Done ✓

TEST 3 Congruence and Similarity

Marks: /80

Time: 1 hour 30 minutes

Name: .... Date: ....

INSTRUCTIONS TO CANDIDATES

Section A (40 marks) **Time: 45 minutes**

1. Answer all the questions in this section.
2. Calculators may not be used in this section.
3. All working must be clearly shown. Omission of essential working will result in loss of marks.
4. The marks for each question is shown in brackets [ ] at the end of each question.

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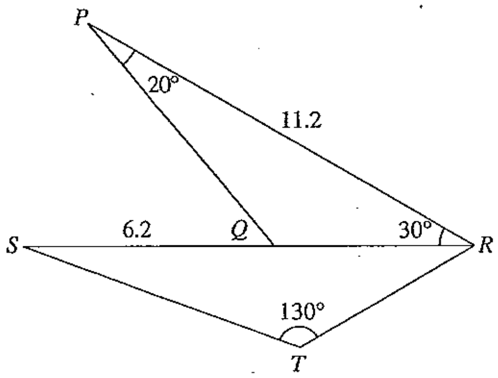
1. In the diagram, triangle PQR is congruent to triangle STR. $P\hat{R}Q = 30°$, $Q\hat{P}R = 20°$, $S\hat{T}R = 130°$, $PR = 11.2$ cm and $SQ = 6.2$ cm. Find the values of

(a) $S\hat{R}T$,

(b) TR.

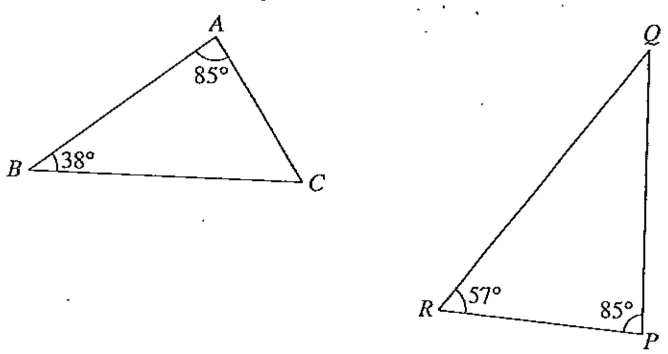


Answer *(a)* $S\hat{R}T =$ .... ° [1]

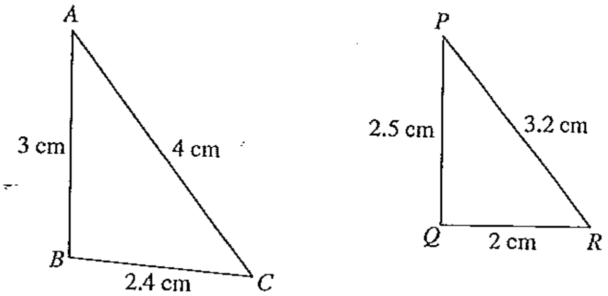
(b) $TR =$ .... cm [2]

2 (a) Are triangles ABC and PQR similar?

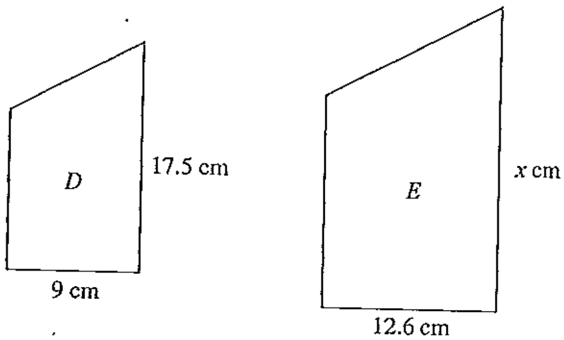
(i)



(ii)



(b) Figures D and E are similar. Find the value of x.



Answer *(a)* (i) .... [1]

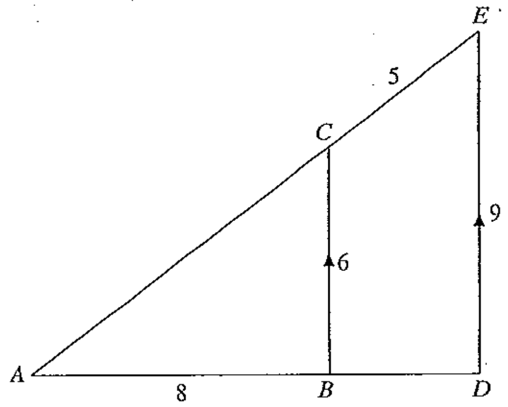
(ii) .... [1]

(b) $x =$ .... [2]

3 In the diagram, $AB = 8$ cm, $BC = 6$ cm, $DE = 9$ cm, $CE = 5$ cm and BC is parallel to DE. Find

(a) AC,

(b) BD.



Answer (a) $AC =$ .... cm [2]

(b) $BD =$ .... cm [2]

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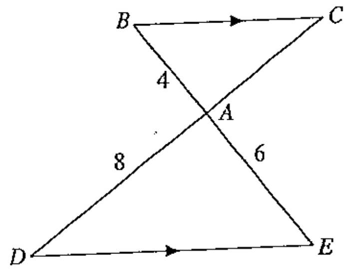
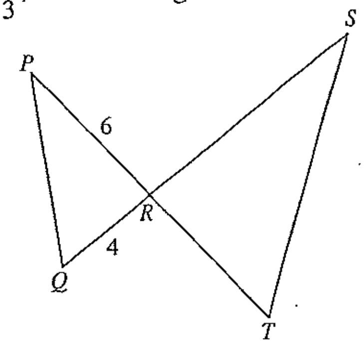
4 (a) In the diagram, BC is parallel to DE. $AB = 4$ cm, $AD = 8$ cm and $AE = 6$ cm. Calculate the length of AC.



(b) Triangle PQR is similar to triangle STR. Given that $\frac{PQ}{ST} = \frac{2}{3}$, find the length of RS.



Answer (a) $AC =$ .... cm [2]

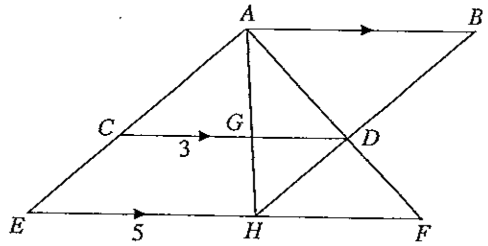
(b) $RS =$ .... cm [2]

---

5 In the diagram, AB is parallel to CD and EF. $CG = 3$ cm, $EH = 5$ cm and $ABHE$ is a parallelogram. Find the numerical value of

(a) $\frac{AG}{AH}$,

(b) $\frac{FH}{AB}$.



Answer (a) .... [1]

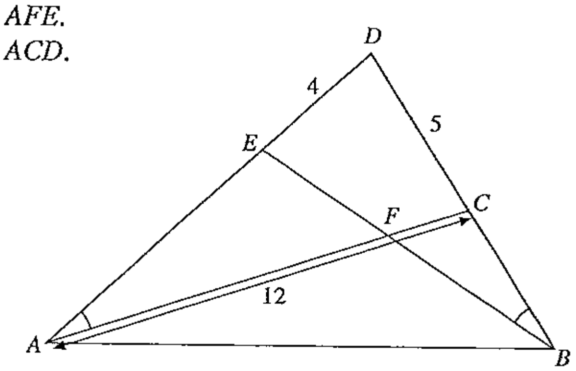
(b) .... [2]

6 In the diagram, $D\hat{A}C = E\hat{B}D$, $AC = 12$ cm, $CD = 5$ cm and $DE = 4$ cm.

(a) Name a triangle that is similar to triangle AFE.

