Student: _____

Teacher: _____

BRIGIDINE COLLEGE RANDWICK

HSC MATHEMATICS EXT 1

YEAR 12

ASSESSMENT TASK

JUNE 2009

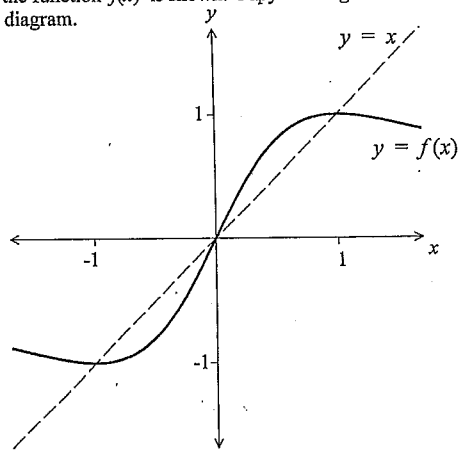
(TIME: 45 MINUTES)

Directions to candidates:

- Write your **name** at the top of this question paper and each of the **3 sections** to be handed in.
- All questions are to be attempted.
- All questions are to be answered on **separate pages** and will be collected **separately at the conclusion of this exam.**
- Pen should be used and all necessary working should be shown for every question.
- Full marks may not be awarded for careless or badly arranged work.

QUESTION 1 *(Start a new page)*

(a) The graph of the function $f(x)$ is shown. Copy the diagram and sketch the graph of $f^{-1}(x)$ on the diagram. (1)



(b) Evaluate

(i) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$ (1)

(ii) $\tan\left(\cos^{-1}\left(\frac{3}{5}\right)\right)$ (2)

(c) A function is given by

$h(x) = \sin^{-1}x + \cos^{-1}x, \quad 0 \le x \le 1$

(i) Find $h'(x)$ (2)

(ii) Sketch the graph of $y = h(x)$ (1)

(d) Find the exact area bounded by the curve $y = \frac{1}{\sqrt{1-x^2}}$, the x-axis and the lines $x = -\frac{1}{2}$ and $x = \frac{1}{\sqrt{2}}$ (3)

QUESTION 2 *(Start a new page)*

(a) Find the quotient and remainder when (2)

$$x^4 - 2x^3 + x^2 - 5x + 7$$

is divided by

$$x^2 + x - 1$$

(b) $(x-2)$ is a factor of the polynomial (1)

$$P(x) = 2x^3 + x + a.$$

Find a.

(c) Consider The function $f(x) = x^3 - \ln(x+1)$ has one root between 0·5 and 1.

(i) Show the root lies between 0·8 and 0·9. (2)

(ii) Hence use the halving-the-interval method to find the value of the root, correct to one decimal place . (2)

QUESTION 3 *(Start a new page)*

An object is moving in simple harmonic motion about the origin. The displacement, x metres, of the object from the origin at time t seconds is given by

$$x = 6\sin\left(2t + \frac{\pi}{4}\right).$$

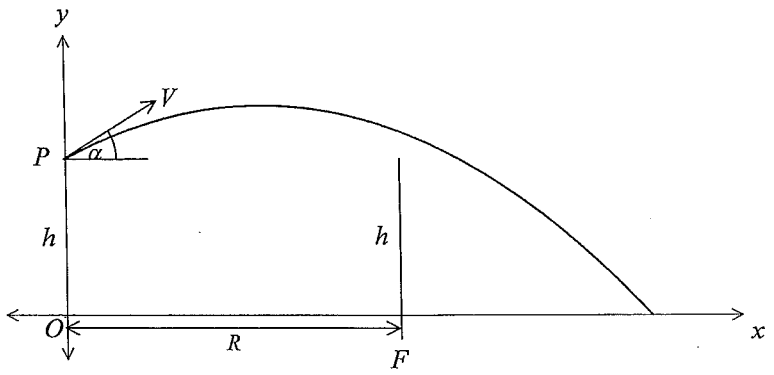
(a) State the period and amplitude of the motion. (2)

(b) Sketch the graph of $x = 6\sin\left(2t + \frac{\pi}{4}\right)$ for $0 \le t \le 2\pi$ (2)

(c) Find the velocity of the object when $t = 0$. (2)

(d) Find the first time after $t = 0$ when the object is a t $x = 3$. (2)

QUESTION 4 *(Start a new page)*



At time t a ball is hit from the point P at a speed of V metres per second and at an angle of α to the horizontal.

