



ĺ						
į		 				
			Stra	dent	Nun	her

2008
TRIAL HIGHER SCHOOL CERTIFICATE
EXAMINATION

Mathematics Extension 1

Afternoon Session Thursday, 14 August 2008

General Instructions

- Reading time 5 minutes
- Working time 2 hours
- Write using blue or black pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

Total marks - 84

- Attempt Questions 1-7
- All questions are of equal value

Disclaimer

Every effort has been made to prepare these 'Trial' Higher School Certificate Examinations in accordance with the Board of Studies documents, Principles for Setting HSC Examinations in a Standards-Referenced Framework (BOS Bulletin, Vol 8, No 9, Nov/Dec 1999), and Principles for Developing Marking Guldelines Examinations in a Standards Referenced Framework (BOS Bulletin, Vol 9, No 3, May 2000). No guarantee or warranty is made or implied that the 'Trial' Examination papers mirror in every respect the actual HSC Examination question paper in any or all courses to be examined. These papers do not constitute 'advice' nor can they be construed as authoritative interpretations of Board of Studies intentions. The CSSA accepts no liability for any reliance use or purpose related to these 'Trial' question papers. Advice on HSC examination issues is only to be obtained from the NSW Board of Studies.

$$\int x^n dx = \frac{1}{n+1} x^{n+1}, \ n \neq -1; \ x \neq 0, \ \text{if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax}, \ a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \ a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \ a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \ a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \ a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, \ a > 0, \ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \ x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx \qquad = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE: $\ln x = \log_e x, x > 0$

Total marks – 84 Attempt Questions 1–7 All questions are of equal value

Answer each question in a SEPARATE writing booklet.

Question 1 (12 marks) Use a SEPARATE writing booklet.

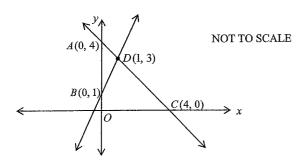
(a) Find the exact value of
$$\int_0^{\frac{\pi}{8}} \sec^2 2x \, dx$$
.

(b) (i) Sketch the graph of
$$y = |2 - x|$$
.

(ii) Using this graph, or otherwise, find the solution to
$$|2-x| < x$$
.

(c) Find the value of k if
$$x+2$$
 is a factor of $P(x) = x^2 + kx + 6$.

(d) A(0, 4), B(0, 1), C(4, 0) and D(1, 3) are points in the plane where D is the point of intersection of the two lines shown. Find, correct to the nearest minute, the size of the acute angle, $\angle BDC$, between the two lines.



(e) Jasi was trying to find the solution to the inequality $\frac{3}{x+1} < 2$. He stated that the solution is all values of x greater than $\frac{1}{2}$. Solve the inequality $\frac{3}{x+1} < 2$ to determine if Jasi's solution is correct. Total marks – 84
Attempt Questions 1–7
All questions are of equal value

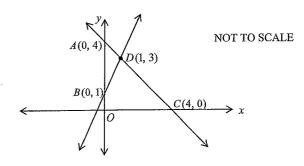
Marks

1

Answer each question in a SEPARATE writing booklet.

Ques	stion 1 (12 marks) Use a SEPARATE writing booklet.	Marks
(a)	Find the exact value of $\int_0^{\frac{\pi}{8}} \sec^2 2x dx$.	2
(b)	(i) Sketch the graph of $y = 2 - x $.	1
	(ii) Using this graph, or otherwise, find the solution to $ 2-x < x$.	2
(c)	Find the value of k if $x + 2$ is a factor of $P(x) = x^2 + kx + 6$.	2

(d) A(0, 4), B(0, 1), C(4, 0) and D(1, 3) are points in the plane where D is the point of intersection of the two lines shown. Find, correct to the nearest minute, the size of the acute angle, $\angle BDC$, between the two lines.



(e) Jasi was trying to find the solution to the inequality $\frac{3}{x+1} < 2$.

He stated that the solution is all values of x greater than $\frac{1}{2}$.

Solve the inequality $\frac{3}{x+1} < 2$ to determine if Jasi's solution is correct.

Marks

1

2

2

Question 2 (12 marks) Use a SEPARATE writing booklet.

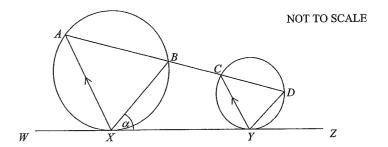
- (a) If α , β and γ are the roots of $2x^3 5x^2 + 3x 5 = 0$ find the value of $\alpha^2 \beta \gamma + \alpha \beta^2 \gamma + \alpha \beta \gamma^2$.
- (b) Let $f(x) = \frac{2x}{\sqrt{1-x^2}}$.
 - (i) For what values of x is f(x) undefined?
 - (ii) Find $\int_0^{\frac{1}{2}} \frac{2x}{\sqrt{1-x^2}} dx$, using the substitution $x = \sin u$.
- (c) (i) Find the derivative of $\sin^{-1} x + \cos^{-1} x$.
 - (ii) Explain why $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$.
- (d) How many different arrangements can be made from the letters of the word EXERCISE if:
 - (i) there are no restrictions?
 - (ii) the letters C and R are at the ends?

Question 3 (12 marks) Use a SEPARATE writing booklet.

Marks

3

(a) In the diagram below, WZ is a common tangent to the two circles and AX is parallel to CY. AD is a straight line through B and C on the circles as shown. Let $\angle BXY = \alpha$.



Copy or trace this diagram into your writing booklet.

- (i) Explain why BX is parallel to DY.
 - Show that BCYX is a cyclic quadrilateral.
- (b) If A and B are both reflex angles, and given $\cos A = \frac{3}{5}$ and $\tan B = \frac{12}{5}$, find the exact value of $\sin(A B)$.
- (c) In the expansion of $(1-kx)^9$ the coefficient of x^6 is half that of the coefficient of x^5 . 3

 Find the value of the constant k.
- (d) Taking x = 2 as the first approximation, use one application of Newton's method to 2 obtain a closer approximation to the solution to $x = \sqrt[3]{9}$.

2

1

3

2

Question 4 (12 marks) Use a SEPARATE writing booklet.