(b) Name a triangle that is similar to triangle ACD.

(c) Calculate the length of BE.



Answer (a) Triangle .... [1]

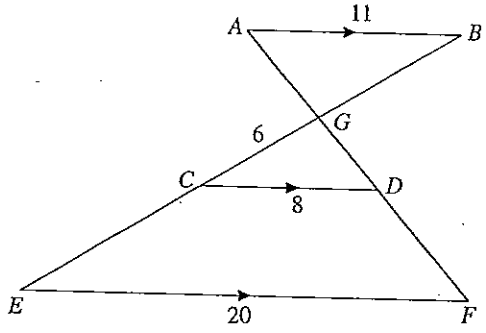
(b) Triangle .... [1]

(c) BE = ... cm [2]

7 In the diagram, AB, CD and EF are parallel. $AB = 11$ cm, $CD = 8$ cm, $EF = 20$ cm and $CG = 6$ cm. Calculate

(a) CE,

(b) BG.



Answer *(a)* CE = ... cm [2]

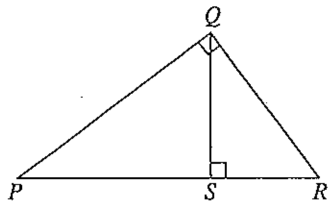
(b) BG = ... cm [2]

---

8 In the diagram, $P\hat{Q}R = Q\hat{S}R = 90°$.

(a) Write down an angle which is equal to $R\hat{P}Q$.

(b) Given that $PR = 20$ cm and $QR = 12$ cm, find the length of SR.



Answer *(a)* .... [1]

(b) SR = ... cm [2]

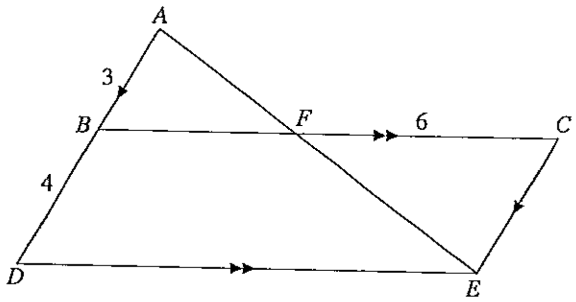
9 In the diagram, AD is parallel to CE and BC is parallel to DE. AFE and BFC are straight lines. $AB = 3$ cm, $BD = 4$ cm and $FC = 6$ cm.

(a) Write down a triangle that is similar to triangle CEF.

(b) Calculate

(i) BF,

(ii) DE.



10

Answer *(a)* Triangle........................ [1]

(b) (i) BF = ... cm [2]

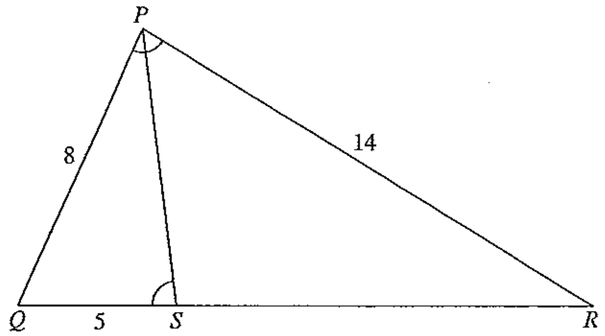
(ii) DE = ... cm [1]

10 (a) In the diagram, $Q\hat{P}R = Q\hat{S}P$, $PR = 14$ cm, $PQ = 8$ cm and $QS = 5$ cm.

(i) Name a pair of similar triangles.

(ii) Write down $\frac{QP}{QR}$ as a fraction.

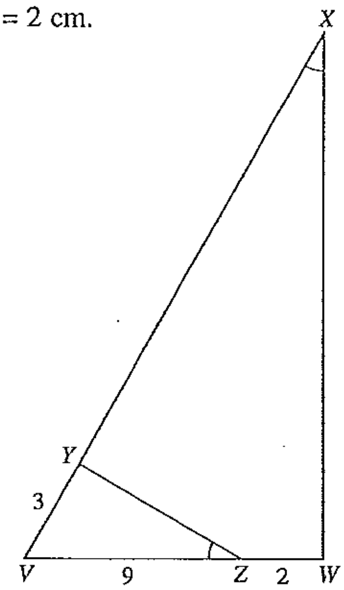
(iii) Find the length of PS.



(b) In the diagram, $V\hat{X}W = V\hat{Z}Y$, $VY = 3$ cm, $VZ = 9$ cm and $ZW = 2$ cm.

(i) Name a pair of similar triangles.

(ii) Calculate the length of XY.



Answer *(a)* (i) .... [1]

(ii) .... [1]

(iii) PS = ... cm [2]

(b) (i) .... [1]

(ii) XY = ... cm [2]

INSTRUCTIONS TO CANDIDATES

Section B (40 marks)

Time: 45 minutes

1. Answer **all** the questions in this section.
2. Calculators may be used in this section.
3. All working must be clearly shown. Omission of essential working will result in loss of marks.
4. The marks for each question is shown in brackets [ ] at the end of each question.

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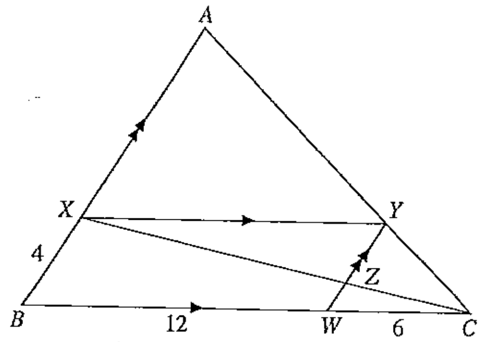
11 In triangle *ABC*, *XY* is parallel to *BC*, *BA* is parallel to *WY* and *XC* meets *WY* at *Z*. *BX* = 4 cm, *BW* = 12 cm and *WC* = 6 cm.

(a) Name two triangles that are each similar to triangle *CWZ*.

(b) Using similar triangles, calculate

(i) *WZ*,

(ii) *AX*.



Answer (a) .... [2]

(b) (i) *WZ* = ... cm [2]

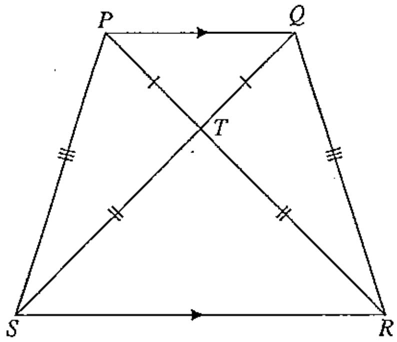
(ii) *AX* = ... cm [3]

12 (a) The diagram shows a quadrilateral $PQRS$ with diagonals PR and QS intersecting at T. $PT = QT$, $ST = RT$, $PS = QR$ and PQ is parallel to SR.

