$1000\times 1.08^{8}$
1998	\$1000 " "	" 7yrs	$\rightarrow \$1000\times 1.08^{7}$
⋮			⋮
2004	\$1000 " "	" 1 yr	$\rightarrow \$1000\times 1.08$
2005	\$1000		$\rightarrow \$1000$ ✓ (No penalty if 2005 left out)

Balance in account on 1 July 2005

$= \$15000\times 1.08^{10} + \underbrace{(1.08^9+1.08^8+1.08^7+\ldots+1.08+1)}_{\text{GP with } a=1,\ r=1.08,\ n=10}\$1000$ ✓

$= \$32383.87$ (from (i))
$+ \$1000\times\frac{(1.08^{10}-1)}{1.08-1}$

$= \$46870.44$ ✓

Accept with working
\$45870.44
\$46870.43
\$45870.43

(7)

(iii) $A = P(1+r)^n$
$53217 = 15000(1+r)^{10}$
$(1+r)^{10} = \frac{53217}{15000}$ ✓
$1+r = \sqrt[10]{\frac{53217}{15000}}$
$r = \sqrt[10]{\frac{53217}{15000}} - 1$ ✓

$= 13.5\%$ (to 1 dp) ✓