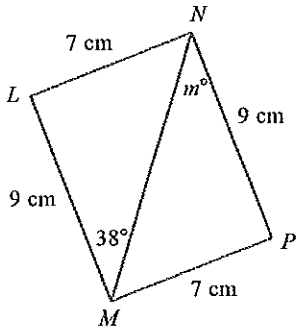
TEST FOR CONGRUENT TRIANGLES.

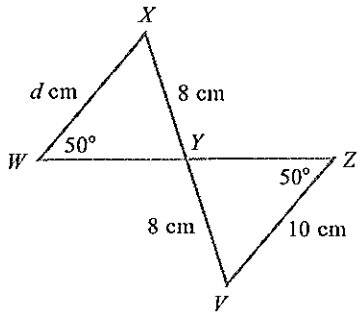
Exercise 11.07

1. Find the value of the pronumeral, giving reasons for each answer.

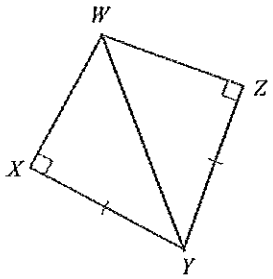
(a)



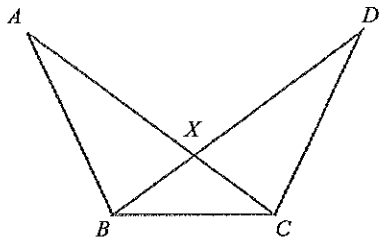
(b)



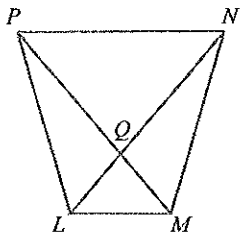
2. (a) Show $\Delta WXY \equiv \Delta WZY$.



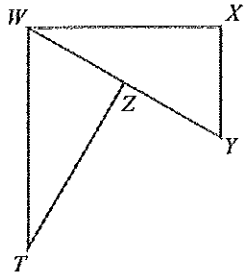
(b) $\angle ABC = \angle DCB$ and $AB = DC$.
Show $\Delta ABC \equiv \Delta BCD$.



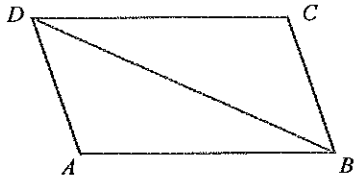
(c) $LQ = MQ$ and $PQ = NQ$.
Show $\Delta PQL = \Delta NQM$.



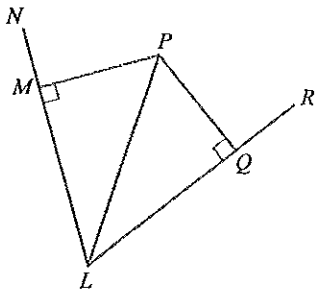
(d) $TZ \perp WY$, $TW \perp WX$, $YX \perp WX$ and $TW \perp WY$.
Show $\Delta TZW \equiv \Delta WXY$.



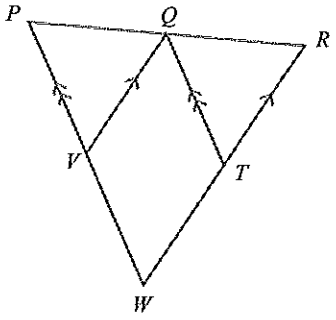
(e) $AB = CD$ and $AD = BC$. Show:
(i) $\Delta ABD \equiv \Delta CDB$
(ii) $AB \parallel DC$ and $AD \parallel BC$



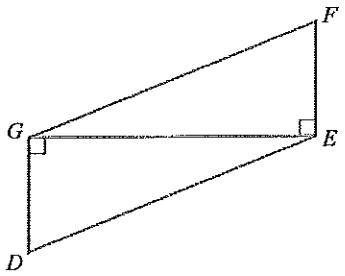
(f) $PM = PQ$, $PM \perp LN$ and $PQ \perp LR$.
Show $\Delta LMP \equiv \Delta LQP$.