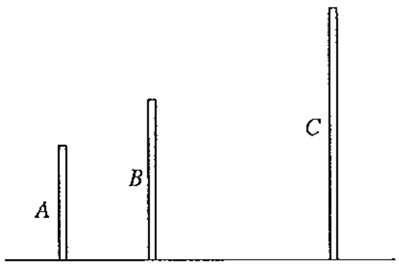
(i) Name a triangle that is congruent to triangle PSQ.

(ii) Name a triangle that is similar to triangle PTQ.

(iii) Given that $PQ = 6$ cm, $PT = 4$ cm and $QS = 13$ cm, find the length of SR.



(b) Three vertical poles A, B and C are shown in the diagram. The heights of poles A and B are 5 m and 7 m respectively. Pole A is 3 m from Pole B and 9 m from Pole C. The top of the three poles are in a straight line. Find the height of Pole C.



Answer (a) (i) Triangle .... [1]

(ii) Triangle .... [1]

(iii) SR = ... cm [3]

(b) .... m [4]

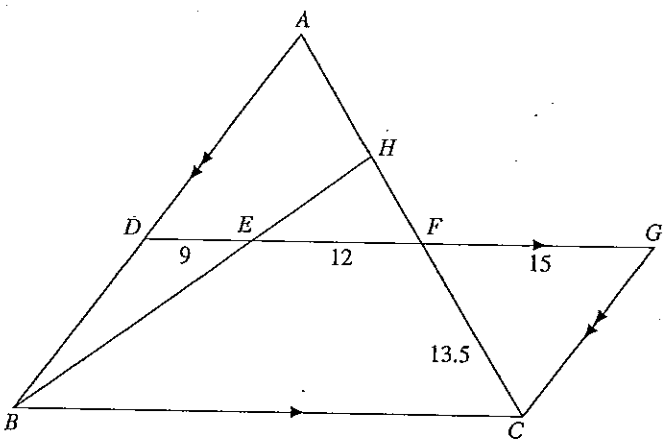
13 In the diagram, DG is parallel to BC and AB is parallel to GC. ADB, $AHFC$ and $DEFG$ are straight lines. $DE = 9$ cm, $EF = 12$ cm, $FG = 15$ cm and $FC = 13.5$ cm.

(a) **(i)** Name a triangle that is similar to triangle CFG.

(ii) Calculate the length of AF.

(b) **(i)** Name a triangle that is similar to triangle HEF.

(ii) Calculate the length of HF.



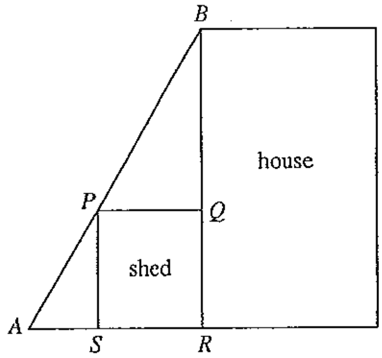
Answer (a) (i) Triangle .... [1]

(ii) AF = ... cm [2]

(b) (i) Triangle .... [1]

(ii) HF = ... cm [3]

14 **(a)** The diagram shows a ladder, AB resting against the roof of the shed $PQRS$ at P. The top of the ladder, B reaches a height of 6.4 m on the wall of the house. The foot of the ladder, A is 3.5 m from the wall of the house. Given that PQ is 2.1 m, find the height of the shed.

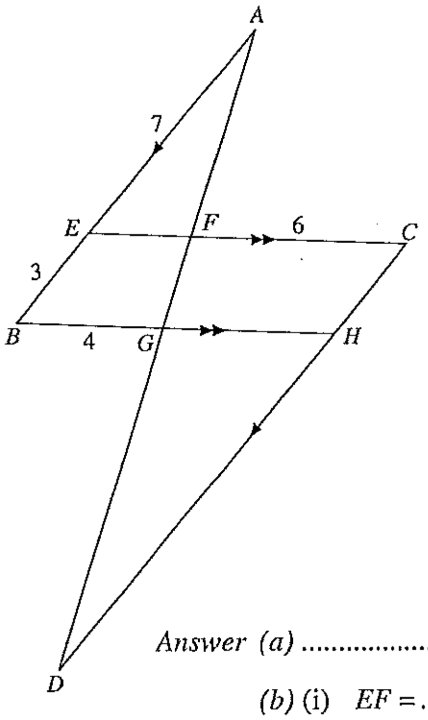


(b) In the diagram, AB is parallel to CD and EC is parallel to BH. $AFGD$, EFC and BGH are straight lines. Given that $AE = 7$ cm, $EB = 3$ cm, $BG = 4$ cm and $FC = 6$ cm, calculate

(i) EF,

(ii) GH,

(iii) DH.



Answer (a) .... m [4]

(b) (i) EF = ... cm [2]

(ii) GH = ... cm [2]

(iii) DH = ... cm [2]

15 In the diagram, AF, BC and DE are parallel. ABD, ACE and DCF are straight lines. $AD = 10$ cm, $BC = 5$ cm and $DE = 8$ cm.

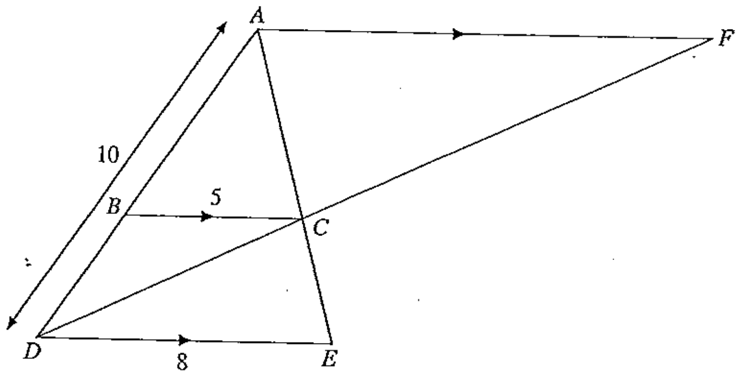
(a) Write down a triangle that is similar to triangle ADE.

(b) Calculate

(i) BD,

(ii) AF.

(c) Write down the numerical value of $\frac{DC}{CF}$.



Answer (a) Triangle .... [1]

(b) (i) $BD =$ .... cm [3]

(ii) $AF =$ .... cm [2]

(c) .... [1]

Total length of wire needed
= Circumference of smaller circle + Circumference of larger circle

Circumference of a circle $= 2\pi r$ where r = radius.

$= 2\pi r_1 + 2\pi r_2$
$= 2\pi(1.396 \times 10^{-1}) + 2\pi(4.188 \times 10^{1})$
$\approx$ **2.64×10^{2} cm** (correct to 3 sig. fig.)

12. (a) (i) Total monthly salary
$= \$735\ 000$ million
$= \$(7.35 \times 10^{5} \times 10^{6})$
$= \$(7.35 \times 10^{5+6})$
$= \$(7.35 \times 10^{11})$

1 million = 1 000 000 $= 10^6$
Use $10^m \times 10^n = 10^{m+n}$

(ii) Average monthly salary per worker
$= \dfrac{\$(7.35 \times 10^{11})}{2.8 \times 10^{8}}$
= **\$2625**

(b) Volume of block
$= (3 \times 10^{-2}) \times (5 \times 10^{-2}) \times (8 \times 10^{-2})$
$=$ **1.2×10^{-4} cm^3**

100 cm = 1 m
$1 \text{ cm} = \dfrac{1}{100} = \dfrac{1}{10^2} = 10^{-2}$ m

(c) $A = \sqrt{\dfrac{2pq}{r^3}}$
$= \sqrt{\dfrac{2 \times 6.25 \times 10^{-8} \times 8.14 \times 10^{-5}}{(3.91 \times 10^{-7})^3}}$
$\approx$ **1.30×10^{4}** (correct to 3 sig. fig.)

