TEST 14 Angle Properties of Triangles and Quadrilaterals

Marks: /60

Time: 1 hour 30 minutes

Name: ..... Date: .....

INSTRUCTIONS TO CANDIDATES

Section A (30 marks) **Time: 45 minutes**

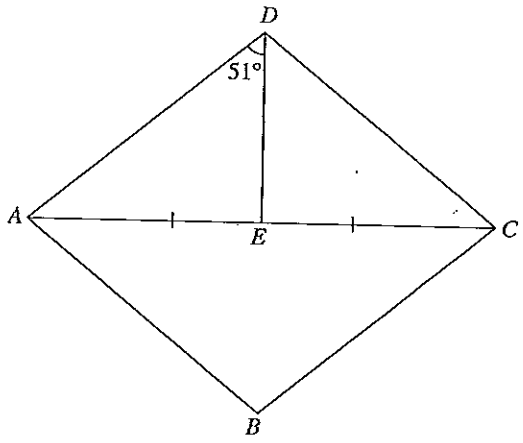
1. Answer all the questions in this section.
2. Calculators may **not** be used in this section.
3. All working must be clearly shown. Omission of essential working will result in loss of marks.
4. The marks for each question is shown in brackets [ ] at the end of each question.

---

1 In the diagram $ABCD$ is a rhombus. Given that $AE = EC$ and $A\hat{D}E = 51°$, calculate

(a) $B\hat{A}E$,

(b) $A\hat{B}C$.



Answer (a) $B\hat{A}E$ = ... ° [1]

(b) $A\hat{B}C$ = ... ° [1]

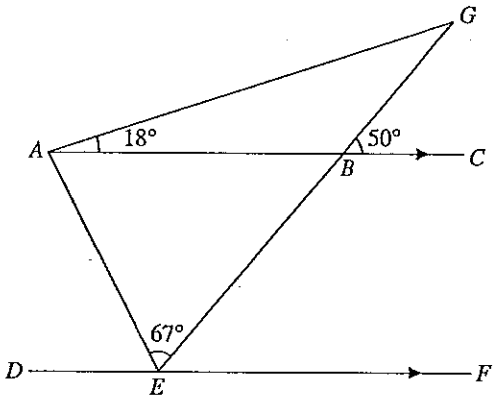
---

2 In the diagram ABC is parallel to DEF, $G\hat{A}B = 18°$, $A\hat{E}B = 67°$ and $G\hat{B}C = 50°$. Calculate

(a) $B\hat{E}F$,

(b) $A\hat{G}B$,

(c) $E\hat{A}B$.



Answer (a) $B\hat{E}F$ = ... ° [1]

(b) $A\hat{G}B$ = ... ° [1]

(c) $E\hat{A}B$ = ... ° [1]

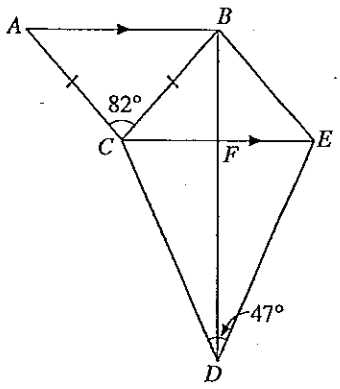
---

3 In the diagram, $BCDE$ is a kite. AB is parallel to CE, $CA = CB$, $A\hat{C}B = 82°$ and $C\hat{D}E = 47°$. Calculate

(a) $A\hat{B}C$,

(b) $C\hat{B}E$,

(c) $B\hat{C}D$.



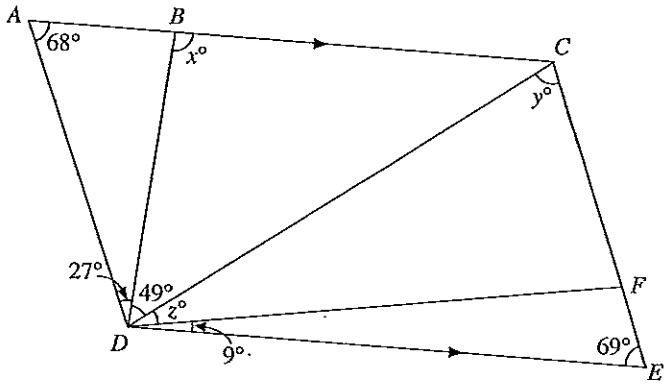
Answer (a) $A\hat{B}C$ = ... ° [1]

(b) $C\hat{B}E$ = ... ° [1]

(c) $B\hat{C}D$ = ... ° [1]

---

4 In the diagram AC is parallel to DE, $A\hat{D}B = 27°$, $B\hat{A}D = 68°$, $B\hat{D}C = 49°$, $D\hat{E}F = 69°$ and $F\hat{D}E = 9°$. Find the values of x, y and z.



Answer $x =$ ..... [1]

$y =$ ..... [1]

$z =$ ..... [1]

---

5 In the diagram PT is parallel to QR, $QP = QS$, $S\hat{P}T = 48°$ and $S\hat{Q}R = 18°$.

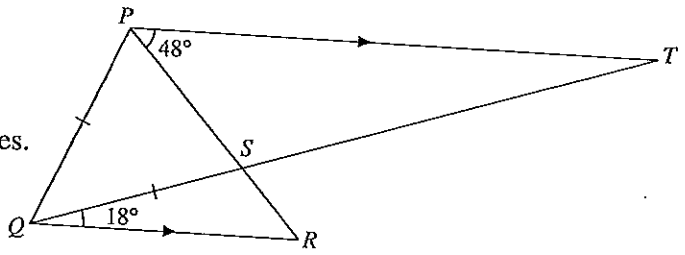
(a) Calculate

(i) $P\hat{R}Q$,

(ii) $R\hat{S}T$,

(iii) $Q\hat{P}R$.

(b) Explain why triangle PQR is isosceles.



