Centre Number

Student Number

CATHOLIC SECONDARY SCHOOLS
ASSOCIATION OF NEW SOUTH WALES

2008
TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics
Extension 1

Afternoon Session
Thursday, 14 August 2008

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using blue or black pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

Total marks – 84

- Attempt Questions 1-7
- All questions are of equal value

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\quad n \neq -1;\ x \neq 0,\ \text{if } n<0$$

$$\int \frac{1}{x}\,dx = \ln x,\quad x>0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax},\quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax,\quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax,\quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax,\quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax,\quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a},\quad a>0,\ -a<x<a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right),\quad x>a>0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE: $\ln x = \log_e x,\quad x>0$

Total marks – 84
Attempt Questions 1–7
All questions are of equal value

Answer each question in a SEPARATE writing booklet.

Marks

Question 1 (12 marks) Use a SEPARATE writing booklet.

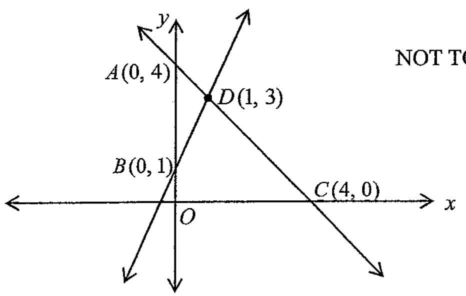
(a) Find the exact value of $\int_0^{\frac{\pi}{8}} \sec^2 2x\,dx$. **2**

(b) (i) Sketch the graph of $y = |2 - x|$. **1**

(ii) Using this graph, or otherwise, find the solution to $|2 - x| < x$. **2**

(c) Find the value of k if $x + 2$ is a factor of $P(x) = x^2 + kx + 6$. **2**

(d) $A(0, 4)$, $B(0, 1)$, $C(4, 0)$ and $D(1, 3)$ are points in the plane where D is the point of intersection of the two lines shown. Find, correct to the nearest minute, the size of the acute angle, $\angle BDC$, between the two lines. **3**



NOT TO SCALE

(e) Jasi was trying to find the solution to the inequality $\frac{3}{x+1} < 2$. **2**

He stated that the solution is all values of x greater than $\frac{1}{2}$.

Solve the inequality $\frac{3}{x+1} < 2$ to determine if Jasi's solution is correct.

Total marks – 84
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All questions are of equal value

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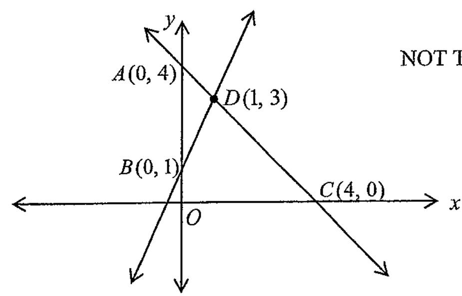

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He stated that the solution is all values of x greater than $\frac{1}{2}$.

Solve the inequality $\frac{3}{x+1} < 2$ to determine if Jasi's solution is correct.

2

Marks

Question 2 (12 marks) Use a SEPARATE writing booklet.

(a) If α, β and γ are the roots of $2x^3 - 5x^2 + 3x - 5 = 0$ find the value of $\alpha^2\beta\gamma + \alpha\beta^2\gamma + \alpha\beta\gamma^2$. **2**

(b) Let $f(x) = \dfrac{2x}{\sqrt{1-x^2}}$.

(i) For what values of x is $f(x)$ undefined? **1**

(ii) Find $\displaystyle\int_0^{\frac{1}{2}} \frac{2x}{\sqrt{1-x^2}}\,dx$, using the substitution $x = \sin u$. **3**

(c) (i) Find the derivative of $\sin^{-1} x + \cos^{-1} x$. **1**

(ii) Explain why $\sin^{-1} x + \cos^{-1} x = \dfrac{\pi}{2}$. **2**

(d) How many different arrangements can be made from the letters of the word EXERCISE if:

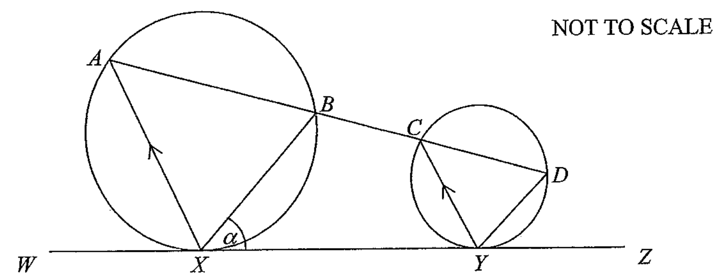
(i) there are no restrictions? **1**

(ii) the letters C and R are at the ends? **2**

Marks

Question 3 (12 marks) Use a SEPARATE writing booklet.