13. (a) Diameter = 1 392 000
$=$ **1.392×10^{6} km**

(b) Mass of Earth : Mass of Sun = 1 : n
$\dfrac{5.976 \times 10^{24} \text{ kg}}{1.99 \times 10^{30} \text{ kg}} = \dfrac{1}{n}$
$n = \dfrac{1.99 \times 10^{30}}{5.976 \times 10^{24}}$
$\approx 333\ 000$ (correct to 3 sig. fig.)
$\therefore$ required ratio = **1 : 333 000.**

(c) In one hour, the moon travels 36 800 km. Given
Distance travelled by the moon in one week
$= 168 \times 36\ 800$
$= 6\ 182\ 400$
$\approx$ **6.18×10^{6} km** (correct to 3 sig. fig.)

1 week = 7 days $= 7 \times 24$ h = 168 h

14. (a) $10^a = 2$, $10^b = 3$, $10^c = 5$ Given
$10^{2a+b-3c}$
$= 10^{2a} \times 10^{b} \div 10^{3c}$

Use $10^{m+n} = 10^m \times 10^n$ and $10^{m-n} = 10^m \div 10^n$

$= (10^a)^2 \times 10^b \div (10^c)^3$

Use $10^{mn} = (10^m)^n$.

$= \dfrac{2^2 \times 3}{5^3}$
$= 0.096$
$=$ **9.6×10^{-2}**

(b) $8093.02 = 8000 + 90 + 3 + \dfrac{2}{100}$
$= 8 \times 10^3 + 9 \times 10^1 + 3 \times 10^0 + 2 \times 10^{-2}$
$= 8 \times 10^3 + 9 \times 10^x + 3 \times 10^y + 2 \times 10^z$
$\therefore$ **$x = 1$, $y = 0$ and $z = -2$.**

(c) (i) Breadth of rectangle
$= \dfrac{5.32 \times 10^{-6} \text{ m}^2}{2.8 \times 10^{-3} \text{ m}}$
$=$ **1.9×10^{-3} m**

Area of rectangle = Length × Breadth
$\text{Breadth} = \dfrac{\text{Area}}{\text{Length}}$

(ii) Perimeter of rectangle
$= 2(2.8 \times 10^{-3} + 1.9 \times 10^{-3})$
$=$ **9.4×10^{-3} m**

Perimeter of rectangle = 2 (Length + Breadth)

15. Speed of light $= 3 \times 10^5$ km/s
Speed of sound $= 1.226 \times 10^3$ km/h } Given

(a) Time taken $= \dfrac{150 \text{ km}}{3 \times 10^5 \text{ km/s}}$
$=$ **5×10^{-4} s**

$\text{Time} = \dfrac{\text{Distance}}{\text{Speed}}$

(b) In 1 hour, (3600 s) sound travels 1.226×10^3 km.
In 1 second, sound travels
$= \dfrac{1.226 \times 10^3}{3600}$
≈ 0.3406
$=$ **3.41×10^{-1} km** (correct to 3 sig. fig.)

1 h = 60 min $= 60 \times 60$ s = 3600 s

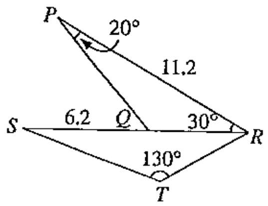
(c) Time taken
$= \dfrac{150 \text{ km}}{3.406 \times 10^{-1} \text{ km/s}}$
$\approx$ **4.40×10^{2} s** (correct to 3 sig. fig.)

$\text{Time} = \dfrac{\text{Distance}}{\text{Speed}}$
Speed of sound $= 3.406 \times 10^{-1}$ km/s

Test 3: Congruence and Similarity

Section A

1.



(a) $S\hat{R}T = P\hat{R}Q$
= **30°**

Teacher's Tip

$\triangle PQR \equiv \triangle STR$ (Given)

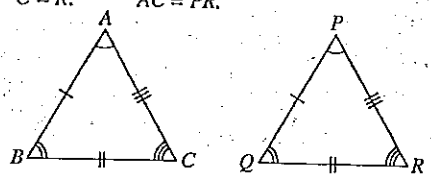
- Use a matching diagram to match the corresponding vertices.
- The symbol '≡' means 'is congruent to'.

(b) $SR = PR = 11.2$ cm
$QR = SR - SQ$
$= 11.2 - 6.2$
$= 5$
$TR = QR = 5$ **cm**

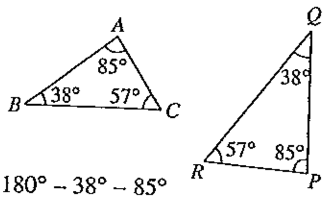
Teacher's Tip

Congruent Triangles

1. Congruent triangles have exactly the same size and shape.
2. If two triangles are congruent, then
   (a) their corresponding angles are equal,
   (b) their corresponding sides are equal.
3. If $\triangle ABC \equiv \triangle PQR$, then
   $\hat{A} = \hat{P}$, $AB = PQ$,
   $\hat{B} = \hat{Q}$, $BC = QR$,
   $\hat{C} = \hat{R}$. $AC = PR$.



2. (a) (i)



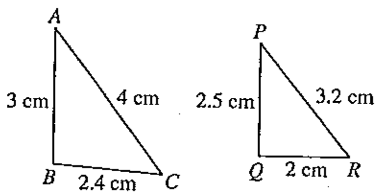
$\hat{C} = 180° - 38° - 85°$
$= 57°$
$\hat{P} = 180° - 57° - 85°$
$= 38°$

Since $\triangle ABC$ and $\triangle PQR$ have all their corresponding angles equal to each other, **$\triangle ABC$ and $\triangle PQR$ are similar.**

Teacher's Tip

If two triangles are similar, then the corresponding angles of the two triangles must be equal.

(ii)



$\frac{AB}{PQ} = \frac{3}{2.5} = 1.2$

$\frac{BC}{QR} = \frac{2.4}{2} = 1.2$

$\frac{AC}{PR} = \frac{4}{3.2} = 1.25$

$\frac{AB}{PQ} = \frac{BC}{QR} \neq \frac{AC}{PR}$

$\therefore$ **$\triangle ABC$ is not similar to $\triangle PQR$.**

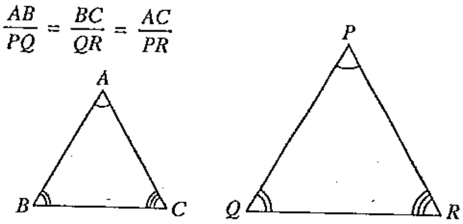
Teacher's Tip

If two triangles are similar, then the ratio of their corresponding sides of the two triangles must be equal.