Answer (a) (i) $P\hat{R}Q =$ ..... ° [1]

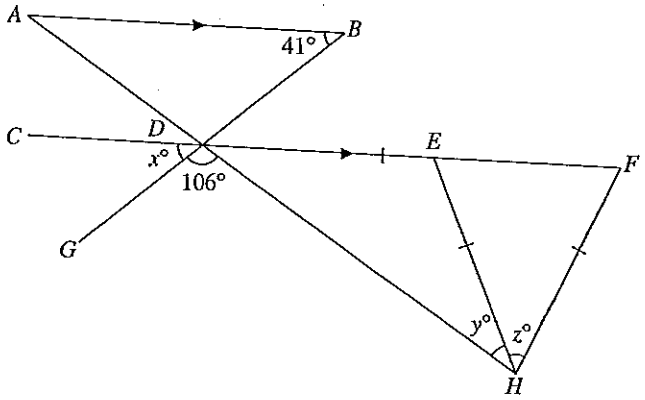
(ii) $R\hat{S}T =$ ..... ° [1]

(iii) $Q\hat{P}R =$ ..... ° [1]

(b) ..... [1]

---

6 In the diagram AB is parallel to $CDEF$, $ED = EH = HF$, $A\hat{B}D = 41°$ and $G\hat{D}H = 106°$. Find the values of x, y and z.



Answer $x =$ .....

$y =$ .....

$z =$ ..... [3]

---

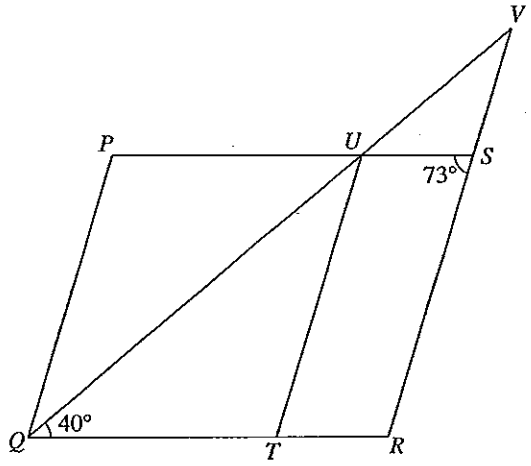
7 In the diagram $PQRS$ is a parallelogram. PQ is parallel to UT, $U\hat{Q}T = 40°$ and $U\hat{S}R = 73°$. RSV, PUS and QTR are straight lines.

Calculate

(a) $P\hat{Q}U$,

(b) $Q\hat{T}U$,

(c) $U\hat{V}S$.



Answer (a) $P\hat{Q}U =$ ..... ° [1]

(b) $Q\hat{T}U =$ ..... ° [1]

(c) $U\hat{V}S =$ ..... ° [1]

---

8 Construct a rhombus whose diagonals are of lengths 10 cm and 14 cm respectively. Measure the length of one side of the rhombus.

Answer

[2]

Answer Length = ... cm [1]

9 Construct a parallelogram $PQRS$ such that $PQ = 10$ cm, $QR = 6.5$ cm and $P\hat{Q}R = 105°$. Measure the length of PR.

Answer [2]

Answer $PR =$ ..... cm [1]

10 Construct a triangle ABC such that $AB = 9$ cm, $A\hat{B}C = 72°$ and $B\hat{A}C = 65°$. Measure and write down the length of AC.

Answer

[2]

Answer $AC =$ ..... cm [1]

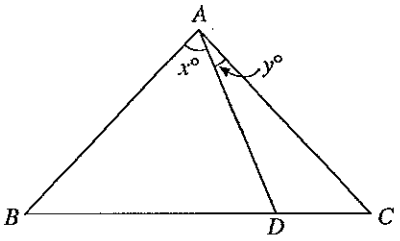
INSTRUCTIONS TO CANDIDATES

Section B (30 marks) **Time: 45 minutes**

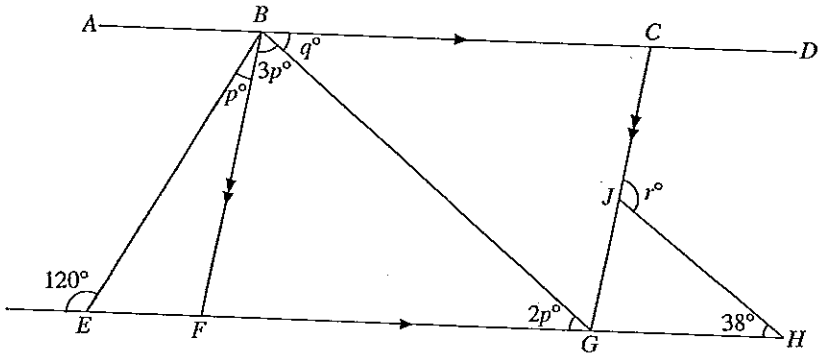
1. Answer all the questions in this section.
2. Calculators may be used in this section.
3. All working must be clearly shown. Omission of essential working will result in loss of marks.
4. The marks for each question is shown in brackets [ ] at the end of each question.

---

11 (a) In the diagram, $AB = AC = BD$. Express y in terms of x.



(b) In the diagram $ABCD$, CJG and $EFGH$ are straight lines. AD is parallel to EH and BF is parallel to CG. Calculate the values of p, q and r.



Answer (a) $y =$ ..... [2]

(b) $p =$ .....

$q =$ .....