(a) In the diagram below, WZ is a common tangent to the two circles and AX is parallel to CY. AD is a straight line through B and C on the circles as shown. Let $\angle BXY = \alpha$.



Copy or trace this diagram into your writing booklet.

(i) Explain why BX is parallel to DY. **3**

(ii) Show that $BCYX$ is a cyclic quadrilateral. **1**

(b) If A and B are both reflex angles, and given $\cos A = \dfrac{3}{5}$ and $\tan B = \dfrac{12}{5}$, find the exact value of $\sin(A - B)$. **3**

(c) In the expansion of $(1 - kx)^9$ the coefficient of x^6 is half that of the coefficient of x^5. Find the value of the constant k. **3**

(d) Taking $x = 2$ as the first approximation, use one application of Newton's method to obtain a closer approximation to the solution to $x = \sqrt[3]{9}$. **2**

Marks

Question 4 (12 marks) Use a SEPARATE writing booklet.

(a) Prove by mathematical induction that $\sum_{r=1}^{n} r \times r! = (n+1)! - 1$. **3**

(b) The acceleration of a particle P, moving in a straight line, is given by $\ddot{x} = 2x - 3$ where x metres is the displacement from the origin O. Initially the particle is at O and its velocity v is 2 metres per second.

(i) Show that the velocity v of the particle is $v^2 = 2x^2 - 6x + 4$. **2**

(ii) Calculate the velocity and acceleration of P at $x = 1$ and briefly describe the motion of P after it moves from $x = 1$. **2**

(c) The rate of change of the number of bees infected by a disease is given by the equation $\frac{dN}{dt} = N(200 - N)$, where N is the number of infected bees in the hive at time t years. There are 200 bees in the hive.

(i) If k is a constant, show that $N = \frac{200}{1 + ke^{-200t}}$ satisfies the above equation. **2**

(ii) If at time $t = 0$ one bee was infected, after how many days will half the colony be infected? **2**

(iii) Show that eventually all the bees will be infected. **1**

Marks

Question 5 (12 marks) Use a SEPARATE writing booklet

(a) Let $P(x) = -2x^3 + px^2 - qx + 5$.

(i) Show that if $P(x)$ is to have any stationary points, then $p^2 - 6q \geq 0$. **2**

(ii) Discuss the situation when $p^2 - 6q = 0$. **1**

(b) A camera, one kilometre away in the horizontal direction from where the Space Shuttle is being launched, is tracking the ascent of the Shuttle. Assume the Shuttle ascends vertically. **3**

Thirty seconds after the launch the Shuttle reaches a height, h, of 3240 metres and it is travelling at a speed of 230 metres per second.

The angle θ is the angle of elevation of the camera as it tracks the Shuttle. At what rate is θ increasing 30 seconds after the Shuttle is launched?

(c) Diana loves to play basketball. From the free throw line she makes 2 out of every 5 baskets that she throws. For every basket that she makes she scores one point.

(i) In her game last week she had 6 free throws. What is the probability that she scored 2 points? **1**

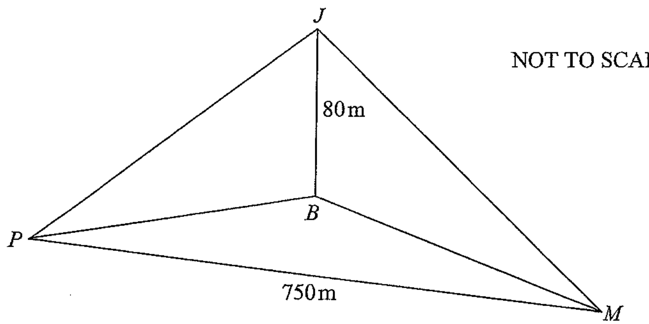
(ii) How many free throws would she need in one game so that the probability that she scores at least one point is 0.9978? **2**

Question 5 continues on page 7

Question 5 (continued) **Marks**

(d) Janus, J, is on top of an 80 metre cliff, watching the Sydney to Hobart yacht race. **3**

From the base of the cliff, B, directly below Janus, *Poseidon, P,* is on a bearing of 202° and *Majorca, M,* is on a bearing of 140°. *Majorca* is 750 m from *Poseidon* on a bearing of 110°.