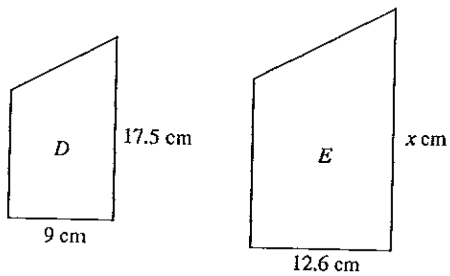
Teacher's Tip

Similar Triangles

1. Similar triangles have the same shape but may vary in size.
2. If two triangles are similar, then
   (a) their corresponding angles are equal,
   (b) their corresponding sides are in the same ratio.
3. If $\triangle ABC$ is similar to $\triangle PQR$, then
   $\hat{A} = \hat{P}$
   $\hat{B} = \hat{Q}$,
   $\hat{C} = \hat{R}$.
   $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$



(b)

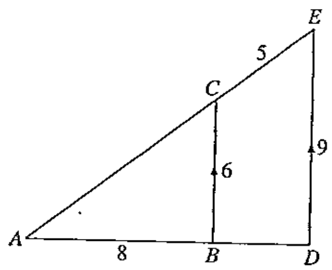


Since D and E are similar,

$\frac{x}{17.5} = \frac{12.6}{9}$ Ratios of corresponding sides are equal.

$x = \frac{12.6}{9} \times 17.5$
$= 24.5$

3.



Teacher's Tip

$\triangle ABC$ is similar to $\triangle ADE$ since
$B\hat{A}C = D\hat{A}E$ ($\hat{A}$ is common.),
$A\hat{B}C = A\hat{D}E$ (corr. $\angle$s, $BC \parallel DE$),
$A\hat{C}B = A\hat{E}D$ (corr. $\angle$s, $BC \parallel DE$).

(a) $\triangle ABC$ is similar to $\triangle ADE$.

$\therefore \frac{AC}{AE} = \frac{BC}{DE}$ Ratios of corresponding sides are equal.

$\frac{AC}{AC+5} = \frac{\cancel{6}^{2}}{\cancel{9}^{3}}$

$3AC = 2(AC+5)$

$3AC = 2AC + 10$

$AC = 10$ cm

(b) $\frac{AD}{AB} = \frac{DE}{BC}$ Ratios of corresponding sides are equal.

$\frac{AD}{8} = \frac{9}{6}$

$AD = \frac{9}{6} \times 8$

$= 12$

$BD = AD - AB$

$= 12 - 8$

$= 4$ cm

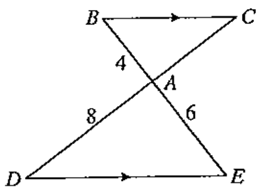
4. (a) **Teacher's Tip**

$\triangle ABC$ is similar to $\triangle AED$ since

$B\hat{A}C = E\hat{A}D$ (vert. opp. $\angle$s),

$A\hat{B}C = A\hat{E}D$ (alt. $\angle$, $BC \parallel DE$),

$A\hat{C}B = A\hat{D}E$ (alt. $\angle$s, $BC \parallel DE$).



$\triangle ABC$ is similar to $\triangle AED$.

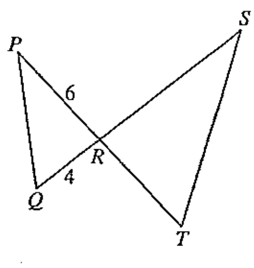
$\therefore \frac{AC}{AD} = \frac{AB}{AE}$ Ratios of corresponding sides are equal.

$\frac{AC}{8} = \frac{4}{6}$

$AC = \frac{4}{6} \times 8$

$= 5\frac{1}{3}$ cm

(b)



Since $\triangle PQR$ is similar to $\triangle STR$,

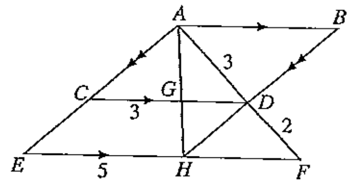
$\frac{RS}{RP} = \frac{ST}{PQ}$ Ratios of corresponding sides are equal.

$\frac{RS}{6} = \frac{3}{2}$ Given $\frac{PQ}{ST} = \frac{2}{3}$

$RS = \frac{3}{2} \times 6$ $\therefore \frac{ST}{PQ} = \frac{3}{2}$

$= 9$ cm

5.



(a) $\triangle ACG$ is similar to $\triangle AEH$.

$\therefore \frac{AG}{AH} = \frac{CG}{EH}$

$= \frac{3}{5}$

(b) $\triangle AGD$ is similar to $\triangle AHF$.

$\therefore \frac{AD}{AF} = \frac{AG}{AH}$

$= \frac{3}{5}$

$\triangle DHF$ is similar to $\triangle DBA$.

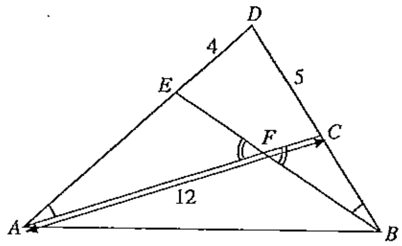
$\frac{FH}{AB} = \frac{DF}{DA}$

$= \frac{AF - AD}{DA}$

$= \frac{5-3}{3}$

$= \frac{2}{3}$

6.



(a) **$\triangle AFE$ is similar to $\triangle BFC$.**

$C\hat{B}F = E\hat{A}F$ (Given)

$B\hat{F}C = A\hat{F}E$ (vert. opp. $\angle$s)

$\therefore A\hat{E}F = B\hat{C}F$

$\therefore \triangle AFE$ is similar to $\triangle BFC$.

(b) **$\triangle ACD$ is similar to $\triangle BED$.**

$E\hat{B}D = C\hat{A}D$ (Given)

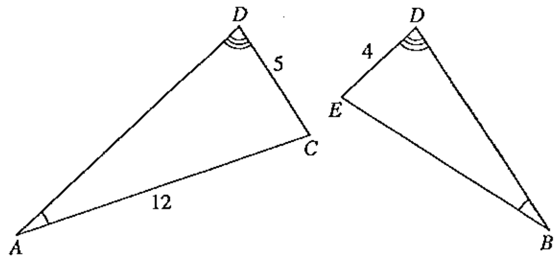
$B\hat{D}E = A\hat{D}C$ ($\hat{D}$ is common.)

$\therefore B\hat{E}D = A\hat{C}D$

$\therefore \triangle ACD$ is similar to $\triangle BED$.

(c) **Teacher's Tip**

If necessary, redraw the two similar triangles ACD and BED to help you visualize.



Since $\triangle ACD$ is similar to $\triangle BED$,

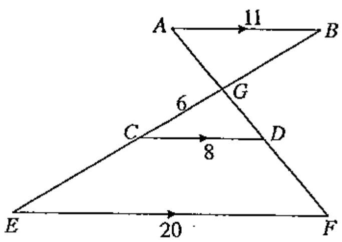
$$\frac{BE}{AC} = \frac{DE}{DC}$$

$$\frac{BE}{12} = \frac{4}{5}$$

$$BE = \frac{4}{5} \times 12$$

$$= 9\frac{3}{5} \text{ cm}$$

7.