$r =$ ..... [3]

---

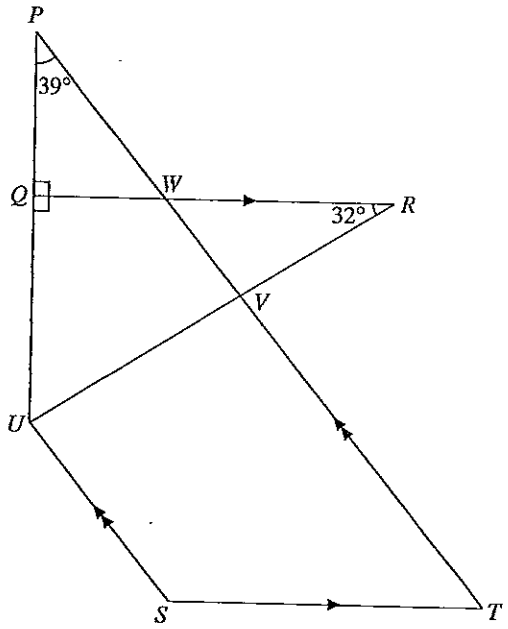
12 (a) In the diagram QR is parallel to ST and SU is parallel to TP. PQU, QWR, UVR and $PWVT$ are straight lines and PQU is perpendicular to QWR. Given that $Q\hat{P}W = 39°$ and $Q\hat{R}U = 32°$, calculate

(i) $P\hat{T}S$,

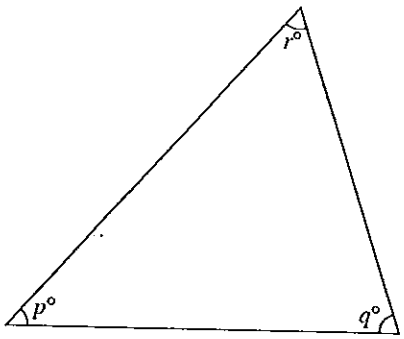
(ii) $U\hat{S}T$,

(iii) $U\hat{V}W$,

(iv) $S\hat{U}V$.



(b) The ratio of $p : q = 2 : 3$ and $q : r = 6 : 5$.
Find the value of p.



Answer (a) (i) $P\hat{T}S$ = ... ° [1]

(ii) $U\hat{S}T$ = ... ° [1]

(iii) $U\hat{V}W$ = ... ° [1]

(iv) $S\hat{U}V$ = ... ° [1]

(b) p = ... [2]

13 In the diagram, $ABCD$ is a parallelogram and $CEFG$ is a rhombus. $B\hat{A}G = 98°$, $A\hat{B}G = 25°$, $C\hat{D}H = 51°$ and $C\hat{E}F = 80°$. Calculate

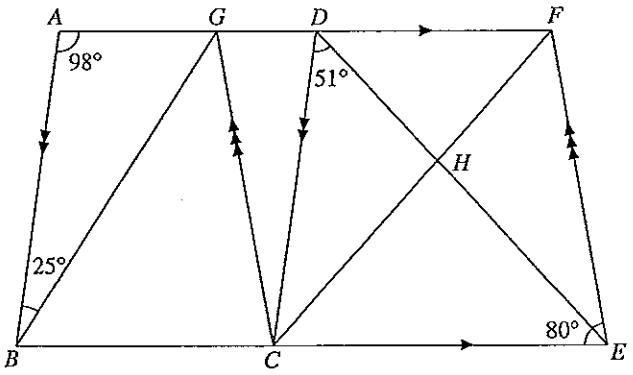
(a) $G\hat{B}C$,

(b) $B\hat{G}C$,

(c) $G\hat{C}D$,

(d) $D\hat{F}C$,

(e) $C\hat{H}E$.



Answer (a) $G\hat{B}C =$ ..... ° [1]

(b) $B\hat{G}C =$ ..... ° [1]

(c) $G\hat{C}D =$ ..... ° [1]

(d) $D\hat{F}C =$ ..... ° [1]

(e) $C\hat{H}E =$ ..... ° [2]

14 **(a)** Construct a triangle ABC such that $AB = 9.6$ cm, $BC = 8.7$ cm and $AC = 9.1$ cm. Measure and write down the size of the smallest angle.

(b) On the same diagram

(i) construct the angle bisector of $B\hat{A}C$,

(ii) construct the angle bisector of $A\hat{C}B$,

(iii) mark down with the letter P, the point of intersection of the two angle bisectors. Measure and write down the length of BP.

Answer (a), (b) (i), (ii), (iii)

[4]

Answer *(a)* Smallest angle = ... ° [1]

(b) BP = ... cm [1]

15 (a) Construct a quadrilateral $PQRS$ such that PQ = 7.8 cm, PR = 12.4 cm, PS = 9.5 cm, RS = 11.5 cm and $P\hat{Q}R = 106°$. Measure and write down the size of $P\hat{S}R$.

(b) On the same diagram, construct

(i) the perpendicular bisector of the line RS,

(ii) the angle bisector of $P\hat{S}R$.

Mark down the point T, where these two lines meet. Measure and write down the length of ST.

Answer (a), (b), (i), (ii) [5]

Answer (a) $P\hat{S}R$ = ... ° [1]

(b) ST = ... cm [1]

Teacher's Tip

To construct a line perpendicular to AB passing through A:

(a) Place set square on line AB and ruler as shown in Figure I.

(b) Slide set square along the length of the ruler until the edge of the set square passes through A as shown in Figure II. Draw the line passing through A and perpendicular to AB.

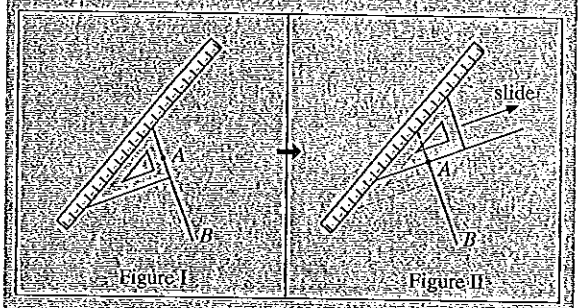
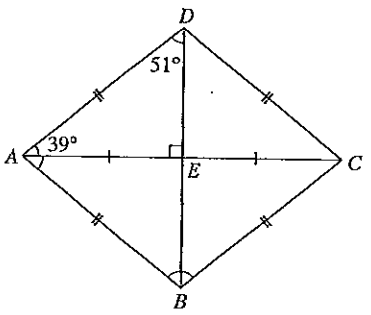


Figure I Figure II

Test 14: Angle Properties of Triangles and Quadrilaterals

Section A

1.



