BRIGIDINE COLLEGE RANDWICK

Year 12 Mathematics **Student**

12 December 2005 **Teacher**

Time 45 Minutes

Show all necessary working.
Neatness may be taken into consideration in the awarding of marks.

There are 6 Questions.

1. Differentiate the following
(leaving answers completely simplified with positive indices)

a. $y = 3x^2 - \dfrac{1}{x}$ *2 marks*

b. $y = -9(4-x)^8$ *2 marks*

c. $y = \sqrt{x^2 - 3x}$ *2 marks*

d. $y = \dfrac{3x^2}{1+x^3}$ *2 marks*

2. Find the equation of a curve, given the gradient function of this curve is $3x^2 - 4x + 5$ and $f(1) = 4$. *3 marks*

3. Consider the curve given by $f(x) = x^3 - 12x + 12$.

a. Find $f'(x)$ and $f''(x)$. $y' = 3x^2 - 12$ *2 mark*

$y'' = 6x$

$0 = 6x$

b. Show there are two stationary values at $x = -2$ and $x = 2$ and determine their nature. *3 mark*

c. Determine any possible points of inflection. *2 mark*

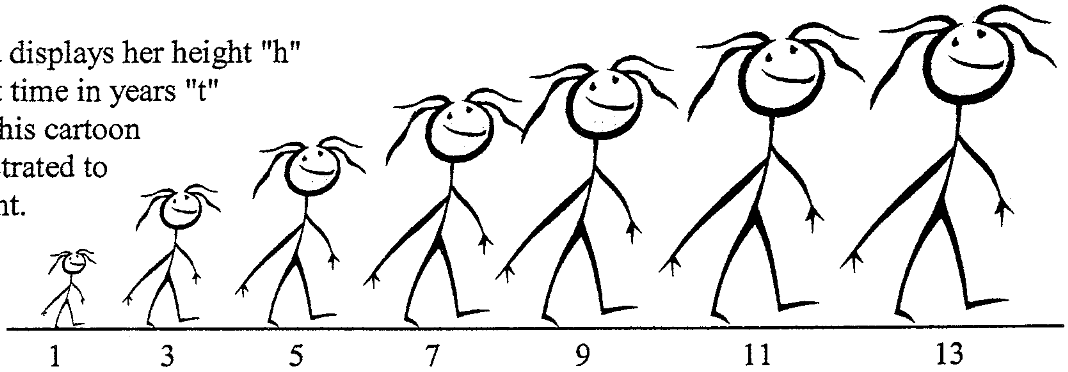
x	$\leftarrow$	0	$\rightarrow$
y''		0	

d. Sketch the curve, showing the above features. *2 mark*

e. Find the equation of the tangent to this curve when $x = -1$. *2 mark*

... please turn over

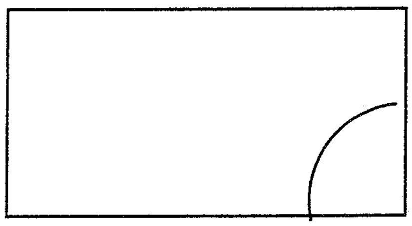
4. Jacinta displays her height "h" against time in years "t" using this cartoon as illustrated to the right.



Comment on $\frac{dh}{dt}$ and $\frac{d^2h}{dt^2}$, justifying your answer. 3 marks

5. This diagram to the right shows a rectangular paddock which has an area of 10 000 m^2.

The paddock requires fencing around the perimeter and also along a circular arc in one corner. The radius of the arc is half the width of the paddock.



$A = 2xy$.

a. Show that the total amount of fencing required is given in metres by

$$L = 4x + \frac{\pi x}{2} + \frac{10\,000}{x}$$, where x is the radius of the circular arc. **3** marks

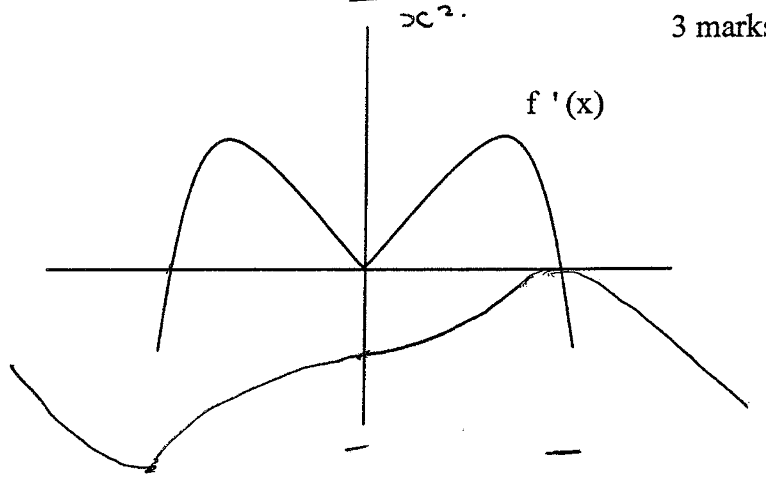
b. Show that when x is approximately 42.4 metres, the paddock will require the least amount of fencing. 3 marks