(a) $\triangle GCD$ is similar to $\triangle GEF$.

$$\therefore \frac{GE}{GC} = \frac{EF}{CD}$$

$$\frac{GE}{6} = \frac{20}{8}$$

$$GE = \frac{20}{8} \times 6$$

$$= 15 \text{ cm}$$

$$CE = 15 - 6$$

$$= 9 \text{ cm}$$

(b) $\triangle AGB$ is similar to $\triangle DGC$.

$$\therefore \frac{BG}{CG} = \frac{AB}{DC}$$

$$\frac{BG}{6} = \frac{11}{8}$$

$$BG = \frac{11}{8} \times 6$$

$$= 8\frac{1}{4} \text{ cm}$$

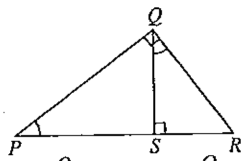
8. **Teacher's Tip**

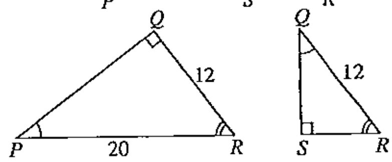
Split $\triangle PQR$ into two similar triangles, $\triangle PQR$ and $\triangle QSR$.

$P\hat{Q}R = Q\hat{S}R = 90°$ (Given)

$P\hat{R}Q = Q\hat{R}S$ ($\hat{R}$ is common.)

$\therefore R\hat{P}Q = R\hat{Q}S$





(a) $R\hat{P}Q = R\hat{Q}S$

(b) $\triangle RPQ$ is similar to $\triangle RQS$.

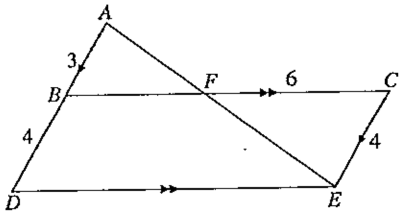
$$\frac{SR}{QR} = \frac{QR}{PR}$$

$$\frac{SR}{12} = \frac{12}{20}$$

$$SR = \frac{12}{20} \times 12$$

$$= 7\frac{1}{5} \text{ cm}$$

9.



(a) $\triangle CEF$ is similar to $\triangle BAF$.

$A\hat{F}B = E\hat{F}C$ (vert. opp. ∠s)

$A\hat{B}F = E\hat{C}F$ (alt. ∠s, $AB \parallel CE$)

$B\hat{A}F = C\hat{E}F$ (alt. ∠s, $AB \parallel CE$)

$\therefore \triangle CEF$ is similar to $\triangle BAF$.

(b) (i) $BDEC$ is a parallelogram since $BD \parallel CE$ and $BC \parallel DE$.

$CE = BD = 4$ cm

Since $\triangle CEF$ is similar to $\triangle BAF$,

$$\frac{BF}{CF} = \frac{AB}{EC}$$

$$\frac{BF}{6} = \frac{3}{4}$$

$$BF = \frac{3}{4} \times 6$$

$$= 4\frac{1}{2} \text{ cm}$$

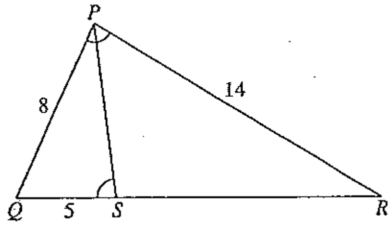
(ii) $DE = BC$ since $BDEC$ is a parallelogram.

$$= BF + FC$$

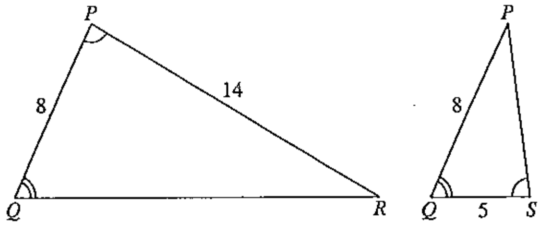
$$= 4\frac{1}{2} + 6$$

$$= 10\frac{1}{2} \text{ cm}$$

10. (a)



(i) **$\triangle PQR$ and $\triangle SQP$ are similar.**



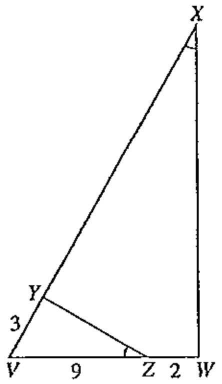
$Q\hat{P}R = Q\hat{S}P$ (Given)
$P\hat{Q}R = S\hat{Q}P$ ($\hat{Q}$ is common.)
$\therefore P\hat{R}Q = S\hat{P}Q$
$\therefore \triangle PQR$ is similar to $\triangle SQP$.

(ii) Since $\triangle PQR$ is similar to $\triangle SQP$,

$$\frac{QP}{QR} = \frac{QS}{QP}$$
$$= \frac{5}{8}$$

(iii) $$\frac{PS}{PR} = \frac{QS}{QP}$$
$$\frac{PS}{14} = \frac{5}{8}$$
$$PS = \frac{5}{8} \times 14$$
$$= 8\frac{3}{4} \text{ cm}$$

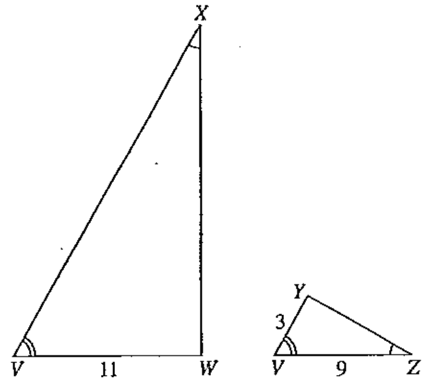
(b)



(i) **$\triangle VXW$ and $\triangle VZY$ are similar.**

$V\hat{X}W = V\hat{Z}Y$ (Given)
$X\hat{V}W = Z\hat{V}Y$ ($\hat{V}$ is common.)
$\therefore V\hat{W}X = V\hat{Y}Z$
$\therefore \triangle VXW$ is similar to $\triangle VZY$.

(ii)

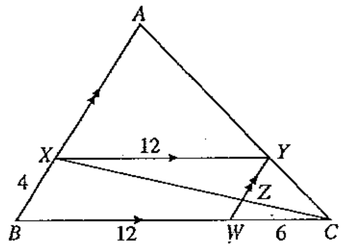


Since $\triangle VXW$ is similar to $\triangle VZY$,

$$\frac{VX}{VZ} = \frac{VW}{VY}$$
$$\frac{VX}{9} = \frac{11}{3}$$
$$VX = \frac{11}{3} \times 9$$
$$= 33 \text{ cm}$$
$$XY = VX - VY$$
$$= 33 - 3$$
$$= 30 \text{ cm}$$

Section B

11.



(a) **$\triangle CBX$ and $\triangle XYZ$.**

$\triangle CBX$ is similar to $\triangle CWZ$.
$W\hat{C}Z = B\hat{C}X$ ($\hat{C}$ is common.)
$C\hat{W}Z = C\hat{B}X$ (corr. $\angle$s, $WZ // BX$)
$C\hat{Z}W = C\hat{X}B$ (corr. $\angle$s, $WZ // BX$)

$\triangle XYZ$ is similar to $\triangle CWZ$.
$C\hat{Z}W = X\hat{Z}Y$ (vert. opp. $\angle$s)
$W\hat{C}Z = Y\hat{X}Z$ (alt. $\angle$s, $XY // WC$)
$Z\hat{C}W = Z\hat{X}Y$ (alt. $\angle$s, $XY // WC$)

Teacher's Tip
Since $\triangle CBX$ is similar to $\triangle CWZ$ and $\triangle CWZ$ is similar to $\triangle XYZ$,
$\therefore \triangle CBX$ and $\triangle XYZ$ are similar to $\triangle CWZ$.