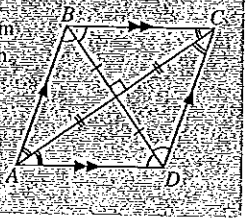
(a) $B\hat{A}E = D\hat{A}E$ ← Since $AE = EC$, DEB is a diagonal. The diagonals of a rhombus bisect each other at right angles.

$= 180° - 51° - 90°$ • $BAE = DAE$ since the diagonal AC bisect DAB.

$= \mathbf{39°}$

Teacher's Tip

1. A rhombus is a parallelogram with all sides equal in length.
2. The diagonals bisect each other at right angles.
3. Opposite angles are equal.
4. The diagonals bisect the angles.

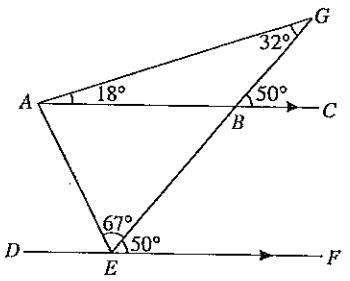


(b) $A\hat{B}C = A\hat{D}C$ Opposite angles are equal.

$= 2 \times 51°$

$= \mathbf{102°}$

2.



(a) $B\hat{E}F = \mathbf{50°}$ (corr. $\angle$s, $AC \parallel DF$)

(b) $A\hat{G}B + 18° = 50°$ (ext $\angle$ = sum of int. opp. $\angle$s)

$A\hat{G}B = 50° - 18° = \mathbf{32°}$

(c) $E\hat{A}B + A\hat{E}F = 180°$ (int. $\angle$s, $AC \parallel DF$)

$E\hat{A}B + 67° + 50° = 180°$

$E\hat{A}B = 180° - 67° - 50°$

$= \mathbf{63°}$

Teacher's Tip

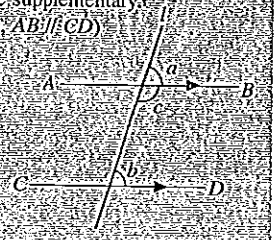
1. When two parallel lines are cut by a transversal l:

(a) the corresponding angles are equal.

$a = b$ (corr. $\angle$s, $AB \parallel CD$)

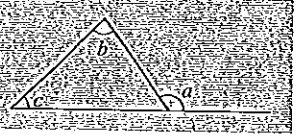
(b) the interior angles are supplementary.

$b + c = 180°$ (int. $\angle$s, $AB \parallel CD$)

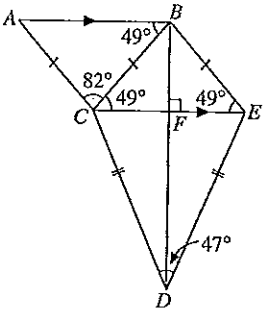


2. The exterior angle of a triangle is equal to the sum of the interior opposite angles.

$a = b + c$ (ext. $\angle$ = sum of int. opp. $\angle$s)



3.



Since $BCDE$ is a kite, $BC = BE$ and $DC = DE$.

(a) $A\hat{B}C = (180° - 82°) \div 2$ (base $\angle$s of isos. $\triangle$)
$= 98° \div 2$
$= \mathbf{49°}$

(b) $B\hat{C}E = 49°$ (alt. $\angle$s, $AB \parallel CE$)
$B\hat{E}C = B\hat{C}E = 49°$ (base $\angle$s of isos. $\triangle$)
$C\hat{B}E = 180° - 49° - 49°$ ($\angle$ sum of $\triangle$)
$= \mathbf{82°}$

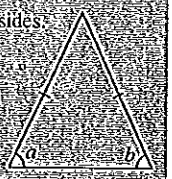
Alternative method:
$C\hat{B}F = 180° - 49° - 90°$ ($\angle$ sum of $\triangle$)
$= 41°$
$C\hat{B}E = 2 \times 41°$
$= \mathbf{82°}$

(c) $D\hat{C}E = (180° - 47°) \div 2$ (base $\angle$s of isos. $\triangle$)
$= 133° \div 2$
$= 66\frac{1}{2}°$
$B\hat{C}D = 49° + 66\frac{1}{2}°$
$= \mathbf{115.5°}$

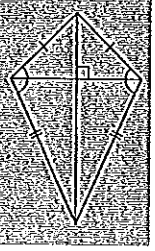
Teacher's Tip

1. The sum of the angles of a triangle is 180°.
$a + b + c = 180°$ ($\angle$ sum of $\triangle$)

2. An isosceles triangle has two equal sides. The two base angles are equal.
$a = b$ (base $\angle$s of isos. $\triangle$)

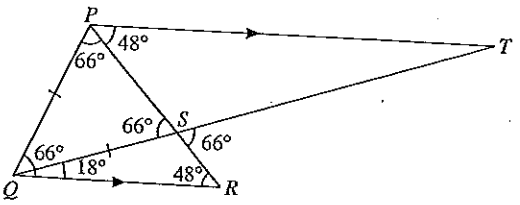


3. A kite has:
(a) two pairs of equal adjacent sides,
(b) one pair of opposite angles equal,
(c) the longer diagonal bisects the other diagonal at right angles.