Copy or trace this diagram into your writing booklet.

Find the angle of depression of *Poseidon, P,* from Janus, J.

End of Question 5

Marks

Question 6 (12 marks) Use a SEPARATE writing booklet

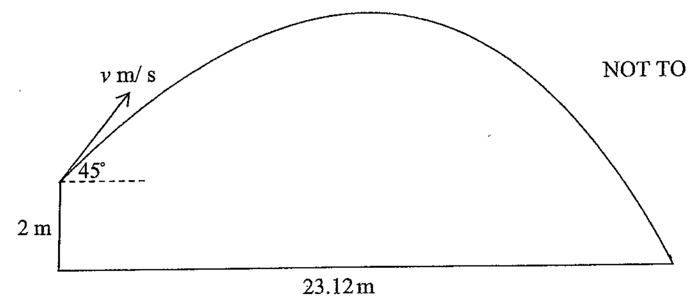
(a) Consider the function given by $f(x)=\dfrac{e^x}{x-1}$.

(i) Determine all vertical and horizontal asymptotes of the graph of $y=f(x)$. **2**

(ii) Find any stationary point(s) and sketch the graph of $y=f(x)$ including any intercepts with the coordinate axes. **3**

(iii) State the largest positive domain for which $f(x)$ has an inverse. **1**

(b) The world record for men's shot-put is 23.12 metres.
You may assume that the shot-put is projected at an initial velocity of v m/s from a height of 2 metres at an angle of projection of 45°, there is no air resistance and that the acceleration due to gravity is $10\,\text{m/s}^2$.



(i) Use integration to show that the equations of motion are **2**

$x=\dfrac{vt}{\sqrt{2}}$ and $y=-5t^2+\dfrac{vt}{\sqrt{2}}+2$.

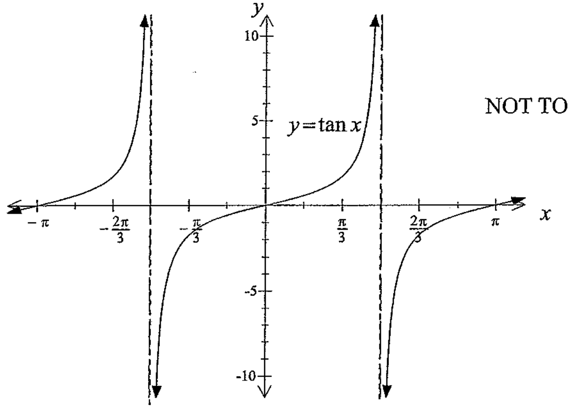
(ii) Find the minimum velocity v m/s at which the shot-put must be projected to achieve the world record distance. **2**

(iii) What is the maximum height that the shot-put reaches in its path if it is projected with this velocity? **2**

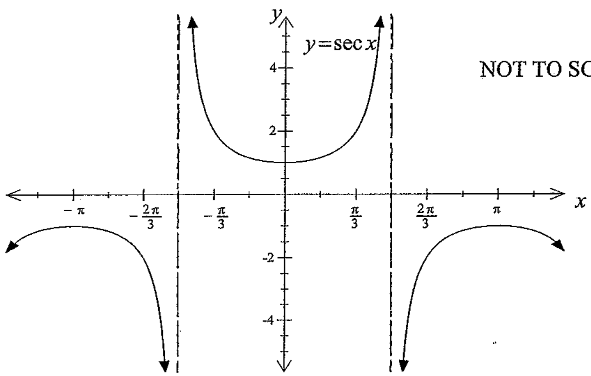
Marks

Question 7 (12 marks) Use a SEPARATE writing booklet

(a) The graphs shown are of $y=\tan x$ and $y=\sec x$ respectively.



NOT TO SCALE



NOT TO SCALE

(i) Prove that $\sec\theta-\tan\theta=\dfrac{1}{\sec\theta+\tan\theta}$. **2**

(ii) Explain why $0<\sec\theta-\tan\theta\le 1$ for $0\le\theta<\dfrac{\pi}{2}$. **2**

(iii) Solve the equation $\sec\theta-\tan\theta=\dfrac{1}{2}$ for $0\le\theta<\dfrac{\pi}{2}$. **3**

Question 7 continues on page 10

Marks

Question 7 (continued)

(b) (i) From a point $A(p,q)$ perpendiculars AP and AQ are drawn to meet the x and y axes at $P(p,0)$ and $Q(0,q)$ respectively. **1**

Find the equation of PQ.