(b) (i) Since $\triangle CWZ$ is similar to $\triangle CBX$,

$$\frac{WZ}{BX} = \frac{CW}{CB}$$

$$\frac{WZ}{4} = \frac{6}{6+12}$$

$$WZ = \frac{6}{18} \times 4$$

$$= 1\frac{1}{3} \text{ cm}$$

(ii) $\triangle AXY$ is similar to $\triangle ABC$.

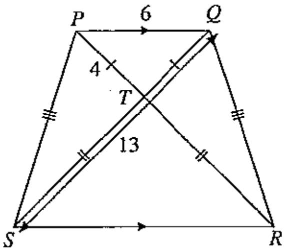
$$\therefore \frac{AX}{AB} = \frac{XY}{BC}$$

$$\frac{AX}{AX+4} = \frac{\cancel{12}^{2}}{\cancel{18}_{3}}$$

$3AX = 2(AX + 4)$
$3AX = 2AX + 8$
$AX = 8$ cm

$XY = BW = 12$ cm since $BWYX$ is a parallelogram.

12. (a)



(i) $\triangle PSQ \equiv \triangle QRP$

$PS = QR$ (Given)
$SQ = RP$ (Given)
$PQ = QP$
All the corresponding sides of both triangles are equal.
$\therefore \triangle PSQ \equiv \triangle QRP$

(ii) **$\triangle PTQ$ is similar to $\triangle RTS$.**

$P\hat{T}Q = R\hat{T}S$ (vert. opp. $\angle$s)
$P\hat{Q}T = R\hat{S}T$ (alt. $\angle$s, $PQ \parallel SR$)
$Q\hat{P}T = S\hat{R}T$ (alt. $\angle$s, $PQ \parallel SR$)
$\therefore \triangle PTQ$ is similar to $\triangle RTS$.

(iii) $QT = PT = 4$ cm
$ST = 13 - 4 = 9$ cm
Since $\triangle PTQ$ is similar to $\triangle RTS$,

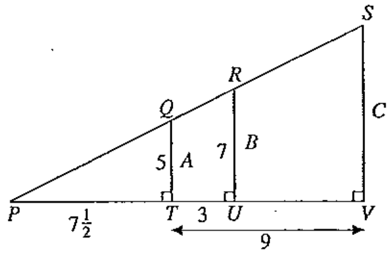
$$\frac{SR}{QP} = \frac{ST}{QT}$$

$$\frac{SR}{6} = \frac{9}{4}$$

$$SR = \frac{9}{4} \times 6$$

$$= 13\frac{1}{2} \text{ cm}$$

(b)



Draw a straight line joining the tops of the 3 poles and extend this line to the horizontal ground.

$\triangle PQT$ is similar to $\triangle PRU$.

$$\frac{PT}{PU} = \frac{QT}{RU}$$

$$\frac{PT}{PT+3} = \frac{5}{7}$$

$7PT = 5(PT + 3)$
$7PT = 5PT + 15$
$2PT = 15$

$$PT = \frac{15}{2} = 7\frac{1}{2} \text{ m}$$

$\triangle PQT$ is similar to $\triangle PSV$.

$$\frac{SV}{QT} = \frac{PV}{PT}$$

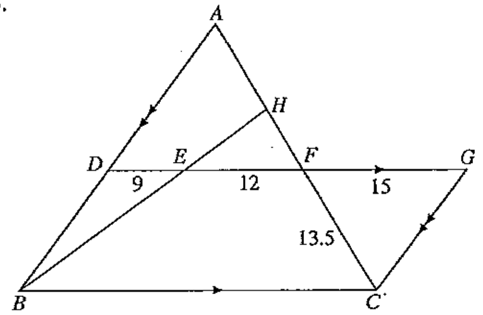
$$\frac{SV}{5} = \frac{7\frac{1}{2} + 9}{7\frac{1}{2}}$$

$$SV = \frac{16\frac{1}{2}}{7\frac{1}{2}} \times 5$$

$= 11$ m

$\therefore$ the height of Pole C is **11 m.**

13.



(a) (i) **$\triangle CFG$ is similar to $\triangle AFD$.**

$C\hat{F}G = A\hat{F}D$ (vert. opp. $\angle$s)
$F\hat{C}G = F\hat{A}D$ (alt. $\angle$s, $GC \parallel AD$)
$F\hat{G}C = F\hat{D}A$ (alt. $\angle$s, $GC \parallel AD$)
$\therefore \triangle CFG$ is similar to $\triangle AFD$.

(ii) Since $\triangle CFG$ is similar to $\triangle AFD$,

$$\frac{AF}{CF} = \frac{DF}{GF}$$

$$\frac{AF}{13.5} = \frac{9+12}{15}$$

$$AF = \frac{21}{15} \times 13.5$$

$$= \mathbf{18.9\ cm}$$

(b) (i) **$\triangle HEF$ is similar to $\triangle HBC$.**

$E\hat{H}F = B\hat{H}C$ ($\hat{H}$ is common.)
$H\hat{E}F = H\hat{B}C$ (corr. $\angle$s, $EF \parallel BC$)
$H\hat{F}E = H\hat{C}B$ (corr. $\angle$s, $EF \parallel BC$)
$\therefore \triangle HEF$ is similar to $\triangle HBC$.

(ii) Since $\triangle HEF$ is similar to $\triangle HBC$,

$$\frac{HF}{HC} = \frac{EF}{BC}$$

$$\frac{HF}{HF + 13.5} = \frac{12^{1}}{36_{3}}$$

$BC = DG$
$= 9 + 12 + 15$
$= 36$ cm

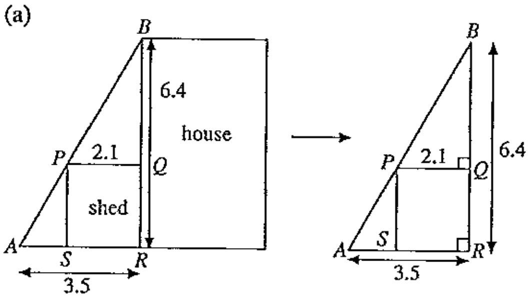
$$3HF = HF + 13.5$$

$$2HF = 13.5$$

$$HF = \frac{13.5}{2}$$

$$= \mathbf{6.75\ cm}$$

14. (a)



$\triangle BPQ$ is similar to $\triangle BAR$.

$$\therefore \frac{BQ}{BR} = \frac{PQ}{AR}$$

$P\hat{B}Q = A\hat{B}R$ ($\hat{B}$ is common.)
$B\hat{P}Q = B\hat{A}R$ (corr. $\angle$s, $PQ \parallel AR$)
$B\hat{Q}P = B\hat{R}A$ (corr. $\angle$s, $PQ \parallel AR$)
$\therefore \triangle BPQ$ is similar to $\triangle BAR$.