4. $x° = 68° + 27°$ (ext. $\angle$ = sum of int. opp. $\angle$s)
$= 95°$
$\therefore x = \mathbf{95}$
$B\hat{C}D = 180° - 95° - 49°$ ($\angle$ sum of $\triangle$)
$= 36°$
$(y° + 36°) + 69° = 180°$ (int. $\angle$s, $AC \parallel DE$)
$y° = 180° - 36° - 69°$
$= 75°$
$\therefore y = \mathbf{75}$
$z° + 9° = 36°$ (alt. $\angle$s, $AC \parallel DE$)
$z° = 36° - 9°$
$= 27°$
$\therefore z = \mathbf{27}$

5.



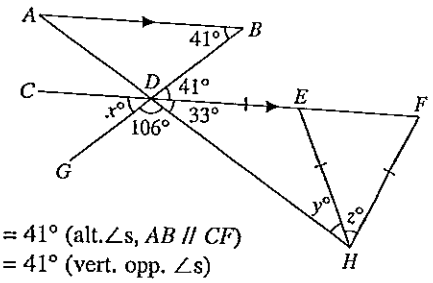
(a) (i) $P\hat{R}Q = 48°$ (alt. $\angle$s, $PT \parallel QR$)

(ii) $R\hat{S}T = 18° + 48°$ (ext. $\angle$ = sum of int. opp. $\angle$s)
$= \mathbf{66°}$

(iii) $Q\hat{P}R = Q\hat{S}P$ (base $\angle$s of isos. $\triangle$)
$= R\hat{S}T$ (vert. opp. $\angle$s)
$= \mathbf{66°}$

(b) $P\hat{Q}R = 180° - 66° - 48°$ ($\angle$ sum of $\triangle$)
$= 66°$
Since $P\hat{Q}R = Q\hat{P}R = 66°$, $\triangle PQR$ is isosceles.

6.



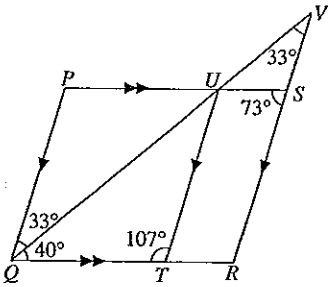
$B\hat{D}E = 41°$ (alt. $\angle$s, $AB \parallel CF$)
$x° = 41°$ (vert. opp. $\angle$s)
$\therefore x = \mathbf{41}$

$y° = E\hat{D}H$ (base $\angle$s of isos. $\triangle$)
$= 180° - 106° - 41°$ (adj. $\angle$s on a str. line)
$= 33°$
$\therefore y = \mathbf{33}$

$H\hat{E}F = 33° + 33°$ (ext. $\angle$ = sum of int. opp. $\angle$s)
$= 66°$

$E\hat{F}H = H\hat{E}F = 66°$ (base $\angle$s of isos. $\triangle$)
$z° = 180° - 66° - 66°$ ($\angle$ sum of $\triangle$)
$= 48°$
$\therefore z = \mathbf{48}$

7.



Given that $PQRS$ is a parallelogram, $PQ \parallel SR$ and $PS \parallel QR$.

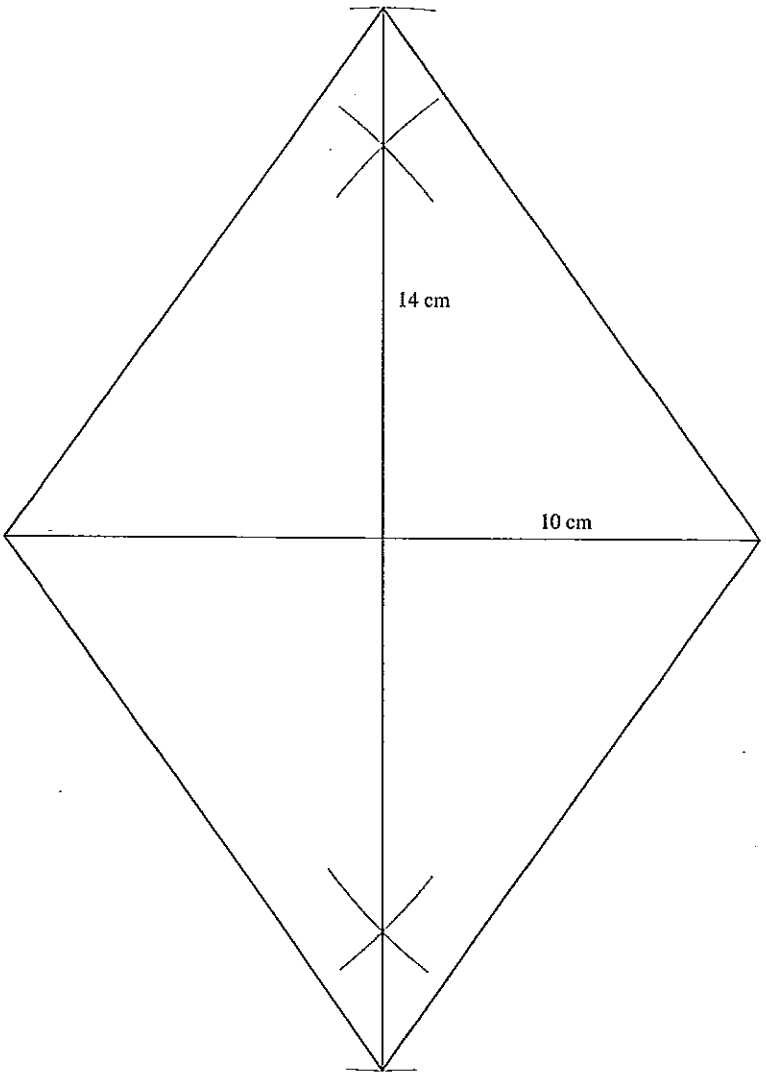
(a) $P\hat{Q}U + 40° = 73°$
$P\hat{Q}U = 33°$

Opposite angles in a parallelogram are equal.