- Prove by mathematical induction that $\sum_{r=1}^{n} r \times r! = (n+1)! -1$.
- 3

2

1

- The acceleration of a particle P, moving in a straight line, is given by $\ddot{x} = 2x 3$ (b) where x metres is the displacement from the origin O. Initially the particle is at O and its velocity ν is 2 metres per second.
 - Show that the velocity ν of the particle is $\nu^2 = 2x^2 6x + 4$.
 - Calculate the velocity and acceleration of P at x=1 and briefly describe 2 the motion of P after it moves from x = 1.
- The rate of change of the number of bees infected by a disease is given by the equation $\frac{dN}{dt} = N(200 - N)$, where N is the number of infected bees in the hive at time t years. There are 200 bees in the hive.
 - If k is a constant, show that $N = \frac{200}{1 + ke^{-200t}}$ satisfies the above equation. 2
 - 2 If at time t = 0 one bee was infected, after how many days will half the colony be infected?
 - Show that eventually all the bees will be infected.

Question 5 (12 marks) Use a SEPARATE writing booklet

- Let $P(x) = -2x^3 + px^2 qx + 5$.
 - Show that if P(x) is to have any stationary points, then $p^2 6q \ge 0$.
 - Discuss the situation when $p^2 6q = 0$.
- A camera, one kilometre away in the horizontal direction from where the Space Shuttle is being launched, is tracking the ascent of the Shuttle. Assume the Shuttle ascends vertically.

Thirty seconds after the launch the Shuttle reaches a height, h, of 3240 metres and it is travelling at a speed of 230 metres per second.

The angle θ is the angle of elevation of the camera as it tracks the Shuttle. At what rate is θ increasing 30 seconds after the Shuttle is launched?

- Diana loves to play basketball. From the free throw line she makes 2 out of every 5 baskets that she throws. For every basket that she makes she scores one point.
 - In her game last week she had 6 free throws. 1 What is the probability that she scored 2 points?
 - How many free throws would she need in one game so that the probability that she scores at least one point is 0.9978?

Question 5 continues on page 7

3

2

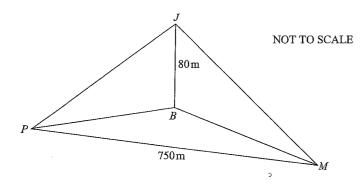
2

2

2

(d) Janus, J, is on top of an 80 metre cliff, watching the Sydney to Hobart yacht race.

From the base of the cliff, B, directly below Janus, Poseidon, P, is on a bearing of 202° and Majorca, M, is on a bearing of 140°. Majorca is 750 m from Poseidon on a bearing of 110°.

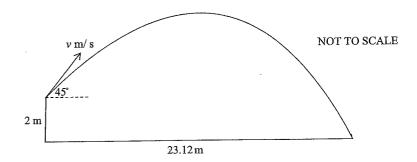


Copy or trace this diagram into your writing booklet.

Find the angle of depression of Poseidon, P, from Janus, J.

End of Question 5

- (a) Consider the function given by $f(x) = \frac{e^x}{x-1}$.
 - (i) Determine all vertical and horizontal asymptotes of the graph of y = f(x).
 - (ii) Find any stationary point(s) and sketch the graph of y = f(x) including any intercepts with the coordinate axes.
 - (iii) State the largest positive domain for which f(x) has an inverse.
- (b) The world record for men's shot-put is 23.12 metres.
 You may assume that the shot-put is projected at an initial velocity of v m/s from a height of 2 metres at an angle of projection of 45°, there is no air resistance and that the acceleration due to gravity is 10 m/s².

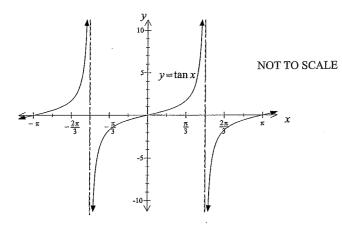


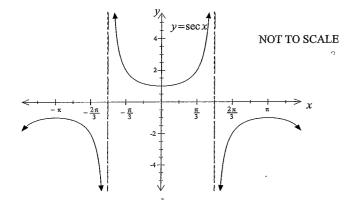
- (i) Use integration to show that the equations of motion are $x = \frac{vt}{\sqrt{2}}$ and $y = -5t^2 + \frac{vt}{\sqrt{2}} + 2$.
- (ii) Find the minimum velocity v m/s at which the shot-put must be projected to achieve the world record distance.
- (iii) What is the maximum height that the shot-put reaches in its path if it is projected with this velocity?

Marks

Question 7 (12 marks) Use a SEPARATE writing booklet

(a) The graphs shown are of $y = \tan x$ and $y = \sec x$ respectively.





- (i) Prove that $\sec \theta \tan \theta = \frac{1}{\sec \theta + \tan \theta}$
- ii) Explain why $0 < \sec \theta \tan \theta \le 1$ for $0 \le \theta < \frac{\pi}{2}$.
- (iii) Solve the equation $\sec \theta \tan \theta = \frac{1}{2}$ for $0 \le \theta < \frac{\pi}{2}$.

2

2

3

Question 7 continues on page 10

Question 7 (continued)

Marks

1

- (b) (i) From a point A(p,q) perpendiculars AP and AQ are drawn to meet the x and y axes at P(p,0) and Q(0,q) respectively.

 Find the equation of PQ.
 - (ii) Show that the condition for the line PQ to be a tangent to the parabola $x^2 = 4ay$ is $aq + p^2 = 0$.
 - (iii) If the points P(p, 0) and Q(0, q) move on the x and y axes respectively such that PQ is a tangent to the parabola $x^2 = 4ay$ then the point A(p, q) traces out a curve as P and Q move.

End of paper



CATHOLIC SECONDARY SCHOOLS ASSOCIATION 2008 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION MATHEMATICS EXTENSION 1

Question 1 (12 marks)

(a) (2 marks)
Outcomes assessed: H5

Lurgetea Perjormance Banas: E2-E3		
	Criteria	Marks
 finds the correct primitive 		1
 evaluates the integral correctly 		1

Sample Answer:

$$\int_{0}^{\frac{\pi}{8}} \sec^{2} 2x \, dx = \frac{1}{2} \left[\tan 2x \right]_{0}^{\frac{\pi}{8}}$$

$$= \frac{1}{2} (\tan \frac{\pi}{4} - \tan 0)$$

$$= \frac{1}{2} (1 - 0)$$

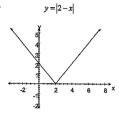
$$= \frac{1}{2}$$

(b) (i) (1 mark)

Outcomes assessed: P4
Targeted Performance Bands: E2-E3

Criteria	Mark
• draws the correct graph of $y = 2 - x $, including intercepts	1

Sample Answer:



ed in this document is intended for the professional assistance of leaching staff, it does not constitute advice to students. Further it is not the intenden or CSS ing outcomes for all possible Total HSC Sample America. Rother the purpose is to provide teachess with information so that they can better expione, understan

audies. or use of CSSA Marking Guidelines in relation to any specific trial exam question or Sa usefulness of any Marking Guidelines provided for the Trial HSC papers.