$$\frac{BQ}{6.4} = \frac{2.1}{3.5}$$

$$BQ = \frac{2.1}{3.5} \times 6.4$$

$$= 3.84 \text{ m}$$

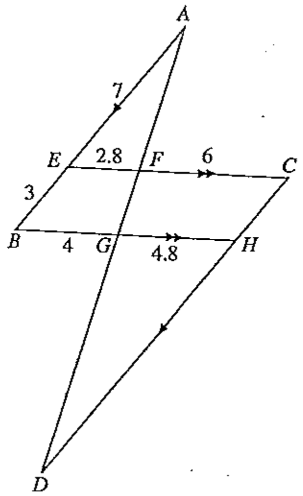
$$QR = BR - BQ$$

$$= 6.4 - 3.84$$

$$= 2.56 \text{ m}$$

$\therefore$ the height of the shed is **2.56 m.**

(b)



(i) $\triangle AEF$ is similar to $\triangle ABG$.

$$\therefore \frac{EF}{BG} = \frac{AE}{AB}$$

$$\frac{EF}{4} = \frac{7}{7+3}$$

$$EF = \frac{7}{10} \times 4$$

$$= \mathbf{2.8\ cm}$$

(ii) $BH = EC$ Since $BHCE$ is a parallelogram.

$$= 2.8 + 6$$

$$= 8.8 \text{ cm}$$

$$GH = BH - BG$$

$$= 8.8 - 4$$

$$= \mathbf{4.8\ cm}$$

(iii) $\triangle ABG$ is similar to $\triangle DHG$.

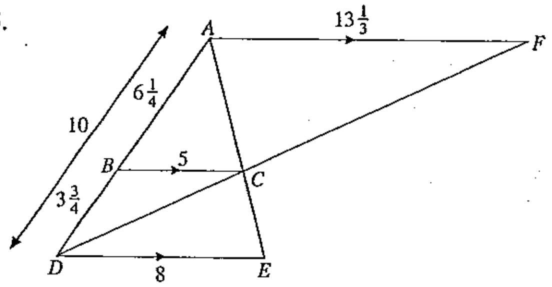
$$\therefore \frac{DH}{AB} = \frac{GH}{GB}$$

$$\frac{DH}{7+3} = \frac{4.8}{4}$$

$$DH = \frac{4.8}{4} \times 10$$

$$= \mathbf{12\ cm}$$

15.



(a) **$\triangle ADE$ is similar to $\triangle ABC$.**

$B\hat{A}C = D\hat{A}E$ ($\hat{A}$ is common.)
$A\hat{B}C = A\hat{D}E$ (corr. $\angle$s, $BC \parallel DE$)
$A\hat{C}B = A\hat{E}D$ (corr. $\angle$s, $BC \parallel DE$)
$\therefore \triangle ABC$ is similar to $\triangle ADE$.

(b) (i) Since $\triangle ABC$ is similar to $\triangle ADE$,

$$\frac{AB}{AD} = \frac{BC}{DE}$$

$$\frac{AB}{10} = \frac{5}{8}$$

$$AB = \frac{5}{8} \times 10$$

$$= 6\frac{1}{4} \text{ cm}$$

$$BD = AD - AB$$

$$= 10 - 6\frac{1}{4}$$

$$= \mathbf{3\frac{3}{4} \text{ cm}}$$

(ii) $\triangle DBC$ is similar to $\triangle DAF$.

$$\therefore \frac{AF}{BC} = \frac{DA}{DB}$$

$$\frac{AF}{5} = \frac{10}{3\frac{3}{4}}$$

$$AF = \frac{10}{3\frac{3}{4}} \times 5$$

$$= \mathbf{13\frac{1}{3} \text{ cm}}$$

(c) $\triangle CDE$ is similar to $\triangle CFA$.

$$\therefore \frac{DC}{CF} = \frac{DE}{AF}$$

$$\frac{DC}{CF} = \frac{8}{13\frac{1}{3}}$$

$$= 8 \times \frac{3}{40}$$

$$= \mathbf{\frac{3}{5}}$$

Test 4: Scales and Maps

1. (a) 1 cm represents 20 cm or 0.2 m.
$\therefore$ 0.6 m is represented by
$\frac{0.6}{0.2}$ cm = 3 cm.
The length of the handle bar on the model is **3 cm**.

> 100 cm = 1 m
> 20 cm = $\frac{20}{100}$ m
> = 0.2 m

Teacher's Tip

A scale of $\frac{1}{20}$ means a length of 1 cm on the model represents an actual length of 20 cm.

(b) 1 cm represents 0.2 m.
$\therefore$ 1 cm^2 represents $(0.2 \text{ m})^2 = 0.04 \text{ m}^2$.
$\therefore$ 12 cm^2 represents $12 \times 0.04 = 0.48 \text{ m}^2$.
The actual area of the front wheel is **0.48 m²**.

Teacher's Tip

The area scale of a map is the square of its linear scale. If the linear scale is $1 : n$, then the area scale is $(1)^2 : (n)^2 = 1 : n^2$.

2. (a) 1 cm represents 200 cm or 2 m.
$\therefore$ 3.5 cm represents $3.5 \times 2 = 7$ m.
The width of the shop is **7 m**.

(b) 2 m is represented by 1 cm.
1 m is represented by $\frac{1}{2}$ cm.
$\therefore$ 1 m^2 is represented by $\left(\frac{1}{2}\text{ cm}\right)^2 = \frac{1}{4} \text{ cm}^2$.
$\therefore$ 54 m^2 is represented by $54 \times \frac{1}{4} = 13.5 \text{ cm}^2$.
The area of the plan representing the shop space is **13.5 cm²**.

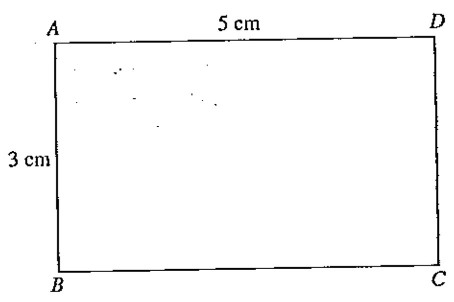
3. (a) 4 cm : 2.5 km — 1 km = 100 000 cm
= 4 cm : 250 000 cm
= 1 : 62 500 — Divide the ratio by 4.
$\therefore$ the R.F. of the map is $\mathbf{\frac{1}{62\,500}}$.

Teacher's Tip

The representative fraction (R.F.) expresses the linear scale of a map $1 : n$ in the form $\frac{1}{n}$.

(b) 1 cm represents 50 000 cm or $\frac{1}{2}$ km.
$\therefore$ 1 cm^2 represents $\left(\frac{1}{2}\text{ km}\right)^2 = \frac{1}{4} \text{ km}^2$.
$\therefore$ 20 cm^2 represents $20 \times \frac{1}{4} = 5 \text{ km}^2$.
The actual area of the wildlife sanctuary is **5 km²**.

(c)



Area of rectangle $ABCD = 5 \times 3 = 15 \text{ cm}^2$
1 cm represents 200 cm or 2 m.
$\therefore$ 1 cm^2 represents $(2 \text{ m})^2 = 4 \text{ m}^2$.
$\therefore$ 15 cm^2 represents $15 \times 4 = 60 \text{ m}^2$.
The actual area of the rectangular plot of land is **60 m²**.