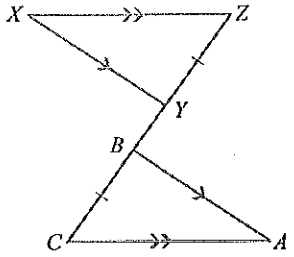
(g) Q is the midpoint of PR.
Show $\Delta PVQ = \Delta QRT$.



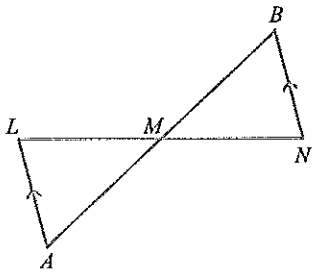
(h) $FE \perp GE$,
$DG \perp GE$ and $FE = GD$.
Show $\Delta DEG \equiv \Delta FGE$.



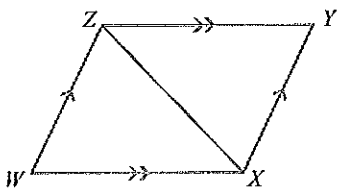
3. $AC \parallel XZ$, $XY \parallel AB$ and $BC = YZ$.
Show that $\Delta ABC \equiv \Delta XYZ$.



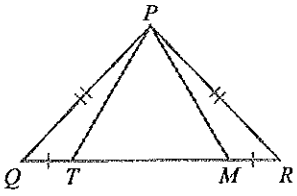
4. $LA \parallel BN$ and AB bisects LN at M.
Prove $\Delta LMA \equiv \Delta NMB$.



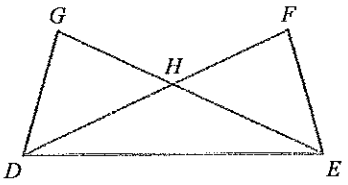
5. $WX \parallel ZY$ and $WZ \parallel XY$. Show:
(a) $\Delta WXZ \equiv \Delta YZX$
(b) $WZ = YX$



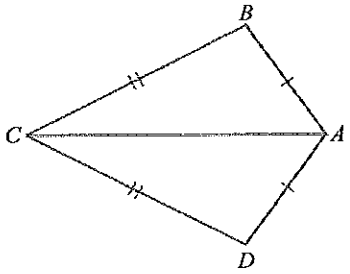
6. ΔPQR is isosceles and $QT = RM$.
Prove ΔPTM is isosceles.
(*Hint:* Show $PT = PM$.)



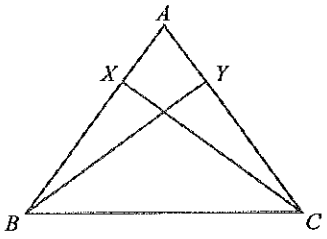
7. $\angle DEG = \angle EDF$ and $GE = FD$.
Show $\Delta DEG \equiv \Delta EDF$.



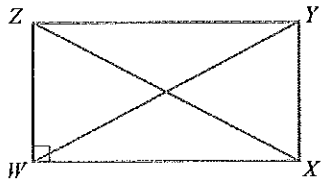
8. $BC = BD$ and $AB = AD$.
Show $\angle BAC = \angle DAC$.



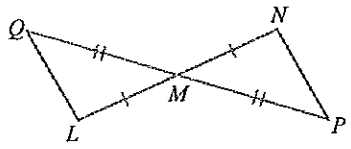
9. $CX \perp AB$, $BY \perp AC$ and $XC = YB$.
Show ΔABC is isosceles.



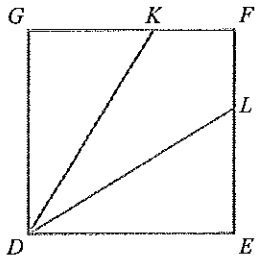
10. Prove that the diagonals of the rectangle $WXYZ$ are equal.
(*Hint:* Show $\Delta WXZ \equiv \Delta XWY$.)



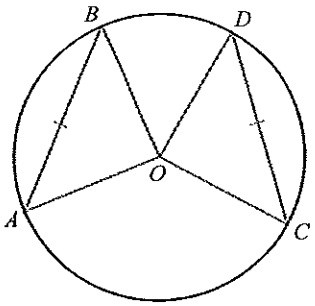
11. $LM = MN$ and $PM = MQ$.
Show $QL \parallel PN$.