3201-2

(e) (2 marks)

Outcomes assessed: PE3
Targeted Performance Bands: E2-E3

Criteria	Marks
 establishes correct quadratic or other correct significant step towards solution 	1
finds full solution	1

Sample Answer:

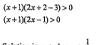
$$\frac{3}{x+1} < 2 \quad \text{multiply by } (x+1)^2$$

$$3(x+1) < 2(x+1)^2$$

$$2(x+1)^2 - 3(x+1) > 0$$

$$(x+1)(2x+2-3) > 0$$

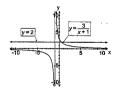
$$(x+1)(2x-1) > 0$$



Solution is x < -1 or $x > \frac{1}{2}$

:. Jasi's solution is only partially correct.

or graphically:



ordinate of intersection $\frac{3}{x+1}$

From the graph $x > \frac{1}{2}$ satisfies the inequality BUT the other branch of the hyperbola for x < -1 also satisfies the inequality.

(b)(ii) (2 marks)

Outcomes assessed: PE2

Targeted Performance Bands: E2-E3

Criteria	Marks
finds the point of intersection or other significant step towards solution	1
writes the correct solution	1

Sample Answer:

the Answer:

point of intersection of
$$y=x$$
 and $y=|2-x|$ is $(1,1)$

... from the graph $|2-x| < x$ when $x > 1$

(c) (2 marks)

Outcomes assessed: PE3

In getta I tijoi mante Banas. B2-B5				
Criteria	Marks			
• uses the factor theorem with substitution of $x = -2$	1			
solves the equation to find k	1			

Sample Answer:

$$P(x) = x^2 - kx + 6$$

 $P(-2) = 4 + 2k + 6 = 0$ as $(x + 2)$ is a factor $k = -5$

(d) (3 marks)

Outcomes assessed: PE2
Targeted Performance Bands: E2-E3

	Criteria	Marks
•	finds the TWO gradients	. 1
•	uses the correct formula for $\tan \theta$	1
•	finds the correct angle	1

Sample Answer:

From the diagram the gradients of the lines are 2 and -1.

$$\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

$$= \left| \frac{2 + 1}{1 - 2} \right|$$

$$= 3$$

$$\therefore \theta = 71^{\circ}34'$$

INSCILLABER

The information contained in this document is intended for the psychosical sociations of teaching stell. It does not consider a fivile to students. Further it is not the intention of the CSSA to psychologous expected mentions of the containing the psychosic process of the psychosic psychosic

Question 2 (12 marks)

(a) (2 marks)
Outcomes assessed: PE3

Targeted Performance Bands: E2-E3					
	Criteria	Mark			
•	finds sum and product of roots	1			
٠	evaluates the expression correctly	1			

$$2x^3 - 5x^2 + 3x - 5 = 0$$

$$\alpha + \beta + \gamma = \frac{5}{2} \quad \alpha\beta + \alpha\gamma + \beta\gamma = \frac{3}{2} \quad \alpha\beta\gamma = \frac{5}{2}$$

$$\alpha^2\beta\gamma + \alpha\beta^2\gamma + \alpha\beta\gamma^2 = \alpha\beta\gamma(\alpha + \beta + \gamma)$$

$$= \frac{5}{2} \times \frac{5}{2}$$

$$= \frac{25}{4}$$

(b) (i) (1 mark)

Outcomes assessed: P4 on Dander E2 E2

Criteria	Mark
gives the correct solutions	1

Sample Answer:

Given
$$f(x) = \frac{2x}{\sqrt{1-x^2}}$$

 $f(x)$ is undefined when $1-x^2 \le 0$
i.e. when $x \le -1$ or $x \ge 1$

ord in this document is intended for the profuscional assistance of reaching staff. It does not constitute advisor to students. Further it is not the intention of the to matching outcomes for all possible Trial HSC caswers. Rother the purpose is to provide teachers with information so that they can better capitors, understan recruitments, as enthinked by the NSP these of Students.

As a single continued to the profuscion of the students of th

DISCLAMBR
The information contained in this document is intended for the professional assistance of tenching staff. It does not constitute solvies to suderats. Further it is not the intention of the CSSA to provide specific marking outcomes for all possible Trial HSC autswerk. Rather the purpose is to provide teachers with information so that they can better explore, understand and apply HSC marking requirements, are stabilished by the NSW Board of Studies.

No guestrate can wrateraje is under or implied with respect to the application or ten or of CSSA Marking Cuitédines in relation to any specific trial exact question or canwer. The CSSA assumes no liability nor responsibility for the accuracy, completeness or usefulness of any Marking Guidelines provided for the Trial HSC paper.

Outcomes assessed: HE6

Criteria	Marks
 rewrites the integral in term of u 	1
finds the new limits	1
• evaluates the integral correctly to at least $-2\left[\frac{\sqrt{3}}{2}-1\right]$ (correct numerical equivalence)	1

Sample Answer:

er:

$$\int_{0}^{\frac{1}{2}} \frac{2x}{\sqrt{1-x^{2}}} dx = \int_{0}^{\frac{\pi}{6}} \frac{2\sin u}{\cos u} \cos u \, du \qquad x = \sin u$$

$$dx = \cos u \, du$$

$$= 2 \int_{0}^{\frac{\pi}{6}} \sin u \, du \qquad x = 0 \Rightarrow u = 0$$

$$= -2 \left[\cos u\right]_{0}^{\frac{\pi}{6}} \qquad x = \frac{1}{2} \Rightarrow u = \frac{\pi}{6}$$

$$= -2 \left[\frac{\sqrt{5}}{2} - 1\right]$$

$$= 2 - \sqrt{3}$$

(c) (i) (1 mark)