(b) $Q\hat{T}U = 180° - (33° + 40°)$ (int. $\angle$s, $PQ \parallel UT$)
$= \mathbf{107°}$

(c) $U\hat{V}S = 33°$ (alt. $\angle$s, $PQ \parallel VR$)

8. Length = **8.6 cm**



Teacher's Tip

Construction steps:

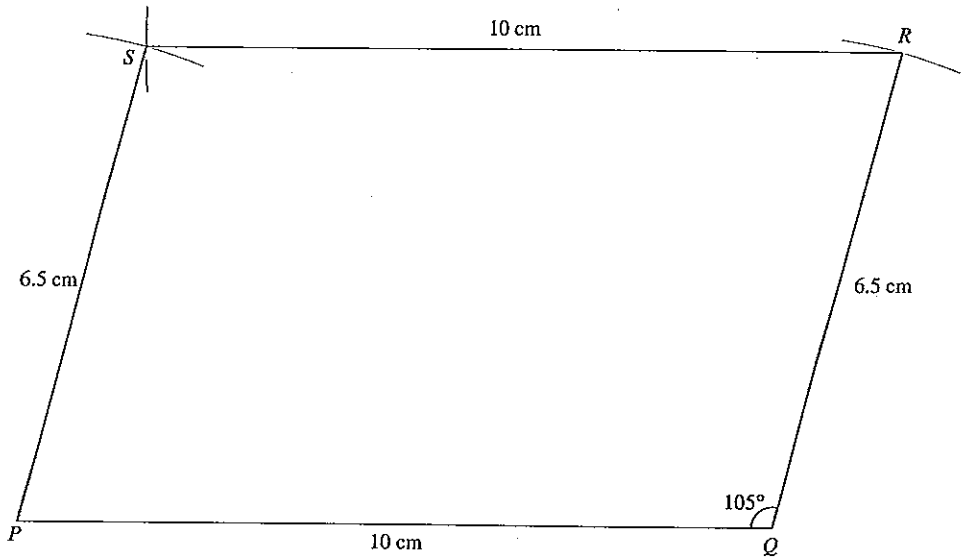
Step 1: Draw a line segment 10 cm long.

Step 2: Construct the perpendicular bisector of the line segment.

Step 3: Using a pair of compasses with centre at the intersection of the two lines and radius 7cm, draw arcs to cut the vertical diagonal at the top and bottom.

Step 4: Join the tip of the diagonals to get the required rhombus.

9. $PR = 13.25$ cm



Teacher's Tip

Construction steps:

Step 1: Draw a line segment PQ of length 10 cm.

Step 2: Using a protractor, construct an angle of 105° at Q with PQ as one side of the angle and produce with other arm of the angle.

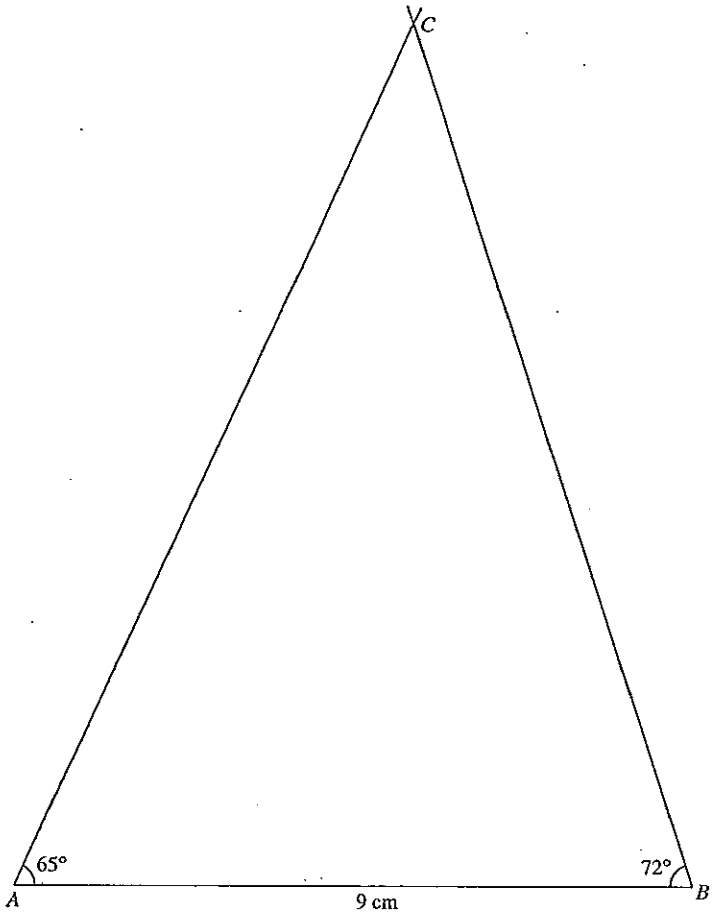
Step 3: With Q as centre and radius 6.5 cm, draw an arc to cut the arm of Q at R.

Step 4: With R as centre and radius 10 cm, draw an arc.

Step 5: With P as centre and radius 6.5 cm, draw an arc to cut the arc from Step 4 at S.

$PQRS$ is the required parallelogram.

10.



$AC = 12.55$ cm

Teacher's Tip

Construction steps:

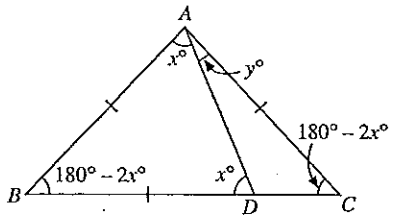
Step 1: Draw a line segment AB, 9 cm long.

Step 2: Use a protractor to draw an angle of 65° and 72° at A and B respectively.

Step 3: Produce the other arms of the angles at A and B to meet at C.

$\triangle ABC$ is the required triangle.