(ii) Show that the condition for the line PQ to be a tangent to the parabola $x^2=4ay$ is $aq+p^2=0$. **3**

(iii) If the points $P(p,0)$ and $Q(0,q)$ move on the x and y axes respectively such that PQ is a tangent to the parabola $x^2=4ay$ then the point $A(p,q)$ traces out a curve as P and Q move. **1**

Find the locus of A.

End of paper

CATHOLIC SECONDARY SCHOOLS ASSOCIATION

2008 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

MATHEMATICS EXTENSION 1

Question 1 (12 marks)
(a) (2 marks)
Outcomes assessed: H5
Targeted Performance Bands: E2-E3

Criteria	Marks
• finds the correct primitive	1
• evaluates the integral correctly	1

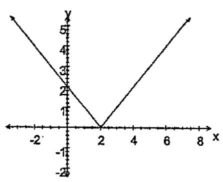
Sample Answer:

$$\int_0^{\frac{\pi}{8}} \sec^2 2x\,dx = \tfrac{1}{2}\big[\tan 2x\big]_0^{\frac{\pi}{8}}$$
$$= \tfrac{1}{2}(\tan\tfrac{\pi}{4} - \tan 0)$$
$$= \tfrac{1}{2}(1-0)$$
$$= \frac{1}{2}$$

(b) (i) (1 mark)
Outcomes assessed: P4
Targeted Performance Bands: E2-E3

Criteria	Mark
• draws the correct graph of $y=\lvert 2-x\rvert$, including intercepts	1

Sample Answer: $y=\lvert 2-x\rvert$



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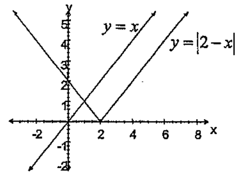
(b)(ii) (2 marks)
Outcomes assessed: PE2
Targeted Performance Bands: E2-E3

Criteria	Marks
• finds the point of intersection or other significant step towards solution	1
• writes the correct solution	1

Sample Answer:

point of intersection of $y=x$ and $y=\lvert 2-x\rvert$ is (1,1)

$\therefore$ from the graph $\lvert 2-x\rvert < x$ when $x>1$



(c) (2 marks)
Outcomes assessed: PE3
Targeted Performance Bands: E2-E3

Criteria	Marks
• uses the factor theorem with substitution of $x=-2$	1
• solves the equation to find k	1

Sample Answer:

$P(x) = x^2 - kx + 6$

$P(-2) = 4 + 2k + 6 = 0$ as $(x+2)$ is a factor

$k = -5$

(d) (3 marks)
Outcomes assessed: PE2
Targeted Performance Bands: E2-E3

Criteria	Marks
• finds the TWO gradients	1
• uses the correct formula for $\tan\theta$	1
• finds the correct angle	1

Sample Answer:

From the diagram the gradients of the lines are 2 and -1.

$$\tan\theta = \left|\frac{m_1 - m_2}{1+m_1m_2}\right|$$
$$= \left|\frac{2+1}{1-2}\right|$$
$$=3$$
$$\therefore \theta = 71°34'$$

(e) (2 marks)
Outcomes assessed: PE3
Targeted Performance Bands: E2-E3

Criteria	Marks
• establishes correct quadratic or other correct significant step towards solution	1
• finds full solution	1

Sample Answer:

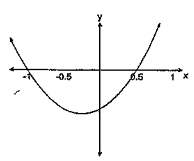
$\frac{3}{x+1} < 2$ multiply by $(x+1)^2$

$3(x+1) < 2(x+1)^2$

$2(x+1)^2 - 3(x+1) > 0$

$(x+1)(2x+2-3) > 0$

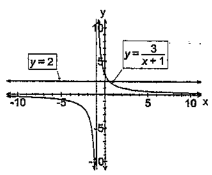
$(x+1)(2x-1) > 0$



Solution is $x<-1$ or $x>\frac{1}{2}$

$\therefore$ Jasi's solution is only partially correct.

or graphically:



ordinate of intersection $\frac{3}{x+1} = 2$

$3 = 2x+2$

$x = \frac{1}{2}$

From the graph $x>\frac{1}{2}$ satisfies the inequality
BUT the other branch of the hyperbola for $x<-1$ also satisfies the inequality.