Outcomes assessed: HE4
Targeted Performance Bands: E2-E3

	Criteria	Mark
differentiates correctly		1

wer:

$$\frac{d}{dx}(\sin^{-1}x + \cos^{-1}x) = \frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-x^2}}$$
= 0

c continced in this document is intended for the professional assistance of teaching staff. It does not constitute subvice to entidents. Further it is not the intended of the desponds marking contennes for all possible This HSC nanvers. Rather the purpose is to provide teachers with information so that they can better explore, understand marking requirements, as entablished by the NSW Board of This MSC nanvers. romes. Or use of CSSA Marking Guidelines in relation to any specific triel exem question or answer. The CSSA tilness of any Marking Guidelines provided for the Trial ESS2 papers

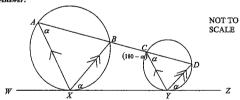
Question 3 (12 marks)

(a) (i) (3 marks)

Outcomes assessed: PE2, PE3

Criteria	Marks
 correct application of alternate segment theorem or other correct step in explanation 	1
identifying corresponding angles or other significant progress in reasoning	1
conclusion with reason	1

Sample Answer:



Given $\angle BXY = \alpha$

 $\therefore \angle BAX = \alpha$ (angle between tangent and chord at the point of contact is equal to

Leave the angle in the alternate segment) $\angle DCY = \alpha$ (corresponding to $\angle BAX$, $AX \parallel CY$) $\angle DYZ = \alpha$ (angle between tangent and chord at the point of contact is equal to the angle in the alternate segment)

∴ ∠DYZ= ∠BXY ∴BX DY (corresponding angles are equal)

(a) (ii) (1 mark)

Outcomes assessed: PE2, PE3

Lar	geted Performance Bands: E2-E3			
	Criteria Criteria	Mai	rk	
•	states the correct reason	1	i	ł

 $\angle BCY = 180 - \alpha$ (BCD is a straight line)

 $\therefore \angle BCY + \angle BXY = 180^{\circ}.$

...BCYX is a cyclic quadrilateral as one pair of opposite angles are supplementary

(c) (ii) (2 marks)

Outcomes assessed: HE4

Turgeteu Ferjormunce Bunus: E2-E3		
	Criteria	Marks
٠	identifies that the primitive is a constant	1
•	uses a suitable substitution, or otherwise, to show that the constant is $\frac{\pi}{2}$	1

Sample Answer:

Since the derivative is zero, $\sin^{-1} x + \cos^{-1} x = C$ (C is a constant)

Let
$$x = 0 \Rightarrow \sin^{-1} 0 + \cos^{-1} 0 = 0 + \frac{\pi}{2}$$

= $\frac{\pi}{2}$

(d) (i) (1 mark)

Outcomes assessed: PE3

Lurgeieu Ferjormance Banas: £2-£3			
	Criteria	Mark	
	correct numerical expression for the answer	1	

Sample Answer:
EXERCISE
$$\Rightarrow$$
 8 letters with 3 Es
Number of arrangements = $\frac{81}{31}$ = 6720

Outcomes assessed: PE3

Targelea Ferjormance Banas: E2-E3	
Criteria	Marks
significant progress towards solution	1
correct numerical expression for answer	1

EXERCISE with C and R at the ends

Number of arrangements =
$$\frac{216!}{3!}$$
 = 240

DISCLAMMER
The laboration contained in this document is intended for the profusional assistance of teaching staff. It does not constitute advice to student. Further it is not the intention of the management of the profusion of

(b) (3 marks)

Outcomes assessed: PE2

	Criteria	Marks
•	determines the correct value of sin A, including the sign	1
•	determines the correct value of $\sin B$ and $\cos B$, including the sign	1
•	expands $sin(A - B)$ and gives a correct numerical expression	1

Sample Answer:

$$\cos A = \frac{3}{5} : \sin A = -\frac{4}{5} (A \text{ is reflex and in the 4th quad})$$

$$\tan B = \frac{12}{5}$$
 .. $\sin B = -\frac{12}{13}$ and $\cos B = -\frac{5}{13}$ (B is reflex and in the 3rd quad)

$$(A - B) = \sin A \cos B - \cos A \sin B$$

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

$$= \left(-\frac{4}{5}\right) \left(-\frac{5}{13}\right) - \left(\frac{3}{5}\right) \left(-\frac{12}{13}\right)$$

$$= \frac{56}{65}$$

(c) (3 marks)

Outcomes assessed: HE7

Targeted Performance Bands: E3-E4	
Criteria	Marks
 writes down at least one correct term in x⁶ or x⁵ 	1
writes down the correct equivalence	1
finds the correct value of k (correct numerical equivalence)	1

Sample Answer:

Consider terms of the expansion of $(1-kx)^9$

Term in
$$x^5 = {}^9C_5(-kx)^5$$
 Term in $x^6 = {}^9C_6(-kx)^6$
= $-{}^9C_5k^5x^5$ = ${}^9C_6k^6x^6$

$$\therefore -{}^{9}C_{5}k^{5} = 2{}^{9}C_{6}k^{6}$$

$$k = -\frac{{}^{9}C_{5}}{2{}^{9}C_{6}}$$

$$= -\frac{1}{2} \times \frac{9!}{5!4!} \times \frac{3!6!}{9!}$$

$$= -\frac{3}{4}$$

DISCLASTOR

The information contained in this document is intended for the professional astriature of reaching staff. If does not constitute advice to students. Further it is not the intention of the CSSA to provide specific numbering quickennes for all possible Trial HSC answers. Astrict the purpose is to provide specific numbers with information to stude they can better explores, understand an poly HSC instruits requirements, as established by the SNP Meand of Situation.

No guanation now rewritely is made or implied with respect to the application or use of CSSA Marking Onlidelines in relation to not specific that exam question or asswers. The CSSA consumes no inhalting or emporation for the occurso, respectively or exploration or desired provided for the Trial HSC papers.