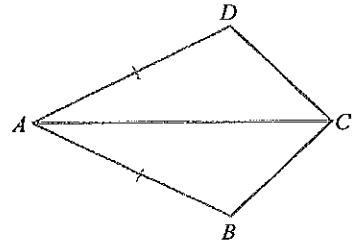
12. $DEFG$ is a square and $KF = LF$.
Show $\Delta DKG \equiv \Delta DLE$.



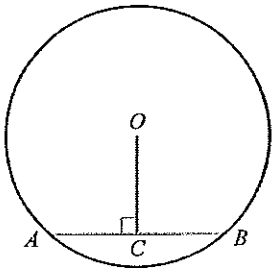
13. O is the centre of the circle and $AB = CD$.
Show $\angle AOB = \angle COD$.



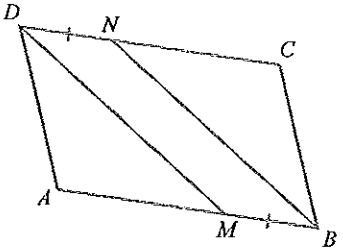
14. $AD = AB$ and CA bisects $\angle DAB$.
Show $CD = CB$.



15. Prove that the line from the centre of a circle to the midpoint of a chord bisects the chord at right angles.
(*Hint:* Draw OC, OA and OB, and show $\Delta OAC \equiv \Delta OBC$.)



16. $ABCD$ is a parallelogram.
Prove $\Delta AMD \equiv \Delta CNB$.



V·CHU Ex 11·07

V. Good work!

1. (a) In $\triangle NLM, \triangle NPM$

$LN = MP$ (given)

$NP = LM$ (given) ✓

NM is common.

$\therefore \triangle NLM \equiv \triangle NPM$ (SSS) ✓

$\therefore m = 36^\circ$, as $\triangle NLM \equiv \triangle NPM$ ✓

(b) In $\triangle WXY, \triangle YZV$

$\angle XWY = \angle YZV$ (given)

$\angle XYW = \angle ZYV$ (vert. opp. $\angle$) ✓

$XY = YV$ (given)

$\therefore \triangle WXY \equiv \triangle YZV$ (AAS) ✓

$\therefore XW = ZV = 10cm$, as $\triangle WXY \equiv \triangle YZV$ ✓

2. (a) In $\triangle WXY, \triangle WZY$

$\angle WXY = \angle WZY$ (given) ✓

WY is common ✓

$XY = ZY$ (given)

$\therefore \triangle WXY \equiv \triangle WZY$ (RHS) ✓

(b) In $\triangle ABC, \triangle BCD$

$AB = DC$ (given) ✓

$\angle ABC = \angle DCB$ (given) ✓

BC is common

$\therefore \triangle ABC \equiv \triangle BCD$ (SAS) ✓

(c) In $\triangle PQL, \triangle NQM$

$LQ = QM$ (given) ✓

$\angle PQL = \angle NQM$ (vert opp. $\angle$)

$PQ = QN$ (given) ✓

$\therefore \triangle PQL \equiv \triangle NQM$ (SAS) ✓

(d) In $\triangle TZW, \triangle WXY$

Let $\angle XWY$ be x

$\therefore \angle ZWT = 90 - x$ (right $\angle$) ✓

$\therefore \angle XYW = 90 - x$.

$\angle WXY = \angle WZT$ (given) ✓

$\angle XYW = \angle ZWT$ (proven)

$WT = WY$ (given)

$\therefore \triangle TZW \equiv \triangle WXY$ (AAS) ✓

(e) (i) In $\triangle ABD, \triangle CDB$

$DC = AB$ (given) ✓

$CD = DA$ (given)

DB is common.

