BRIGIDINE COLLEGE RANDWICK

Year 12 Mathematics **Student**

21 June 2010 **Teacher**
Time 40 minutes

Show all necessary working.
Neatness may be taken into consideration in the awarding of marks.

- there are 7 questions -

1. Given that the $\log_a 2 = 0.23$ and $\log_a 3 = 0.33$, find the $\log_a 18$. 2 m

2. Differentiate the following with respect to x

   a. $-3e^{x^2}$ 2 m

   b. $\ln(\sin x)$ *(answer as a simplified trigonometric expression)* 2 m

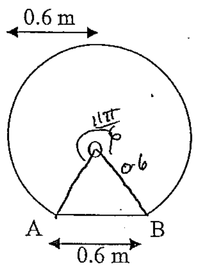
3. Determine the exact value of the following

   a. $\int_0^{\pi/3} \sec^2 x \, dx$ 2 m

   b. $\int_0^2 x^2 e^{-x^3} \, dx$ 3 m

4. A table-top is in the shape of a circle with a small segment removed as shown to the right.

   The circle has centre O and radius 0.6 metres.
   The length of the straight edge AB is also 0.6 metres.



   a. Explain why $\angle AOB = \frac{\pi}{3}$. 1 m

   b. Find the exact area of the table-top. 3 m

5. The area under the curve $y = \frac{1}{\sqrt{x}}$, for $1 \le x \le e^2$ is rotated about the x axis. 4 m
   Find the exact volume of the solid of revolution.

6. A particle is moving in a straight line starting from the origin. At time t seconds the particle has a displacement of x metres from the origin and a velocity v m/s. The displacement is given by $x = 2t - 3\ln(t+1)$ at any time t.

   a. Find an expression for v. 1 m

   b. Find the initial velocity. 1 m

   c. Find when the particle comes to rest. 2 m

   d. Show that this particle is always accelerating. 2 m

7. a. Using a scale of 1 cm represents 1 unit, neatly sketch the curves $y = \cos\frac{\pi}{2}x$ and $y = \ln x$ for $0 \le x \le 4$. 2 m

   b. Determine the number of solutions there are to the equation $\cos\frac{\pi}{2}x = \ln x$ in the domain $0 \le x \le 4$. 1 m

   c. Determine the exact area trapped by the curves $y = \cos\frac{\pi}{2}x$ and $y = \ln x$ and the ordinates $x = 1$ and $x = 3$. 3 m

IRIL 21/6/10

1) $\log_a 18$

$= \log_a 2(3)(3)$

$= \log_a 2 + 2\log 3$ ✓

$\doteqdot 0.23 + 2(0.33)$ ✓

$= 0.89$ 2

2a) if $y = -3e^{x^2}$ ✓

$y' = -3e^{x^2}(2x)$

$= -6x\,e^{x^2}$ ✓ 2

b) if $y = \ln(\sin x)$

$y' = \frac{1}{\boxed{\sin x}} \cos x$ ✓

$= \cot x$ ✓ 2

3a) $\int_0^{\frac{\pi}{3}} \sec^2 x \, dx$

$= \tan x \Big|_0^{\frac{\pi}{3}}$ ✓

$= \tan \frac{\pi}{3} - \tan 0$ ✓

$= \sqrt{3}$ $\sqrt{3}$, 2, 1, $\frac{\pi}{3}$ 2

b) $-\frac{1}{3}\int_0^2 e^{-x^3} (-3)x^2 \, dx$ ✓

$= -\frac{1}{3} e^{-x^3} \Big|_0^2$ ✓

$= -\frac{1}{3}\left[e^{-8}\right] - -\frac{1}{3}e^0$

$= -\frac{1}{3}e^{-8} + \frac{1}{3}$ ✓ 3

4) Triangle AOB

a) equilateral

$60° \Rightarrow \frac{\pi}{3}$ ✓

b) $\frac{1}{2}(\ ,\ )(\ )[\quad - \sin\quad]$

NB $\nearrow$ if $\frac{\pi}{3}$

$\therefore$ Acircle − Ars vs

$\frac{1}{2}(0.6)(0.6)\left[\frac{5\pi}{3} - \sin\frac{5\pi}{3}\right]$

central angle $360 - 60$

$\frac{9}{50}\left[\frac{5\pi}{3} - -\sin\frac{\pi}{3}\right]$

$\frac{9}{50}\left[\frac{5\pi}{3} + \frac{\sqrt{3}}{2}\right]$

$\frac{9}{50}\left[\frac{10\pi + 3\sqrt{3}}{6}\right]$

$\frac{3}{100}\left[10\pi + 3\sqrt{3}\right] \ m^2$ 4

correct method ✓

formula ✓

correct answer ✓

5) $V = \pi\int_1^{e^2}\left[\frac{1}{\sqrt{x}}\right]^2 dx$ ✓

$= \pi\int_1^{e^2} \frac{1}{x}\, dx$ ✓

$= \pi \ln x \Big|_1^{e^2}$ ✓

$= \pi\left[\ln e^2 - \ln 1\right]$ 4

$= \pi[2\ln e - 0]$

$= 2\pi \ u^3$ ✓

8

IRIL 21/6/10

6) $x = 2t - 3\ln(t+1)$

a) $v = 2 - \frac{3}{t+1}$ ✓

$t = 0$

b) $v = 2 - 3.$

$= -1 \, m/s$ ✓

c) REST $v = 0$

$0 = 2 - \frac{3}{t+1}$

$2 = \frac{3}{t+1}$ ✓

$2t + 2 = 3$

$2t = 1$

$t = \frac{1}{2}\, s$ ✓

$\therefore$ comes to REST $\frac{1}{2}$ s

d) $v = 2 - \frac{3}{t+1}$

$= 2 - 3(t+1)^{-1}$

$v' = 3(t+1)^{-2}$ ✓

$= \frac{3}{(t+1)^2}$ $\frac{\text{pos}}{\text{pos}}$ ✓

$a > 0$

$\therefore$ constant

work ⇐

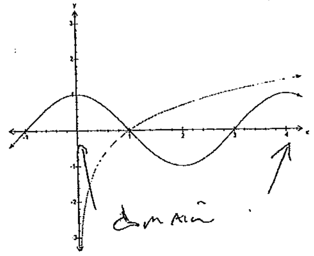
statement ⇐

6

7) $y = \cos\frac{\pi}{2}x$

$\frac{2\pi}{\frac{\pi}{2}} \Rightarrow 4$

a)



2

1 mark each curve

b) $\therefore$ only 1 solution 1

c) $\int_1^3 \left[\ln x - \cos\frac{\pi}{2}x\right] dx$ ✓

$\therefore$ 4 marks

beyond your COURSE

method will be given LATER