Outcomes assessed: PE3_HE7

Targeted Performance Bands: E2-E3

Criteria	Marks
identifies the function	1
 uses Newton's Method to find correct approximation (correct numerical equivalence) 	1

Sample Answer:

$$x = \sqrt[3]{9}$$

$$\therefore x^3 = 9 \Rightarrow f(x) = x^3 - 9 \therefore f'(x) = 3x^2$$
Let $x_1 = 2 \therefore x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} \Rightarrow x_2 = 2 - \frac{-1}{12}$

$$x_2 = 2\frac{1}{12}$$

Question 4 (12 marks) (a) (3 marks)

Outcomes assessed: HE2

Rands: F3_F4

Criteria	Marks
 establishes the truth of S(1) 	1
• establishes the correct relationship between $S(k)$ and $S(k+1)$	1
deduces the required result	1

Sample Answer:

Let
$$S(n)$$
 be the statement $\sum_{i=1}^{n} r \times r! = (n+1)! - 1$

Consider S(1):

LHS =
$$1 \times 1!$$
; RHS = $(1+1)! - 1 = 1$.
Hence $S(1)$ is true

If S(k) is true:

$$\sum_{r=1}^{k} r \times r! = (k+1)! - 1$$
 *

RTP
$$S(k+1)$$
 is true i.e. to prove $\sum_{r=1}^{k+1} r \times r! = (k+2)! - 1$

$$LHS = \sum_{r=1}^{k} r \times r! + (k+1)(k+1)!$$

$$= (k+1)! - 1 + (k+1)(k+1)! \quad \text{if } S(k) \text{, using } *$$

$$= (k+1)! (1+k+1) - 1$$

$$= (k+1)! (k+2) - 1$$

Hence if S(k) then S(k+1) is true. Thus since S(1) is true it follows by induction that S(n) is true for positive integral n.

DISCLAMER
The information contained in this document is intended for the professional assistance of teaching staff. It does not constitute achieve to students. Further it is not the intention of the CSSA to provide specific marking concerns for all youshite Irial HSC asswers. Rather the purpose is to provide teachers with information so that they can bester capture, understand and apply HSC marking requirements, as established by the NSW Born of Studies.

**The Administration of the Constitution of the CSSA by the CS

(c) (i) (2 marks)

Outcomes assessed: HE3

Targetea Performance Bands: E3-E4	
Criteria	Marks
differentiates correctly	1
proves correct result	1

$$N = \frac{200}{1 + ke^{-200t}} = 200 \left(1 + ke^{-200t} \right)^{-1}$$

$$\frac{dN}{dt} = -200(1 + ke^{-200t})^{-2} (-200ke^{-200t})$$

$$= \frac{200}{1 + ke^{-200t}} \left(\frac{200ke^{-200t}}{1 + ke^{-200t}} \right)$$

$$= N \left(\frac{200 + 200ke^{-200t}}{1 + ke^{-200t}} \right)$$

$$= N \left(\frac{200(1 + ke^{-200t})}{(1 + ke^{-200t})} - \frac{200}{1 + ke^{-200t}} \right)$$

$$= N(200 - N)$$

(c) (ii) (2 marks)

Outcomes assessed: HE3
Toracted Parformance Rands: E3-E4

Targetea Performance Banas: E3-E4	
Criteria	Marks
• finds the value of k	1
finds t (correct numerical equivalence)	1

Sample Answer:

when
$$t=0$$
, $N=1$ i.e. $1=\frac{200}{1+k}$ $\therefore k=199$

$$N=\frac{200}{1+199e^{-200t}}$$
 half the colony infected i.e. $N=100$

$$100=\frac{200}{1+199e^{-200t}}$$

$$1+199e^{-200t}=2$$

$$e^{-200t}=\frac{1}{199}$$

$$-200t=\log_*\frac{1}{199}$$

$$\therefore t=-\frac{1}{200}\log_e\frac{1}{199}$$

$$t=0.0265 \text{ years or } 9.66 \text{ days}$$

ained in this document is intended for the professional satistance of teaching staff. It does not constitute advice to students. Further it is not the intention of the efficient staffing outcomes for all possible Trial ISSC canvers. Rather the purpose is to provide teachers with information so that they can better explore, understand any provincements, are exhibited by the NSP Weard of Studies, many is made or implied with respect to the application or use of CSSA Marking Guidelines in staff one to provide teachers or responsibility for the concessy, completions or useful for the concess, completions or customers of early Minching Guidelines provided for the This ISSC papers.

(b) (i) (2 marks)

Outcomes assessed: HE5
Targeted Performance Bands: E2-E3

	good I cijo. manco Santagi E2 E5	
_	Criteria	Marks
٠	sets up the correct differential equation or significant progress towards result	1
•	finds the desired equation	1

Sample Answer:

$$\ddot{x} = 2x - 3$$

$$\therefore \frac{d}{dx} \left(\frac{1}{2}v^2\right) = 2x - 3$$

$$\frac{1}{2}v^2 = x^2 - 3x + c$$
when $x = 0, v = 2$

$$\therefore \frac{1}{2}v^2 = x^2 - 3x + 2$$

$$\therefore v^2 = 2x^2 - 6x + 4$$

(b) (ii) (2 marks)

Outcomes assessed: HE5

describes the motion

l ar	geted Performance Bands: E3-E4			
	Criteria Criteria	Mar	ks	
•	calculates the correct velocity and acceleration	1		

at
$$x = 1$$
, $v = 0$ and $\ddot{x} = -1$ m/s²

After the object comes to rest at x=1 it then moves towards the origin, and will continue moving in a negative direction.

ment is intended for the professional assistance of tenching staff. It does not constitute advice to students. Further it is not the intention of the cones for all possible Trial ISIG nawvers. Rather the purpose is to provide tenches with information so that they can better explore, understand is established by the NSW Board of Shalles. ne or use of CSSA Marking Guidelines in relation to any specific trial exem question or answer. The CSSA fulness of any Marking Guidelines provided for the Trial HSC papers.

(c) (iii) (1 mark)

Outcomes assessed: HE7

argeted Pet	formance Bands: E3-E4	
	Critoria	

L_	Criteria	Mark
•	shows that the limiting value of N is 200	1

as
$$t \to \infty$$
 $e^{-200t} \to 0$
 $\therefore N \to \frac{200}{1+0} = 200$ i.e. eventually all the bees will be infected.