P is h metres above O.

(a) Write expressions for $\dot{x}$, $\dot{y}$, x and y at $t = 0$. (1)

(b) The equations of motion of the ball are (3)

$$\ddot{x} = 0, \quad \ddot{y} = -g \ .$$

Using calculus, show the position of the ball at time t is given by

$$x = Vt\cos\alpha$$

$$y = Vt\sin\alpha - \frac{1}{2}gt^2 + h$$

(c) Hence show the trajectory of the ball is given by (2)

$$y = h + x\tan\alpha - x^2\frac{g}{2V^2\cos^2\alpha}$$

(c) A fence, F, is R metres from the point O and is h metres high. If the ball clears the fence, show that (2)

$$V^2 \ge \frac{gR}{2\sin\alpha\cos\alpha}$$

END OF EXAMINATION PAPER

HSC Mathematics Ext 1: June 2009 Suggested Solutions

Question 1

(a) Graph – see attached sheet.

(b) (i) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$ or 60°

(ii) $\tan\left(\cos^{-1}\left(\frac{3}{5}\right)\right)$

$= \frac{4}{3}$

(c) $h(x) = \sin^{-1}x + \cos^{-1}x;\ 0 \le x \le 1$

(i) $h'(x) = \frac{1}{\sqrt{1-x^2}} + \frac{-1}{\sqrt{1-x^2}}$

$= 0$

(ii) As $h'(x) = 0$ for $0 \le x \le 1$ then $h(x) = \text{const.}$

Now $h(0) = \sin^{-1}0 + \cos^{-1}0$

$= 0 + \frac{\pi}{2}$

$= \frac{\pi}{2}$

$\therefore h(x) = \frac{\pi}{2}$ for $0 \le x \le 1$

Graph: see attached sheet.

(d) Area $= \int_{-1/2}^{1/\sqrt{2}} \frac{1}{\sqrt{1-x^2}}\,dx$

$= \left[\sin^{-1}x\right]_{-1/2}^{1/\sqrt{2}}$

$= \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) - \left(\sin^{-1}\left(-\frac{1}{2}\right)\right)$

$= \frac{\pi}{4} - \left(-\frac{\pi}{6}\right)$

$= \frac{5\pi}{12}$ unit^2

Question 2

(a)
$$\begin{array}{r} x^2-3x+5 \\ x^3+x-1\ \overline{)\ x^5-2x^3+x^2-5x+7} \\ x^5+x^3-x^2 \\ \hline -3x^3+2x^2-5x \\ -3x^3-3x^2+3x \\ \hline 5x^2-8x+7 \\ 5x^2+5x-5 \\ \hline -13x+12 \end{array}$$

$Q(x) = x^2-3x+5$

$R(x) = -13x+12$

(b) If $(x-2)$ is a factor of $P(x)$ then

$P(2) = 0$

$2\times 2^3 + 2 + a = 0$

$18 + a = 0$

$a = -18$

(c) $f(x) = x^3 - \ln(x+1)$

(i) $f(0.8) = -0.0757\ldots$

< 0

$f(0.9) = 0.0871\ldots$

> 0

Thus as $f(0.8)$ and $f(0.9)$ have different sign, then for some value a, $0.8 < a < 0.9$, $f(a) = 0$ and 'a' is the root of $f(x) = 0$.