11. (a)



$A\hat{D}B = B\hat{A}D = x°$ (base $\angle$s of isos. $\triangle$)

$A\hat{B}D = 180° - x° - x°$ ($\angle$ sum of $\triangle$)

$= 180° - 2x°$

$A\hat{C}D = A\hat{B}D = 180° - 2x°$ (base $\angle$s of isos. $\triangle$)

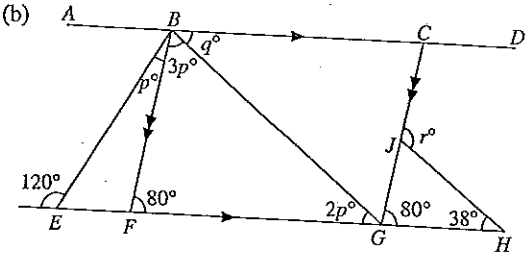
$A\hat{D}B = D\hat{A}C + A\hat{C}D$ (ext. $\angle$ = sum of int. opp. $\angle$s)

$x° = y° + (180° - 2x°)$

$x = y + 180 - 2x$

$y = 3x - 180$

(b)



$(p + 3p)° + 2p° = 120°$ (ext. $\angle$ = sum of int. opp. $\angle$s)

$6p = 120$

$p = \frac{120}{6} = \mathbf{20}$

$q° = 2p°$ (alt. $\angle$s, $AD \parallel EH$)

$= 2 \times 20°$

$= 40°$

$\therefore q = \mathbf{40}$

$J\hat{G}H = B\hat{F}G$ (corr. $\angle$s, $BF \parallel CG$)

$= 180° - (3p° + q°)$ (int. $\angle$s, $AD \parallel EH$)

$= 180° - 3(20°) - 40°$

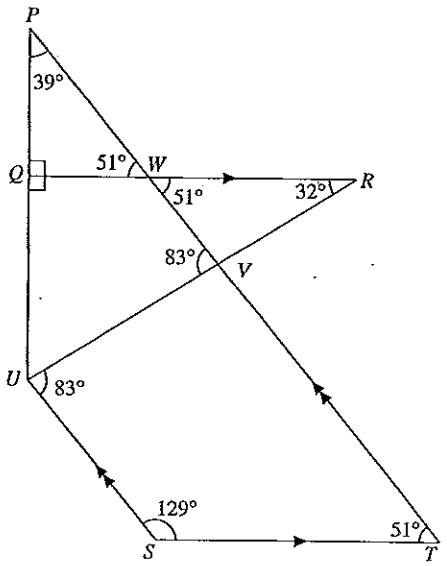
$= 80°$

$r° = 80° + 38°$ (ext. $\angle$s = sum of int. opp. $\angle$s)

$= 118°$

$\therefore r = \mathbf{118}$

12. (a)



(i) $P\hat{T}S = P\hat{W}Q$ (corr. $\angle$s, $QR \parallel ST$)

$= 180° - 90° - 39°$ ($\angle$ sum of $\triangle$)

$= \mathbf{51°}$

(ii) $U\hat{S}T = 180° - 51°$ (int. $\angle$s, $SU \parallel TV$)

$= \mathbf{129°}$

(iii) $V\hat{W}R = 51°$ (vert. opp. $\angle$s)

$U\hat{V}W = 51° + 32°$ (ext. $\angle$ = sum of int. opp. $\angle$s)

$= \mathbf{83°}$

(iv) $S\hat{U}V = \mathbf{83°}$ (alt. $\angle$s, $SU \parallel TP$)

(b) $p : q : r$

$2 : 3$

$\underline{\quad 6 : 5}$

$4 : 6 : 5$

$4 + 6 + 5 = 15$ units

15 units — 180°

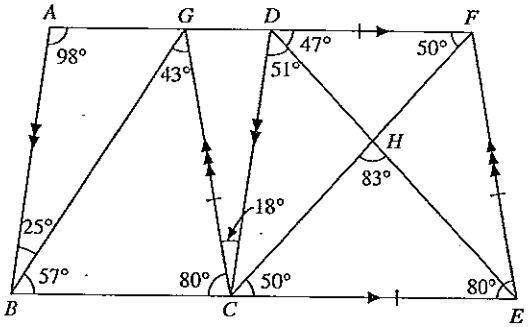
1 unit — $\frac{180°}{15} = 12°$

4 units — $4 \times 12 = 48°$

$p° = 48°$

$\therefore p = \mathbf{48}$

13.



(a) $G\hat{B}C + 25° = 180° - 98°$ (int. $\angle$s, $AD \parallel BC$)

$G\hat{B}C = \mathbf{57°}$

(b) $B\hat{C}G = 80°$ (corr. $\angle$s, $CG \parallel EF$)

$B\hat{G}C = 180° - 57° - 80°$ ($\angle$ sum of $\triangle$)

$= \mathbf{43°}$

(c) $G\hat{C}D + 80° = 98°$

$G\hat{C}D = \mathbf{18°}$

Opp. $\angle$s of a parallelogram are equal.

(d) $F\hat{C}E = (180° - 80°) \div 2$ (base $\angle$s of isos. $\triangle$)

$= 50°$

$D\hat{F}C = \mathbf{50°}$ (alt. $\angle$s, $AF \parallel BE$)

(e) $H\hat{D}F + 51° = 98°$ (corr. $\angle$s, $AB \parallel DC$)