Question 5 (12 marks)

Outcomes assessed: H5, HE7 proeted Performance Rands: F2_F3

	Criteria	Marks
•	differentiates and determines the correct quadratic	1
•	identifies that $\Delta \ge 0$ and sets up the correct inequality	1

Sample Answer:

$$P(x) = -2x^3 + px^2 - qx + 5$$

 $P'(x) = -6x^2 + 2px - q = 0$ for stationary points

For this quadratic to have real solutions

 $\Delta \ge 0$ i.e. $4p^2 - 24q \ge 0$

 $p^2-6q\geq 0$

(a) (ii) (1 mark)

Outcomes assessed: H5. HE7

Targeted Performance Bands; E3-E4	
Criteria	Mark
gives the correct conclusion	1 1

Sample Answer:

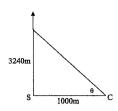
When $p^2 - 6q = 0$ P'(x) has a double root and there is only one stationary point which would be a horizontal point of inflexion.

ment is intended for the professional swistence of teaching staff. It does not constitute advice to students. Further it is not the intention of the connect for all possible This HSC saswers. Rather the purpose is to provide teachers with information so that they can better explore, underscade as entablished by the SWFW Board of Chilano, and the contract of the contract of the provided in the contract of the implied with respect to the application or use of CSSA Marking Guidelines in relation to any specific trial exam question or answer. The CSSA for the accuracy, completeness or usefullows of any Marking Guidelines growted for the This HSC paper.

Outcomes assessed: HE5, HE7
Tayantad Payformanca Rands: E3-E4

Criteria	Marks
• establishes $\frac{dh}{d\theta}$	1
• evaluating θ at $t=30$ s, or other significant progress towards the result such as correct use of chain rule	1
correct answer	1

Sample Answer:



$$\frac{dh}{dt} = 230 \text{m/s}$$

$$\tan \theta = \frac{h}{1000}$$

$$h = 1000 \tan \theta$$

$$\frac{dh}{d\theta} = 1000 \sec^2 \theta$$

at
$$t = 30$$
 seconds, $h = 3240$ m $\Rightarrow \tan \theta = \frac{3240}{1000}$ $\Rightarrow \theta = 1.271$ radians

$$\frac{d\theta}{dt} = \frac{d\theta}{dh} \times \frac{dh}{dt}$$
$$= \frac{\cos^2 1.271}{1000} \times 230$$
$$= 0.02 \text{ rads/sec}$$

(d) (3 marks)

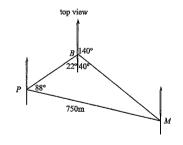
Outcomes assessed: PE6, HE7
Targeted Performance Bands: E2-E3

Criteria	Marks
 some progress towards result, e.g. finds the missing angles 	1
 significant progress towards result, e.g. finding BP (correct numerical equivalence) 	1
finds the angle of depression	1

Sample Answer:

∠BPM =
$$110^{\circ} - 22^{\circ} = 88^{\circ}$$
∠PBM = $202^{\circ} - 140^{\circ} = 62^{\circ}$
∴∠PMB = 30°
∴ $\frac{BP}{\sin 30^{\circ}} = \frac{750}{\sin 62^{\circ}}$
∴ $\frac{BP}{\sin 62^{\circ}} = \frac{750\sin 30^{\circ}}{\sin 62^{\circ}}$

$$= 424.71 \quad (2 \text{ d.p.})$$
angle of elevation = $\tan^{-1}\left(\frac{80}{BP}\right)$
= $10^{\circ}40^{\circ}$
∴ angle of depression is $10^{\circ}40^{\circ}$



Question 6 (12 marks)

(a) (i) (2 marks)
Outcomes assessed: H5

Targeted Performance Bands: E3-E4	
Criteria	Marks
finds the vertical asymptote	1
finds the horizontal asymptote	1

Sample Answer:

$$f(x) = \frac{e^x}{x - 1}$$
vertical asymptote at $x = 1$
horizontal asymptote: $x \to \infty$, $f(x) \to \infty$; $x \to -\infty$, $f(x) \to 0^ \therefore y = 0$ is a horizontal asymptote as $x \to -\infty$

r strances. On or use of CSSA Marking Guidelines in relation to any specific trial exam question or austrofulness of any Marking Guidelines provided for the Trial HSC papers.

(c) (i) (1 mark)

Outcomes assessed: HE3 Targeted Performance Bands: E2-E3

Criteria	Mark
a given the second of	TATAL W
gives the correct answer (correct numerical equivalence)	1
	. 1

Sample Answer:

$$P(\text{a basket}) = \frac{2}{5}$$

$$P(\text{2 points}) = {}^{6}C_{2} \left(\frac{2}{5}\right)^{2} \left(\frac{3}{5}\right)^{4}$$

$$= \frac{972}{3125} \text{ (or 0.31104)}$$

(c) (ii) (2 marks)

Outcomes assessed: HE3, HE7

Tuigeteu I cijoimance Banas: E3-E4	
Criteria	Marks
sets up correct inequality or other significant progress	1
gives the correct solution, rounding to the nearest whole number	1

P(at least one) =
$$1 - P(\text{none})$$

= $1 - 0.6^{\circ}$

 $\therefore 1 - 0.6^n \ge 0.9978$

 $0.6^n \le 0.0022$

take logs of both sides

i.e. $n \ln 0.6 \le \ln 0.0022$

 $n \ge \frac{\ln 0.0022}{}$ ln 0.6

 $n \ge 11.979$ $n \ge 12$

Diana would need 12 free throws.

ment is intended for the professional assistance of teaching staff. It does not constitute advice to students. Further it is not the intention of the other for all possible This PSC convers. Rather the purpose is to provide teachers with information so that they can better explore, understand as rathicitied by the ISSW Droad of Studies.

replied with respect to the spillcotton or use of CSSA Marking Guidelines in relation to my specific trial exam quantion or surver. The CSSA

(a) (ii) (3 marks)
Outcomes assessed: PE5, PE6, H5
Targeted Performance Bands: E3-E4

	Criteria	Marks
•	finds the stationary point	1
•	identifies intercept	1
•	sketches the correct function	1

Sample Answer:

$$f'(x) = \frac{(x-1)e^x - e^x}{(x-1)^2}$$

$$= \frac{e^x(x-2)}{(x-1)^2}$$

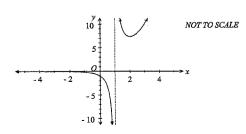
$$f'(x) = 0 \text{ when } x = 2 \text{ and } y = e^2$$

testing nature:



 $\therefore (2, e^2)$ is a minimum point

when
$$x = 0$$
, $y = \frac{e^0}{0-1} = -1$ $\therefore y$ intercept is $(0,-1)$



.Hoard of Mudres. application or uso of CSSA Marking Guidelines in relation to any specific trial exam question or resis or usefulness of any Marking Guidelines provided for the Trial HSC papers.