$10000\,x^{-1}$

$= -\frac{10000}{x^2}$.

6. Sketch the diagram of f '(x) to the right on to your exam page. 3 marks

Below your diagram, sketch the curve f (x).



2005

1a) $y = 3x^2 - x^{-1}$

$y' = 6x + x^{-2}$ ✓

$y' = 6x + \frac{1}{x^2}$ ✓

b) $y = -9(4-x)^8$

$y' = -9 \times 8(4-x)^7 \times -1$ ✓

$y' = 72(4-x)^7$ ✓

c) $y = \sqrt{x^2 - 3x}$

$y = (x^2-3x)^{\frac{1}{2}}$

$y' = \frac{1}{2}(x^2-3x)^{-\frac{1}{2}} \times (2x-3)$

must have some neg power AND correctly finished

$y' = \frac{2x-3}{2\sqrt{x^2-3x}}$ ✓

d) $y = \frac{3x^2}{1+x^3}$

$y' = \frac{6x(1+x^3) - 3x^2(3x^2)}{(1+x^3)^2}$ ✓

$y' = \frac{6x + 6x^4 - 9x^4}{(1+x^3)^2}$

$y' = \frac{6x - 3x^4}{(1+x^3)^2}$ ✓

$= \frac{3x(2-x^3)}{(1+x^3)^2}$

2) $y' = 3x^2 - 4x + 5$

$y = \frac{3x^3}{3} - \frac{4x^2}{2} + 5x + C$

$y = x^3 - 2x^2 + 5x + C$ ✓

$f(1) = 4 \Rightarrow$ when $x=1, y=4$

$4 = 1^3 - 2\times 1^2 + 5\times 1 + C$

$C = 0$ ✓

$\therefore y = x^3 - 2x^2 + 5x$ ✓

Q3 a) $f(x) = x^3 - 12x + 12$

$f'(x) = 3x^2 - 12$ ✓

$f''(x) = 6x$ ✓

b) st pts, $y' = 0$

$3x^2 - 12 = 0$

$x^2 = 4$

$x = \pm 2$ ✓

when $x = 2$: $f''(2) = 6\times 2 > 0$ $\therefore$ MIN ✓

$x = -2$: $f''(-2) = 6\times -2 < 0$ $\therefore$ MAX ✓

when $x=2$, $y = -4$

$x = -2$, $y = 28$

c) possible pts of inflex occur when $y'' = 0$

$6x = 0$

$x = 0$, $y = 12$ ✓

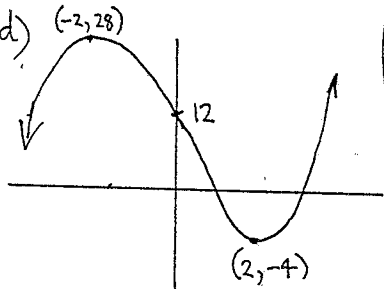
Test:

x	←	0	→
y''	$-$	0	$+$

✓

$\therefore$ because concavity changes sign $(0, 12)$ is a pt of inflex.

d)



must show all 3 points labelled on the graph for 2 marks

1 mark = graph with no markings.

e) $f'(x) = 3x^2 - 12$

when $x = -1$

m of tang $= 3\times(-1)^2 - 12$

$= -9$

need a point.

when $x = -1$

$y = (-1)^3 - 12\times(-1) + 12$

$y = 23$

$\therefore m = -9$, $(-1, 23)$

$y - 23 = -9(x+1)$

$y - 23 = -9x - 9$

$y = -9x + 14$

(or $9x + y - 14 = 0$)

1 mark for either correct tangent gradient or correct point.