Question 2 (12 marks)
(a) (2 marks)
Outcomes assessed: PE3
Targeted Performance Bands: E2-E3

Criteria	Mark
• finds sum and product of roots	1
• evaluates the expression correctly	1

Sample Answer:

$2x^3 - 5x^2 + 3x - 5 = 0$

$\alpha+\beta+\gamma = \frac{5}{2}$ $\alpha\beta+\alpha\gamma+\beta\gamma = \frac{3}{2}$ $\alpha\beta\gamma = \frac{5}{2}$

$\alpha^2\beta\gamma + \alpha\beta^2\gamma + \alpha\beta\gamma^2 = \alpha\beta\gamma(\alpha+\beta+\gamma)$

$= \frac{5}{2}\times\frac{5}{2}$

$= \frac{25}{4}$

(b) (i) (1 mark)
Outcomes assessed: P4
Targeted Performance Bands: E2-E3

Criteria	Mark
• gives the correct solutions	1

Sample Answer:

Given $f(x) = \frac{2x}{\sqrt{1-x^2}}$

$f(x)$ is undefined when $1-x^2 \le 0$

i.e. when $x\le -1$ or $x \ge 1$

(b) (ii) (3 marks)
Outcomes assessed: HE6
Targeted Performance Bands: E2-E3

Criteria	Marks
• rewrites the integral in term of u	1
• finds the new limits	1
• evaluates the integral correctly to at least $-2\left[\frac{\sqrt{3}}{2}-1\right]$ (correct numerical equivalence)	1

Sample Answer:

$$\int_0^{\frac{1}{2}} \frac{2x}{\sqrt{1-x^2}}\,dx = \int_0^{\frac{\pi}{6}} \frac{2\sin u}{\cos u}\cos u\,du \qquad x=\sin u,\quad dx=\cos u\,du$$

$$= 2\int_0^{\frac{\pi}{6}} \sin u\,du \qquad x=0 \Rightarrow u=0,\quad x=\tfrac{1}{2} \Rightarrow u=\tfrac{\pi}{6}$$

$$= -2\left[\cos u\right]_0^{\frac{\pi}{6}}$$

$$= -2\left(\cos\frac{\pi}{6} - \cos 0\right)$$

$$= -2\left[\tfrac{\sqrt{3}}{2}-1\right]$$

$$= 2-\sqrt{3}$$

(c) (i) (1 mark)
Outcomes assessed: HE4
Targeted Performance Bands: E2-E3

Criteria	Mark
• differentiates correctly	1

Sample Answer:

$$\frac{d}{dx}\left(\sin^{-1}x+\cos^{-1}x\right) = \frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-x^2}}$$

$$= 0$$

(c) (ii) (2 marks)
Outcomes assessed: HE4
Targeted Performance Bands: E2-E3

Criteria	Marks
• identifies that the primitive is a constant	1
• uses a suitable substitution, or otherwise, to show that the constant is $\frac{\pi}{2}$	1

Sample Answer:

Since the derivative is zero, $\sin^{-1}x+\cos^{-1}x = C$ (C is a constant)

Let $x=0 \Rightarrow \sin^{-1}0+\cos^{-1}0 = 0+\frac{\pi}{2}$

$$= \tfrac{\pi}{2}$$

(d) (i) (1 mark)
Outcomes assessed: PE3
Targeted Performance Bands: E2-E3

Criteria	Mark
• correct numerical expression for the answer	1

Sample Answer:

EXERCISE $\Rightarrow$ 8 letters with 3 Es

Number of arrangements $= \dfrac{8!}{3!} = 6720$

(d) (ii) (2 marks)
Outcomes assessed: PE3
Targeted Performance Bands: E2-E3

Criteria	Marks
• significant progress towards solution	1
• correct numerical expression for answer	1

Sample Answer:

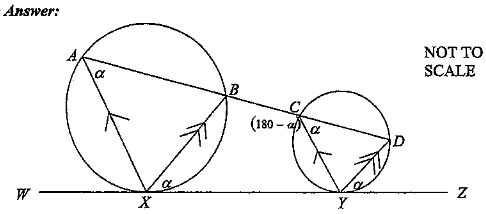
EXERCISE with C and R at the ends

Number of arrangements $= \dfrac{2!6!}{3!} = 240$

Question 3 (12 marks)
(a) (i) (3 marks)
Outcomes assessed: PE2, PE3
Targeted Performance Bands: E2-E3