(a) (iii) (1 mark)
Outcomes assessed: HE\$

Targeted Performance Bands: E2-E3

Criteria	Marks
gives correct domain	1

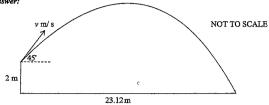
For an inverse function domain is $x \ge 2$

(b) (i) (2 marks)

Outcomes assessed: HE3
Targeted Performance Ra

Turgeteu Terformance Bunus: E2-E3	
Criteria Criteria	Marks
 finds the correct differential equations for the horizontal motion 	1
finds solves the correct differential equations for the vertical motion	1





Horizontal

$$\ddot{x} = 0$$

$$\dot{x} = c_1$$
at $t = 0$, $\dot{x} = v\cos 45^\circ \Rightarrow c_1 = v\cos 45^\circ$

$$\therefore \dot{x} = \frac{v}{\sqrt{2}}$$

$$x = \frac{vt}{\sqrt{2}} + c_2$$
at $t = 0$, $x = 0 \Rightarrow c_2 = 0$

$$x = \frac{vt}{\sqrt{2}}$$

Vertical

$$\ddot{y} = -10
\dot{y} = -10t + c_3
\text{at } t = 0, \dot{y} = v \sin 45^\circ \Rightarrow c_3 = v \sin 45^\circ
\therefore \dot{y} = -10t + \frac{v}{\sqrt{2}}
y = -5t^2 + \frac{vt}{\sqrt{2}} + c_4
\text{at } t = 0, y = 2 \Rightarrow c_4 = 2
y = -5t^2 + \frac{vt}{\sqrt{2}} + 2$$

nent is intended for the professional assistance of teaching staff. It does not constitute advice to students. Further it is not the intention of the omes for all possible Trial HSC answers. Rather the purpose is to provide teachers with information so that they can better explore, understan

Question 7 (12 marks)

(a) (i) (2 marks)
Outcomes assessed: H5, PE2
Translad Porformance Bands: E3-E4

Targetea Performance Banas: E5-E4	
Criteria	Marks
• Showing $\frac{\sec^2\theta - \tan^2\theta}{\sec\theta + \tan\theta}$ or other significant progress	1
completing the proof	1

Sample Answer:

RTP
$$\sec \theta - \tan \theta = \frac{1}{\sec \theta + \tan \theta}$$

LHS
$$= \frac{(\sec \theta - \tan \theta)(\sec \theta + \tan \theta)}{(\sec \theta + \tan \theta)}$$

$$= \frac{\sec^2 \theta - \tan^2 \theta}{\sec \theta + \tan \theta}$$

$$= \frac{1}{\sec \theta + \tan \theta}$$

$$= \frac{1}{\sec \theta + \tan \theta}$$
= RHS

(a) (ii) (2 marks)
Outcomes assessed: PE2, HE7

Tar,	geted Performance Bands: E3-E4	
	Criteria Criteria	Marks
•	establishing $\sec \theta + \tan \theta \ge 1$ or other significant progress	1
	justifying the inequality	1

Sample Answer:

From the graphs
$$\sec\theta \ge 1$$
 and $\tan\theta \ge 0$ for $0 \le \theta < \frac{\pi}{2}$
 $\therefore \sec\theta + \tan\theta \ge 1$
 $\therefore 0 < \frac{1}{\sec\theta + \tan\theta} \le 1$
i.e. $0 < \sec\theta - \tan\theta \le 1$ using (i)

(b) (ii) (2 marks)

Outcomes assessed: HE3

1 ar	getea Perjormance Banas: E5-E4	
	Criteria	Marks
•	determines the Cartesian equation of motion or other significant progress	1
•	substitutes and solves for v	1

Sample Answer:

$$t = \frac{\sqrt{2}x}{v}$$
.. $y = -5 \times \frac{2x^2}{v^2} + x + 2$
at world record range $x = 23.12$ and $y = 0$

$$\therefore 0 = -\frac{10 \times 23.12^2}{v^2} + 25.12$$

$$\frac{5345.344}{v^2} = 25.12$$

$$v^2 = 212.7923567$$

$$v = 14.59 \text{ m/s} (2 \text{ decimal places})$$

(b) (iii) (2 marks)

Outcomes assessed: HE3

Targeted Performance Bands: E3-E4				TA T /
	Targeted.	Pertormance	Bands:)	E3-E4

Luigeteu I erjoimance Danas. 255-254	
Criteria	Marks
• finds the value of t in terms of v	1
substitutes into y and solves	1

maximum height when $\dot{y} = 0$

i.e.
$$t = \frac{v}{10\sqrt{2}}$$

 $y = -5 \times \frac{v^2}{10^2 \times 2} + \frac{v^2}{10 \times 2} + 2$
 $= \frac{v^2}{20} - \frac{v^2}{40} + 2$
 $= \frac{v^2}{40} + 2$
 $= \frac{212.7923567}{40} + 2$
 $= 7.32 \text{ m (2 decimal places)}$

18

ument is intended for the professional assistance of teaching staff. It does not constitute advice to students. Further it is not the intention of the comes for all possible Trial HSC earwers. Rather the purpose is to provide teachers with information so that they can better explore, understand as established by the NSV board of Studets.

The purpose is to be application or use of CSSA Marking Guidelines in relation to any specific trial exam question or surver. The CSSA

(a) (iii) (3 marks)

Outcomes assessed: HE7

Criteria	Marks
• progress towards obtaining a quadratic equation e.g. establishing $2\cos\theta = 1 + \sin\theta$	1
• obtaining the correct quadratic in terms of $\sin\theta$ or equivalent progress	1
correct solution	1

Sample Answer:

$$\sec \theta - \tan \theta = \frac{1}{2}$$
i.e.
$$\frac{1}{\sec \theta + \tan \theta} = \frac{1}{2}$$

$$\therefore \sec \theta + \tan \theta = 2$$

$$\frac{1}{\cos \theta} + \frac{\sin \theta}{\cos \theta} = 2$$
i.e.
$$2\cos \theta = 1 + \sin \theta$$

Square and solve the quadratic:

$$4\cos^2\theta = 1 + 2\sin\theta + \sin^2\theta$$

$$4\left(1 - \sin^2\theta\right) = 1 + 2\sin\theta + \sin^2\theta$$

$$5\sin^2\theta + 2\sin\theta - 3 = 0$$

$$(5\sin\theta - 3)(\sin\theta + 1) = 0$$
since $\sin\theta$ is positive in the interval, $0 \le \theta < \frac{\pi}{2}$
i.e. $\theta = 0.644$ radians

OR

DISCLADER
The information contained in this document is intended for the professional assistance of leaching staff. It does not constitute advice to students, Further 11 is not the intention of the CSSA to provide specific marking outcomes for all possible Trial HSC answers. Rather the purpose is to provide steakness with information so that they can better capions, understand an popyl HSC camping requirements, are established by the NN by Board of Studies.