(ii) $x_3 = \frac{0.8+0.9}{2} = 0.85$

$f(0.85) = -0.00106\ldots$

< 0

$\therefore$ The root of $f(x) = 0$ is between 0.85 and 0.9 and is closer to 0.9 than 0.8

Thus the value of the root to 1dp is 0.9.

Question 3

$x = 6\sin(2t + \pi/4)$

$= a\sin(nt + \alpha)$

$a = 6,\ n = 2,\ \alpha = \pi/4$

(a) $T = \frac{2\pi}{n} = \frac{2\pi}{2} = \pi$

Amplitude $= a = 6$

(b) Graph: see attached sheet.

(c) $\dot{x} = 6\cos(2t + \pi/4) \times 2$

$= 12\cos(2t + \pi/4)$

$t = 0$ $\dot{x} = 12\cos\pi/4$

$= 6\sqrt{2}$ or $\frac{12}{\sqrt{2}}$

(8.49 to 2dp)

(d) $x = 3$ $6\sin(2t + \pi/4) = 3$

$\sin(2t + \pi/4) = \frac{1}{2}$

$2t + \pi/4 = \pi/6, 5\pi/6, \ldots$

$2t = \frac{\pi}{6} - \frac{\pi}{4}, \frac{5\pi}{6} - \frac{\pi}{4}, \ldots$

$= -\frac{\pi}{12}, \frac{7\pi}{12}, \ldots$

As $t > 0$,

$t = \frac{7\pi}{24}$

$(= 0.91629\ldots)$

Question 4

(a) $x(0) = 0$ $\quad y(0) = h$

$\dot{x}(0) = V\cos\alpha$ $\quad \dot{y}(0) = V\sin\alpha$

(b) bookwork

(c) bookwork

(d) If the ball clears the fence $y \ge h$ for $x = R$

$$h + R\tan\alpha - \frac{R^2 g}{2V^2\cos\alpha} \ge h$$

$$R\tan\alpha - \frac{R^2 g}{2V^2\cos\alpha} \ge 0$$

$$2RV^2\cos\alpha\tan\alpha - R^2 g \ge 0$$

$$2RV^2\cos\alpha\sin\alpha \ge R^2 g$$

$$2V^2\cos\alpha\sin\alpha \ge Rg$$

$$V^2 \ge \frac{Rg}{2\cos\alpha\sin\alpha}$$

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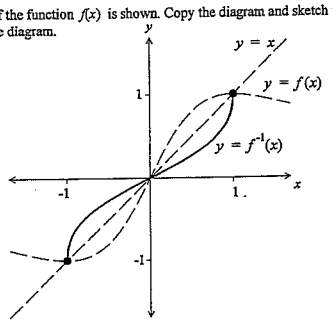
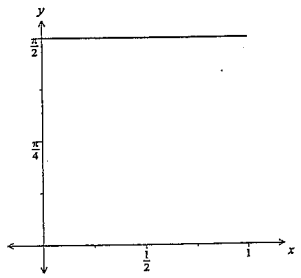
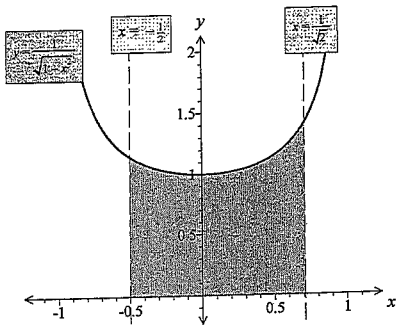


(c) $h(x) = \sin^{-1}x + \cos^{-1}x$

(ii) Sketch the graph of $y = h(x)$



(d) Find the exact area bounded by the curve $y = \frac{1}{\sqrt{1-x^2}}$, the x-axis and the lines $x = -\frac{1}{2}$ and $x = \frac{1}{\sqrt{2}}$



QUESTION 3

(b) Sketch the graph of $x = 6\sin\left(2t + \frac{\pi}{4}\right)$ for $0 \le t \le 2\pi$ (2)