Criteria	Marks
• correct application of alternate segment theorem or other correct step in explanation	1
• identifying corresponding angles or other significant progress in reasoning	1
• conclusion with reason	1

Sample Answer:



Given $\angle BXY = \alpha$

$\therefore \angle BAX = \alpha$ (angle between tangent and chord at the point of contact is equal to the angle in the alternate segment)

$\angle DCY = \alpha$ (corresponding to $\angle BAX$, $AX \parallel CY$)

$\angle DYZ = \alpha$ (angle between tangent and chord at the point of contact is equal to the angle in the alternate segment)

$\therefore \angle DYZ = \angle BXY$

$\therefore BX \parallel DY$ (corresponding angles are equal)

(a) (ii) (1 mark)
Outcomes assessed: PE2, PE3
Targeted Performance Bands: E2-E3

Criteria	Mark
• states the correct reason	1

Sample Answer:

$\angle BCY = 180 - \alpha$ (BCD is a straight line)

$\therefore \angle BCY + \angle BXY = 180^\circ$

$\therefore BCYX$ is a cyclic quadrilateral as one pair of opposite angles are supplementary

(b) (3 marks)
Outcomes assessed: PE2
Targeted Performance Bands: E2-E3

Criteria	Marks
• determines the correct value of $\sin A$, including the sign	1
• determines the correct value of $\sin B$ and $\cos B$, including the sign	1
• expands $\sin(A-B)$ and gives a correct numerical expression	1

Sample Answer:

$\cos A = \dfrac{3}{5} \quad \therefore \sin A = -\dfrac{4}{5}$ (A is reflex and in the 4th quad)

$\tan B = \dfrac{12}{5} \quad \therefore \sin B = -\dfrac{12}{13}$ and $\cos B = -\dfrac{5}{13}$ (B is reflex and in the 3rd quad)

$$\sin(A-B) = \sin A\cos B - \cos A\sin B$$

$$= \left(-\frac{4}{5}\right)\left(-\frac{5}{13}\right) - \left(\frac{3}{5}\right)\left(-\frac{12}{13}\right)$$

$$= \frac{56}{65}$$

(c) (3 marks)
Outcomes assessed: HE7
Targeted Performance Bands: E3-E4

Criteria	Marks
• writes down at least one correct term in x^6 or x^5	1
• writes down the correct equivalence	1
• finds the correct value of k (correct numerical equivalence)	1

Sample Answer:

Consider terms of the expansion of $(1-kx)^9$

Term in $x^5 = {}^9C_5(-kx)^5 = -{}^9C_5k^5x^5$ Term in $x^6 = {}^9C_6(-kx)^6 = {}^9C_6k^6x^6$

$$\therefore -{}^9C_5k^5 = 2\,{}^9C_6k^6$$

$$k = -\frac{{}^9C_5}{2\,{}^9C_6}$$

$$= -\frac{1}{2}\times\frac{9!}{5!4!}\times\frac{3!6!}{9!}$$

$$= -\frac{3}{4}$$

(d) (2 marks)
Outcomes assessed: PE3, HE7
Targeted Performance Bands: E2-E3

Criteria	Marks
• identifies the function	1
• uses Newton's Method to find correct approximation (correct numerical equivalence)	1

Sample Answer:

$x=\sqrt[3]{9}$

$\therefore x^3=9 \quad \Rightarrow f(x)=x^3-9 \quad \therefore f'(x)=3x^2$

Let $x_1=2 \quad \therefore x_2=x_1-\dfrac{f(x_1)}{f'(x_1)} \quad \Rightarrow x_2=2-\dfrac{-1}{12}$

$x_2=2\dfrac{1}{12}$

Question 4 (12 marks)
(a) (3 marks)
Outcomes assessed: HE2
Targeted Performance Bands: E3-E4

Criteria	Marks
• establishes the truth of $S(1)$	1
• establishes the correct relationship between $S(k)$ and $S(k+1)$	1
• deduces the required result	1

Sample Answer:

Let $S(n)$ be the statement $\sum_{r=1}^{n} r\times r!=(n+1)!-1$

Consider $S(1)$: $LHS=1\times1!$; $RHS=(1+1)!-1=1$.
Hence $S(1)$ is true

If $S(k)$ is true: $\sum_{r=1}^{k} r\times r!=(k+1)!-1 \quad *$

RTP $S(k+1)$ is true i.e. to prove $\sum_{r=1}^{k+1} r\times r!=(k+2)!-1$

$LHS=\sum_{r=1}^{k} r\times r! + (k+1)(k+1)!$

$=(k+1)!-1+(k+1)(k+1)!$ if $S(k)$, using $*$

$=(k+1)!\left(1+k+1\right)-1$

$=(k+1)!(k+2)-1$

Hence if $S(k)$ then $S(k+1)$ is true. Thus since $S(1)$ is true it follows by induction that $S(n)$ is true for positive integral n.

