BRIGIDINE COLLEGE RANDWICK

Year 12 Mathematics — **Student** ____________

8 December 2009 — **Teacher** ____________

Time 40 Minutes

Show all necessary working.
Neatness may be taken into consideration in the awarding of marks.

There are 7 Questions.

1. Differentiate the following
(leaving answers completely simplified with positive indices)

   a. $3(2x^2 + 5)^6$ — 2 m

   b. $\dfrac{x^2}{2\sqrt{x}}$ — 2 m

2. Write down the equation of the tangent to the curve $y = \sqrt{x} + 1$ when $x = 4$ in general form. — 3 m

3. Given that $\dfrac{d^2y}{dx^2} = 6x$ and when $x = 2$, $\dfrac{dy}{dx} = 8$ and $y = 6$, find y in terms of x. — 3 m

4. Consider the curve given by $f(x) = x(x - 3)^2$.

   a. Determine its x and y intercepts. — 2 m

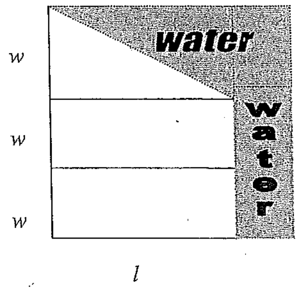
   b. Show that there exists stationary values at $x = 1$ and $x = 3$ and determine their nature. — 3 m

   c. Show there existence of a point of inflection. — 2 m

   d. Neatly sketch this f(x) in domain $-1 \leq x \leq 4$. — 2 m

continued on back

5. A farmer needs to separate his land into **3 sections.**
These sections are made from 2 equal rectangles and a triangle bounded by water (as shown to the right)



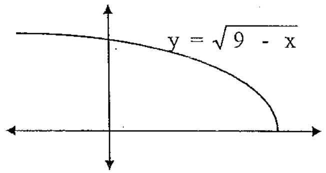
Each section is to have equal widths w and length l.

The amount of fencing available to this farmer is P metres.

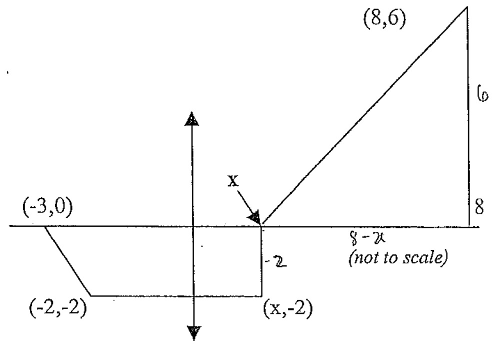
   a. Show that the Area of this field may be given by $A = \dfrac{5}{6}Pw - \dfrac{5}{2}w^2$. — 2 m

   b. Show that if this farmer is to maximize the area of each section $w = l$. — 4 m

6. The area trapped between the curve $y = \sqrt{9 - x}$ and the coordinate axes is rotated about the y-axis, determine the resultant volume. — 4 m



7. Consider this diagram to the right. — 3 m



The trapezium and triangle are traced out by the curve f(x).

If $\displaystyle\int_{-3}^{8} f(x)\,dx = 0$,

determine the value of **x**, leaving answer in exact form.

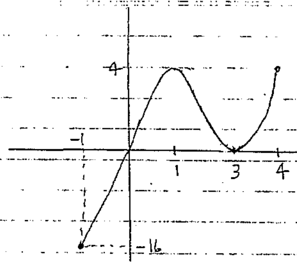
- end of exam -

Yr 12 Dec 2009 Task 1 (MATHEMATICS)

Solutions	Marking scheme
Q1 a) $y' = 3\times 6(2x^2+5)^5 \times 4x$	(2marks) $72x(2x^2+5)^5$
$= 72x(2x^2+5)^5$	(1mark) showing $6(2x^2+5)^5 \times 4x$ or $72(2x^2+5)^5$
b) $y = \frac{x^2}{2\sqrt{x}} = \frac{x^{\frac{3}{2}}}{2}$	2marks $= \frac{3\sqrt{x}}{4}$ or $\frac{3}{4}x^{\frac{1}{2}}$
$= \frac{1}{2}x^{\frac{3}{2}}$	(1mark) $y' = \frac{3x^{\frac{3}{2}}}{4x}$
$y' = \frac{1}{2}\times\frac{3}{2}x^{\frac{1}{2}}$	or $y = \frac{1}{2}x^{\frac{3}{2}}$ or 1mistake
$= \frac{3}{4}\sqrt{x}$	or showing $y = \Box x^{\frac{3}{2}}$ pre derivative,
$= \frac{3\sqrt{x}}{4}$	
Q2 $y = \sqrt{x}+1$ at $x=4$	
$\therefore$ pt $y = \sqrt{4}+1 = 3$	(3marks) $0 = x - 4y + 8$
pt $= (4,3)$	(2marks) $y = \frac{1}{4}x + 2$
$y' = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$	or either correct gradient or correct point and then finding general form.
$m = \frac{1}{2\sqrt{4}} = \frac{1}{4}$	(1mark) $m = \frac{1}{4}$ OR pt$(4,3)$
$\therefore y - 3 = \frac{1}{4}(x-4)$	
$y - 3 = \frac{1}{4}x - 1$	
$4y - 12 = x - 4$	
$0 = x - 4y + 8$	
Q3 $y'' = 6x$	
$y' = 3x^2 + C_1$	(3marks) $y = x^3 - 4x + 6$
$8 = 3\times 2^2 + C_1$	
$C_1 = -4$	(2marks) ~~getting one const wrong~~
$y' = 3x^2 - 4$	$y = x^3 - 4x + C_2$
$y = x^3 - 4x + C_2$	
$6 = 2^3 - 4\times 2 + C_2$	(1mark) finding $C_1 = -4$
$C_2 = 6$	
$y = x^3 - 4x + 6$	