2nd mark correctly substituting + simplify

Q4

$\frac{dh}{dt} > 0$ ✓

$\frac{d^2h}{dt^2} < 0$ ✓

her height was increasing at a decreasing rate ✓

Q5 a) Rectangle perim:

$= 2y + 4x$

Circular Bit: $\frac{1}{4} \times 2 \times \pi \times x$

$= \frac{\pi x}{2}$

$\therefore$ Total Fencing needed

$= 2y + 4x + \frac{\pi x}{2}$ ✓

one mark for either bit

NOTE: students must show how they got $\frac{\pi x}{2}$ because it was given to them in the question. (ie $\frac{2\pi x}{4}$ somewhere.)

Area $= 10000\text{m}^2 = y \times 2x$

$y = \frac{10000}{2x} = \frac{5000}{x}$

$\therefore L = 2 \times \frac{5000}{x} + 4x + \frac{\pi x}{2}$

$L = \frac{10000}{x} + 4x + \frac{\pi x}{2}$ ✓

b) $L = 10000x^{-1} + 4x + \frac{\pi x}{2}$

$\frac{dL}{dx} = -10000x^{-2} + 4 + \frac{\pi}{2}$ ✓

$\frac{d^2L}{dx^2} = 20000x^{-3}$

st pts: $4 + \frac{\pi}{2} - \frac{10000}{x^2} = 0$

$4x^2 + \frac{\pi}{2}x^2 - 10000 = 0$

$x^2(4 + \frac{\pi}{2}) = 10000$

$x^2 = \frac{10000}{4+\frac{\pi}{2}} = 1795.0755$

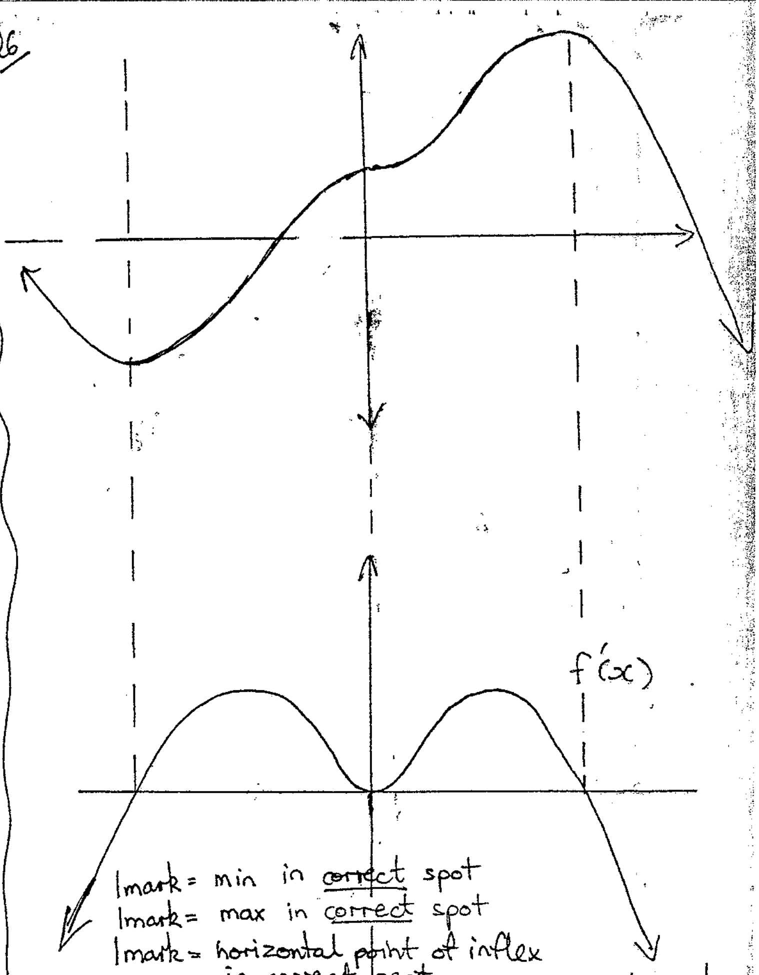
$x = 42.4\text{ m}$ ✓

Test: $L'' = \frac{20000}{x^3}$

$= \frac{20000}{(42.4)^3} > 0$ ✓

$\therefore$ min amount of fencing needed.

Q6



1 mark = min in correct spot

1 mark = max in correct spot

1 mark = horizontal point of inflex in correct spot.

NOTE: correct curve must be shown for full marks.

(Interpret curve as $y = f(x)$ and correctly draws $y = f'(x)$ gets 1 mark).