$\therefore \triangle ABD \equiv \triangle CDB$ (SSS) ✓

(f) In $\triangle LMP, \triangle LQP$

$\angle PML = \angle PQL$ (given) ✓

PL is common

$PM = PQ$ (given) ✓

$\therefore \triangle LMP \equiv \triangle LQR$ (RHS) ✓

(g) In $\triangle PVQ, \triangle QRT$

$\angle QPV = \angle RQT$ (corresp. $\angle$, $PV \parallel QT$) ✓

$\angle PQV = \cancel{\angle QTR}$ (alt. $\angle$, $QV \parallel RT$) X

$\angle QRT$ (Corresponding $\angle$s, $VQ \parallel TR$)

$PQ = QR$ (given) ✓

$\therefore \triangle PVQ \equiv \triangle QRT$ (AAS) ✓

(h) In $\triangle DEG, \triangle FGE$

$FE = GD$ (given) ✓

$\angle EGD = \angle FEG$ (given) ✓

GE is common ✓

$\therefore \triangle DEG \equiv \triangle FGE$ (SAS) ✓

(ii) $\angle ADB = \angle CBD$ (matching $\angle$s from congruency) ✓

$\therefore \angle ADE = \angle CBD$, alt $\angle$s, ($\because AD \parallel BC$) PTD

$\angle CDB = \angle DBA$ (matching $\angle$s from congruency) ✓

$\therefore \angle CDB = \angle DBA$, alt $\angle$s, ($\because AB \parallel DC$)

3. In $\triangle ABC$, $\triangle XYZ$

$\angle XZY = \angle BCA$, (alt $\angle$s, XZ // CA) ✓

$\therefore \angle XYZ = \angle ABC$, (alt $\angle$s, XY // BA) & supplementary angles.

$CB = YZ$ (given) ✓

$\therefore \triangle ABC \equiv \triangle XYZ$ (AAS) ✓

4. In $\triangle LMA$, $\triangle NMB$

$\angle LMA = \angle BMN$ (vert opp. $\angle$ are equal)

$\angle LAM = \angle MBN$ (alt $\angle$, AL // BN) ✓

$LM = MN$ (AB bisects LN at M) ✓

$\therefore \triangle LMA \equiv \triangle NMB$ (AAS) ✓

5. (a) In $\triangle WXZ$, $\triangle YZX$

$\angle WZX = \angle YXZ$ (alt $\angle$s, WZ // XY) ✓

$\angle YZX = \angle ZXW$ (alt $\angle$s, ZY // WX) ✓

ZX is common ✓

$\therefore \triangle WXZ \equiv \triangle YZX$ (AAS) ✓

(b) $WZ = YX$ (matching sides from congruency test) ✓

8. In $\triangle ABC$, $\triangle ADC$

$BC = CD$ (given) ✓

$BA = AD$ (given)

CA is common ✓

$\therefore \triangle ABC \equiv \triangle ADC$ (SSS) ✓

$\therefore \angle BAC = \angle DAC$, (matching angles from congruency test) ✓

9. In $\triangle BXC$, $\triangle CYB$

$\angle BXC = \angle CYB$ (given) ✓

BC is common ✓

$XC = BY$ (given) ✓

$\therefore \triangle BXC \equiv \triangle CYB$ (RHS) ✓

$\therefore \angle XBC = \angle YCB$ (matching $\angle$ from congruency) ✓

$\therefore \angle XBC$ and $\angle YCB$ are base $\angle$ for $\triangle ABC$ ✓

$\therefore \triangle ABC$ is isos. (equal base $\angle$s, $\angle XBC = \angle YCB$)

10. In $\triangle WXZ$, $\triangle XWY$

$WZ = YX$ (properties of rectangle) ✓

$\angle ZWX = \angle YXW$ (properties of rectangle) ✓

WX is common ✓

$\therefore \triangle WXZ \equiv \triangle XWY$ (SAS) ✓

$\therefore ZX = YW$ (matching sides from congruency test) ✓

$\therefore$ The diagonals of rec. WXYZ are equal

11. In $\triangle MLQ$, $\triangle PMN$

$LM = MN$ (given) ✓

$\angle QML = \angle NMP$ (vert opp $\angle$) ✓

$QM = MP$ (given) ✓

$\therefore \triangle MLQ \equiv \triangle PMN$ (SAS) ✓

$\therefore \angle MQL = \angle NPM$ (matching angles from congruency test) ✓

$\therefore QL // NP$ (alt $\angle$s, $\angle MQL = \angle NPM$) ✓

PTO

4. In $\triangle LMA$ and $\triangle NMB$
$\angle ALM = \angle BNM$ (Alternate $\angle$'s)
$\therefore AM = BM$ (LN bisects AB at M)
$\angle LMA = \angle BMN$ (Vert opp)
$\triangle LMA \equiv \triangle NMB$ (AAS)