(b) (i) (2 marks)
Outcomes assessed: HE5
Targeted Performance Bands: E2-E3

Criteria	Marks
• sets up the correct differential equation or significant progress towards result	1
• finds the desired equation	1

Sample Answer:

$\ddot{x}=2x-3$

$\therefore \dfrac{d}{dx}\left(\tfrac{1}{2}v^2\right)=2x-3$

$\tfrac{1}{2}v^2=x^2-3x+c$

when $x=0, v=2 \quad \Rightarrow c=2$

$\therefore \tfrac{1}{2}v^2=x^2-3x+2$

$\therefore v^2=2x^2-6x+4$

(b) (ii) (2 marks)
Outcomes assessed: HE5
Targeted Performance Bands: E3-E4

Criteria	Marks
• calculates the correct velocity and acceleration	1
• describes the motion	1

Sample Answer:

at $x=1$, $v=0$ and $\ddot{x}=-1$ m/s^2

After the object comes to rest at $x=1$ it then moves towards the origin, and will continue moving in a negative direction.

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(c) (i) (2 marks)
Outcomes assessed: HE3
Targeted Performance Bands: E3-E4

Criteria	Marks
• differentiates correctly	1
• proves correct result	1

Sample Answer:

$N=\dfrac{200}{1+ke^{-200t}}=200\left(1+ke^{-200t}\right)^{-1}$

$\dfrac{dN}{dt}=-200(1+ke^{-200t})^{-2}(-200ke^{-200t})$

$=\dfrac{200}{1+ke^{-200t}}\left(\dfrac{200ke^{-200t}}{1+ke^{-200t}}\right)$

$=N\left(\dfrac{200+200ke^{-200t}-200}{1+ke^{-200t}}\right)$

$=N\left(\dfrac{200(1+ke^{-200t})}{(1+ke^{-200t})}-\dfrac{200}{1+ke^{-200t}}\right)$

$=N(200-N)$

(c) (ii) (2 marks)
Outcomes assessed: HE3
Targeted Performance Bands: E3-E4

Criteria	Marks
• finds the value of k	1
• finds t (correct numerical equivalence)	1

Sample Answer:

when $t=0$, $N=1$ i.e. $1=\dfrac{200}{1+k} \quad \therefore k=199$

$N=\dfrac{200}{1+199e^{-200t}}$ half the colony infected i.e. $N=100$

$100=\dfrac{200}{1+199e^{-200t}}$

$1+199e^{-200t}=2$

$e^{-200t}=\dfrac{1}{199}$

$-200t=\log_e\dfrac{1}{199}$

$\therefore t=-\dfrac{1}{200}\log_e\dfrac{1}{199}$

$t=0.0265$ years or 9.66 days

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(c) (iii) (1 mark)
Outcomes assessed: HE7
Targeted Performance Bands: E3-E4

Criteria	Mark
• shows that the limiting value of N is 200	1

Sample Answer:

as $t\to\infty \quad e^{-200t}\to 0$

$\therefore N\to\dfrac{200}{1+0}=200$ i.e. eventually all the bees will be infected.

Question 5 (12 marks)
(a) (i) (2 marks)
Outcomes assessed: H5, HE7
Targeted Performance Bands: E2-E3

Criteria	Marks
• differentiates and determines the correct quadratic	1
• identifies that $\Delta\geq 0$ and sets up the correct inequality	1

Sample Answer:

$P(x)=-2x^3+px^2-qx+5$

$P'(x)=-6x^2+2px-q=0$ for stationary points

For this quadratic to have real solutions $\Delta\geq 0$

i.e. $4p^2-24q\geq 0$

$p^2-6q\geq 0$

(a) (ii) (1 mark)
Outcomes assessed: H5, HE7
Targeted Performance Bands: E3-E4

Criteria	Mark
• gives the correct conclusion	1

Sample Answer:

When $p^2-6q=0$ $P'(x)$ has a double root and there is only one stationary point which would be a horizontal point of inflexion.