No guarantee nor warranty is made or implied with respect to the application or use of CSSA. Marking Childelines in relation to may specific trial exam question or enswer. The CSSA assessmen so including our responsibility for the accumple, completions or user places for the control of the trial HSC pages.

•	progress towards use of auxiliary angle method e.g. establishing $2\sin\theta + \cos\theta = 2$	1
•	obtaining the correct value for R or α using the auxiliary angle method	1
٠	correct solution	1

$$\begin{split} \sec\theta - \tan\theta &= \frac{1}{2} \\ \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta} &= \frac{1}{2} \\ 2 - 2\sin\theta &= \cos\theta \\ 2\sin\theta + \cos\theta &= 2 \end{split}$$

let
$$2\sin\theta + \cos\theta \equiv R\sin(\theta + \alpha)$$
 where R is positive and α is acute i.e. $2\sin\theta + \cos\theta \equiv R\sin\theta\cos\alpha + R\cos\theta\sin\alpha$ $\Rightarrow R\cos\alpha = 2$ and $R\sin\alpha = 1$

$$2 \sin^{-1} 2 \cos^{-1} 2$$

OR

From simplifying to $2\cos\theta = 1 + \sin\theta$ i.e. $2\cos\theta - \sin\theta = 1$ let $2\cos\theta - \sin\theta = R\cos(\theta + \alpha)$ where R is positive and α is acute i.e. $2\cos\theta - \sin\theta = R\cos\theta\cos\alpha - R\sin\theta\sin\alpha$ $\Rightarrow R\cos\alpha = 2$ and $R\sin\alpha = 1$ i.e. $\tan\alpha = \frac{1}{2}$ and $R = \sqrt{5}$

$$\frac{2}{\sqrt{5}\cos(\theta + \tan^{-1}\frac{1}{2}) = 1} = 1$$

$$\cos(\theta + \tan^{-1}\frac{1}{2}) = \frac{1}{\sqrt{5}}$$

$$(\theta + \tan^{-1}\frac{1}{2}) = \cos^{-1}\frac{1}{\sqrt{5}}$$

$$\theta = \cos^{-1}\frac{1}{\sqrt{5}} - \tan^{-1}\frac{1}{2}$$

$$\theta = 0.644 \text{ radians}$$

21

The information contained in this document is intended for the productional assistance of tracting staff. It does not constitute advice to students. Purther it is not the intention of the CSSA to provide openition marking outcomes for all possible in PLISC asserts. Rather the purpose is not provide touches with information on that they can better explore, understand and apply PLISC marking requirements, as entablished by the NSW Board of Studies.

No gazamete nor warranty is made or implied with respect to the application or use of CSSA Marking Guidelines in relation to any specific total exam question or answer. The CSSA stansons no liability nor responsibility for the accessor, completeness or usualizates of any Marking Guidelines provided for the Trial ISSC papers.

(b) (iii) (1 mark)

Outcomes assessed: HE7
Targeted Performance Bands: E3-E4

Targeted Performance Bands: E3-E4

	Criteria	Mark
 finds t 	e locus	1

Sample Answer:

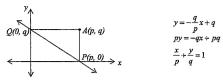
Coordinates of
$$A \Rightarrow x = p$$
 and $y = q$
 \therefore since $aq + p^2 = 0$ from (ii) then $ay + x^2 = 0$
i.e. $x^2 = -ay$ is the locus of A

(b) (i) (1 mark)

Outcomes assessed: HE7 Targeted Performance Bands: E2-E3

		Criteri:	a	Mark	ŗ
•	finds the correct equation of PQ in	any form	O.	1	Τ

Sample Answer:



(b) (ii) (3 marks)

Outcomes assessed: HE7

Targeted Performance Bands: E3-E4	
Criteria	Marks
 determines the quadratic or other significant progress towards the solution 	1
• uses $\Delta = 0$ when PQ is a tangent to the curve	1
establishes the relationship	1

Sample Answer:

If
$$PQ$$
 is a tangent to the parabola then there is one point of intersection.

solve
$$\frac{x}{p}+\frac{y}{q}=1$$
 and $y=\frac{x^2}{4a}$ simultaneously
$$\frac{x}{p}+\frac{x^2}{4aq}=1$$

$$4aqx+px^2=4apq$$

$$px^2+4aqx-4apq=0$$
 for PQ to be a tangent then $\Delta=0$ in this quadratic (i.e. only one root/solution) i.e. $16a^2q^2+16ap^2q=0$
$$16aq(aq+p^2)=0$$
 i.e. $aq+p^2=0$

2

DISCLAMBER.

COSA to provide specific making contenses in instead for the professional sentence of tending setf. If does not constitute derite to students. Further it is not the intension of the COSA to provide specific making contenses for all possible Trail BCC scawers. Rather the pumper is to provide tenders with information to so that they can better explore, understand and apply HSC marking requirements, as established by the NSW bosed of Studies.

No guarantees new remarkly is made or implicable with respect to the specification or common of CSSA between Cost and the provided in relations to any specific trial occurs construction or tensive. The CSSA is provided as the provided in the contract of the provided in the provided in the contract of the provided in the pr

23

The information contained in this document is intended for the professional assistance of teaching staff. It does not constitute advice to students. Further it is not the intention of the CSSA to provide specific marking outdomes for all possible Trial HSC answers. Rather the purpose is to provide teachers with information so that they can better explore, understand exply HSC marking requirements, as established by the RSV PSD mont of Statistic.

No generates now warranty is made or implied with respect to the application or two or CSSA Marking Guidelines in relation to any specific trial exam question or narwer. The CSSA statistics by the RSV processing of the CSSA Marking Guidelines in relation to any specific trial exam question or narwer. The CSSA statistics of the CSSA Marking Guidelines in relation to any specific trial exam question or narwer. The CSSA statistics of the CSSA Marking Guidelines in the CSSA Mark