5. a) In $\triangle WXZ$ and $\triangle YZX$
$\angle ZXW = \angle YZX$ (Alternate $\angle$'s)
$\angle WZX = \angle YXZ$ (Alternate $\angle$'s)
$ZX = ZX$ (Common)
$\therefore \triangle WXZ \equiv \triangle YZX$ (AAS)

b) $WZ = YX$ ($\because$ corresponding sides of $\equiv \triangle$)

6. In $\triangle PQT$ and $\triangle PMR$
$PQ = PR$ (Given)
$\angle PQT = \angle PRM$ (Base $\angle$'s of isosceles $\triangle$)
$QT = MR$ (Given)
$\therefore \triangle PQT \equiv \triangle PMR$ (SAS)
$\therefore PT = PM$ ($\because$ corresponding sides of $\equiv \triangle$)
$\therefore \triangle PTM$ isosceles

7. In $\triangle DEG$ and $\triangle EDF$
$DE = DE$ (Common)
$\angle DEG = \angle EDF$ (Given)
$GE = FD$ (Given)
$\therefore \triangle DEG \equiv \triangle EDF$ (SAS)

8. a) In $\triangle ABC$ and $\triangle ADC$
$BC = CD$ (Given)
$BA = DA$ (Given)
$CA = CA$ (Common)
$\therefore \triangle ABC \equiv \triangle ADC$ (SSS)
$\therefore \angle BAC = \angle DAC$ (Corresponding $\angle$'s of $\equiv \triangle$)

12. In $\Delta DKG, \Delta DLE$

$GD = DE$ (properties of a square) ✓

$\angle KGD = \angle LED$ (properties of a square) ✓

$GF = FE$ (properties of a square) ✓

$KF = FL$ (given)

$\therefore GF - KF = FE - FL$ ✓

$\therefore \Delta DKG \equiv \Delta DLE$ (SAS) ✓

13. In $\Delta AOB, \Delta DOC$

$AB = DC$ (given) ✓

$AO = OC$ (O is the centre of the circle) ✓ (equal radii)

$BO = DO$ (O is the centre of the circle) ✓

$\therefore \Delta AOB \equiv \Delta DOC$ (SSS) ✓

$\therefore \angle AOB = \angle COD$ (matching $\angle$s from congruency test) ✓

14. In $\Delta ADC, \Delta ABC$

$AD = AB$ (given)

$\angle DAC = \angle BAC$ (CA bisects $\angle DAB$)

AC is common.

$\therefore \Delta ADC \equiv \Delta ABC$ (SAS)

$\therefore CD = CB$ (matching sides from congruency)

15. In $\Delta OAC, \Delta OBC$

$AO = OB$ (O is the midpoint of the circle ✓

$AC = CB$ (O is the midpoint of the circle)

OC is the midpoint. ✓

$\therefore \Delta OAC \equiv \Delta OBC$ (SSS) ✓

$\therefore \angle OCA + \angle OCB = 180°$ (straight $\angle$) ✓

Let $\angle OCA$ and $\angle OCB = x$ respectively.

$\therefore 2x = 180°$

$\therefore x = 90°$ or right angle. ✓

16. In $\Delta AMD, \Delta CNB$

$DA = CB$ (properties of a parallelogram) ✓

$\angle DAM = \angle NCB$ (properties of a parallelogram)

$\therefore DC = AB$ (properties of a parallelogram) ✓

$\therefore DN = MB$ (given)

$\therefore NC = DC - DN$ and $AM = AB - MB$ ✓

$\therefore NC = AM$

$\therefore \Delta AMD \equiv \Delta CNB$ (SAS) ✓

Q6 & 7 missing ?