Solutions	Marking scheme
Q4a when $x=0$, $y = 0(0-3)^2$	(2marks) y int $= 0$
$y = 0$	x int $= 0, 3$
when $y=0$, $0 = x(x-3)^2$	(1mark) correct y int or x int
$x = 0, 3$	
b) $f(x) = x(x-3)^2$	
$f'(x) = 1\times(x-3)^2 + 2(x-3)^1 \times x$	(3marks) correctly finding $x=1$, and $x=3$ and testing
$= (x-3)^2 + 2x(x-3)$	
st pts: $0 = (x-3)^2 + 2x(x-3)$	(2marks) correctly finding $x=1$ and $x=3$ and one mistake in the testing procedure.
$0 = (x-3)[(x-3)+2x]$	
$= (x-3)(3x-3)$	
$x = 3$, $x = 1$	(1mark) correctly finding $x=1$ and 3
when $x=3$ when $x=1$	
$y' = (x-3)^2 + 2x(x-3)$	
$= (x-3)[x-3+2x]$	
$= (x-3)(3x-3)$	
$y'' = 1\times(3x-3) + 3x(x-3)$	
$= 3x - 3 + 3x - 9$	
$= 6x - 12$	
when $x=3$ when $x=1$	
$y'' = 6\times 3 - 12 > 0$ $y'' = 6\times 1 - 12 < 0$	
$\therefore$ min $\therefore$ MAX	
c) $y'' = 6x - 12 = 0$	
$x = 2$, $y = 2$	
$\therefore$ pt of inflex at $(2,2)$	(1mark) find point
x: $\leftarrow$ 2 $\rightarrow$; y'': neg 0 pos	(1mark) TEST

d)



(2marks) see sketch, must show absolute max and local max stopping at $y=4$

(1mark) correct shape passing through origin.

Solutions	Marking scheme
Q5a) $A = 2wl + \frac{1}{2}\times w \times l$	
$= \frac{5}{2}wl$ ... ①	(1mark) statement for Perimeter and area
$P = 3w + 3l$	
$\frac{P}{3} = w + l$	
$l = \frac{P}{3} - w$ ... ②	(1mark) Link
sub in ① $A = \frac{5}{2}w(\frac{P}{3} - w)$	
$= \frac{5Pw}{6} - \frac{5w^2}{2}$	
b) $A = \frac{5Pw}{6} - \frac{5w^2}{2}$ (P = perim of fencing = const)	
$\frac{dA}{dw} = \frac{5P}{6} - \frac{10w}{2} = 0$	(1mark) attempt $\frac{dA}{dw}$
$5P - 30w = 0$	
$30w = 5P$	(1mark) A'
$w = \frac{P}{6}$	
$A'' = -\frac{10}{2} < 0$ $\therefore$ max area when $w = \frac{P}{6}$	(1mark) show max
~~so~~ $P = 6w$ sub in ②	
$l = \frac{6w}{3} - w$	
$l = 2w - w$	
$l = w$ as required	(1mark) show $l = w$

Solutions	Marking scheme
Q6 $y = \sqrt{9-x}$	(1mark)
$y^2 = 9 - x$	$V = \pi\int (\ )^2\, dy$
$x = 9 - y^2$	
$x^2 = (9-y^2)^2$	
$x^2 = 81 - 18y^2 + y^4$	
when $x=0$, $y = \sqrt{9-0} = 3$	
So $V = \pi\int_0^3 81 - 18y^2 + y^4\, dy$	(1mark) correct integral
$V = \pi\left[81y - 6y^3 + \frac{y^5}{5}\right]_0^3$	(1mark) integration
$= \pi\left[(81\times 3 - 6\times 3^3 + \frac{3^5}{5}) - (0)\right]$	
$= \cancel{237 \cdot 6\pi}$ or $\cancel{237\frac{3}{5}\pi}$ or $\cancel{\frac{1188\pi}{5}}$	(1mark)
$129 \cdot 6\pi$ or $129\frac{3}{5}\pi$ or $\frac{648\pi}{5}$	
Q7 $A_\Delta = \frac{1}{2}\times(8-x)\times 6$	(1mark) AREA = AREA
$A_{TRAP} = \frac{1}{2}\times 2\times((x+3)+(x+2))$	
$= 2x + 5$	Area TRAP $\therefore$ (1mark)
~~Since $\int_{-3}^{8} f(x) = 0$ then~~	Area TRAP
$A_\Delta = A_{TRAP}$	
$\frac{1}{2}\times(8-x)\times 6 = 2x + 5$	
$3(8-x) = 2x + 5$	
$24 - 3x = 2x + 5$	
$19 = 5x$	Answer (1mark)
$\frac{19}{5} = x$	
$\therefore x = 3.8$	