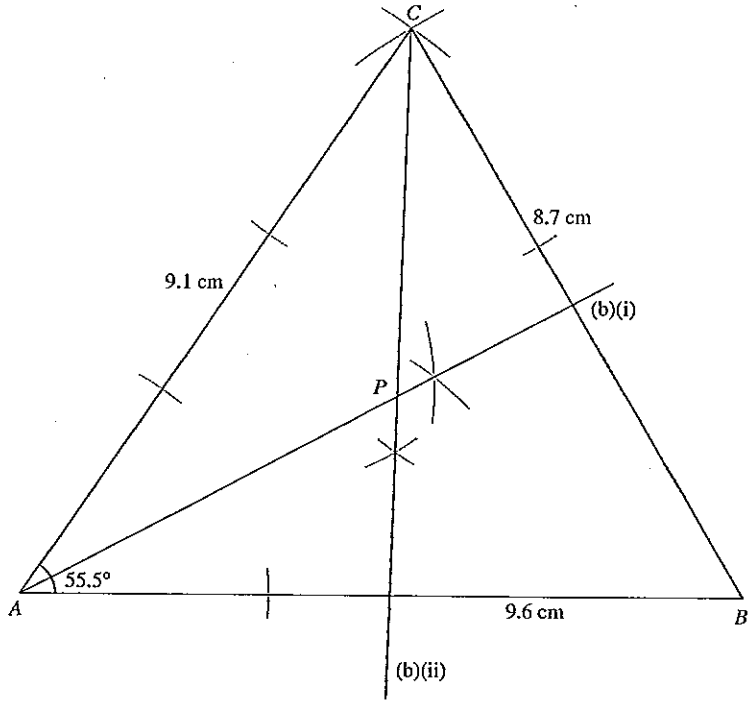
$H\hat{D}F = 47°$

$C\hat{H}E = D\hat{H}F$ (vert. opp. $\angle$s)

$= 180° - 47° - 50°$ ($\angle$ sum of $\triangle$)

$= \mathbf{83°}$

14. (a), (b)



(a) Smallest angle = $B\hat{A}C$ The smallest angle is opposite the shortest side.
= **55.5°**

(b) BP = **5.3 cm**

Teacher's Tip

Construction steps:

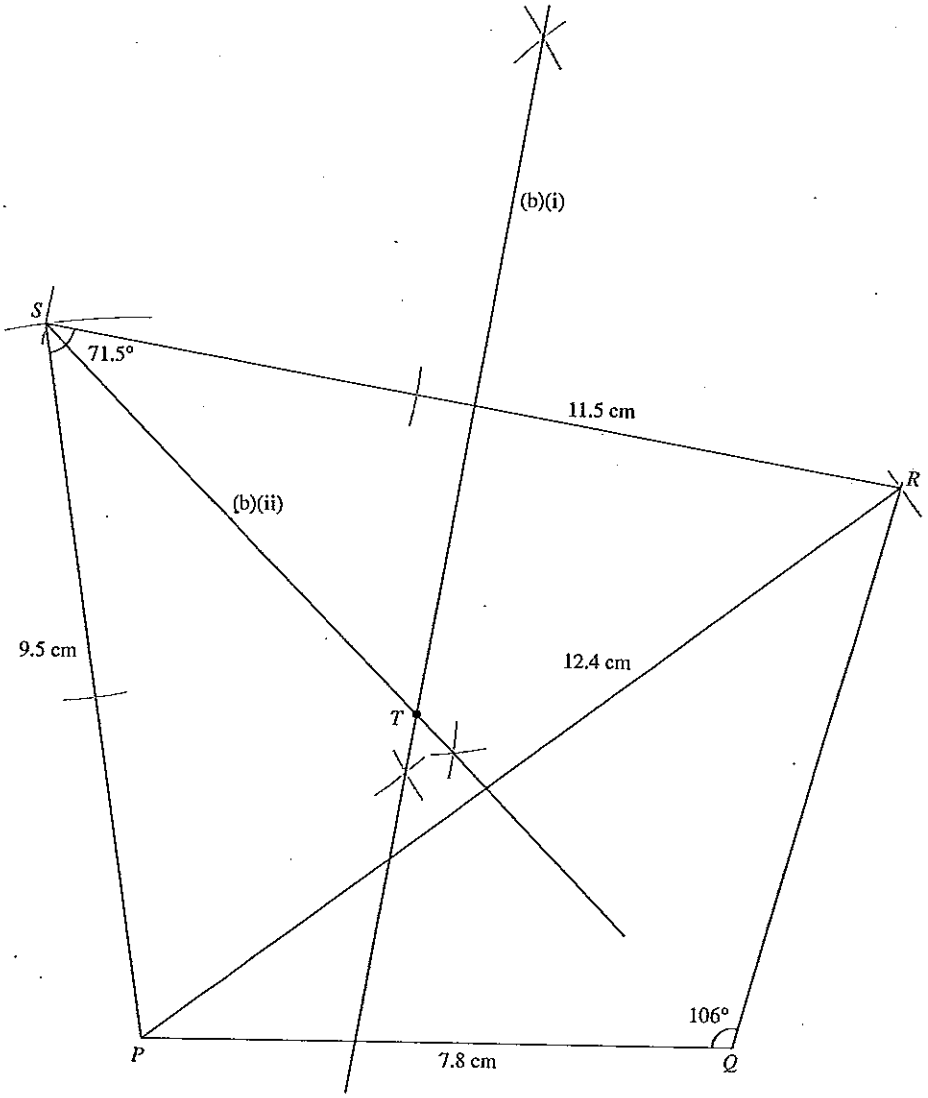
Step 1: Draw a line segment AB, 9.6 cm long.

Step 2: With B as centre and radius 8.7 cm, draw an arc.

Step 3: With A as centre and radius 9.1 cm, draw an arc to cut the arc from Step 2 at C.

$\triangle ABC$ is the required triangle.

15. (a), (b)



(a) $P\hat{S}R = 71.5°$

(b) $ST = 7.1$ cm

Teacher's Tip

Construction steps:

Step 1: Draw a line segment PQ, 7.8 cm long.

Step 2: Using a protractor, draw an angle of 106° at Q and produce the arm of the angle.

Step 3: With P as centre and radius 12.4 cm, draw an arc to cut the arm of Q at R.

Step 4: With P as centre and radius 9.5 cm, draw an arc.

Step 5: With R as centre and radius 11.5 cm, draw an arc to cut the arc from Step 4 at S.

$PQRS$ is the required quadrilateral.