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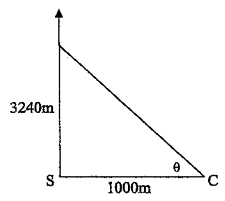
(b) (3 marks)
Outcomes assessed: HE5, HE7
Targeted Performance Bands: E3-E4

Criteria	Marks
• establishes $\frac{dh}{d\theta}$	1
• evaluating θ at $t = 30$ s, or other significant progress towards the result such as correct use of chain rule	1
• correct answer	1

Sample Answer:



$\frac{dh}{dt} = 230\text{m/s}$

$\tan\theta = \frac{h}{1000}$

$h = 1000\tan\theta$

$\frac{dh}{d\theta} = 1000\sec^2\theta$

at $t = 30$ seconds, $h = 3240$ m $\Rightarrow \tan\theta = \frac{3240}{1000} \Rightarrow \theta = 1.271$ radians

$$\frac{d\theta}{dt} = \frac{d\theta}{dh} \times \frac{dh}{dt}$$
$$= \frac{\cos^2 1.271}{1000} \times 230$$
$$= 0.02 \text{ rads/sec}$$

(c) (i) (1 mark)
Outcomes assessed: HE3
Targeted Performance Bands: E2-E3

Criteria	Mark
• gives the correct answer (correct numerical equivalence)	1

Sample Answer:

$$P(\text{a basket}) = \frac{2}{5}$$
$$P(\text{2 points}) = {}^6C_2\left(\frac{2}{5}\right)^2\left(\frac{3}{5}\right)^4$$
$$= \frac{972}{3125} \quad (\text{or } 0.31104)$$

(c) (ii) (2 marks)
Outcomes assessed: HE3, HE7
Targeted Performance Bands: E3-E4

Criteria	Marks
• sets up correct inequality or other significant progress	1
• gives the correct solution, rounding to the nearest whole number	1

Sample Answer:

$P(\text{at least one}) = 1 - P(\text{none})$
$= 1 - 0.6^n$

$\therefore 1 - 0.6^n \geq 0.9978$

$0.6^n \leq 0.0022$

take logs of both sides

i.e. $n\ln 0.6 \leq \ln 0.0022$

$n \geq \frac{\ln 0.0022}{\ln 0.6}$

$n \geq 11.979$

$n \geq 12$

Diana would need 12 free throws.

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(d) (3 marks)
Outcomes assessed: PE6, HE7
Targeted Performance Bands: E2-E3

Criteria	Marks
• some progress towards result, e.g. finds the missing angles	1
• significant progress towards result, e.g. finding BP (correct numerical equivalence)	1
• finds the angle of depression	1

Sample Answer:

$\angle BPM = 110^\circ - 22^\circ = 88^\circ$

$\angle PBM = 202^\circ - 140^\circ = 62^\circ$

$\therefore \angle PMB = 30^\circ$

$\therefore \frac{BP}{\sin 30^\circ} = \frac{750}{\sin 62^\circ}$

$\therefore BP = \frac{750\sin 30^\circ}{\sin 62^\circ}$

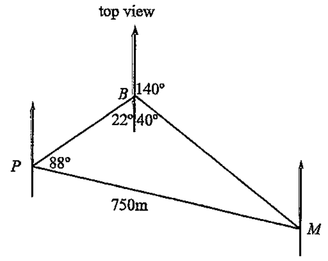
$= 424.71$ (2 d.p.)

angle of elevation $= \tan^{-1}\left(\frac{80}{BP}\right)$

$= 10^\circ 40'$

$\therefore$ angle of depression is $10^\circ 40'$

top view



Question 6 (12 marks)
(a) (i) (2 marks)
Outcomes assessed: H5
Targeted Performance Bands: E3-E4

Criteria	Marks
• finds the vertical asymptote	1
• finds the horizontal asymptote	1

Sample Answer:

$f(x) = \frac{e^x}{x-1}$

vertical asymptote at $x = 1$

horizontal asymptote : $x \to \infty,\ f(x) \to \infty;\quad x \to -\infty,\ f(x) \to 0^-$

$\therefore\ y = 0$ is a horizontal asymptote as $x \to -\infty$

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(a) (ii) (3 marks)
Outcomes assessed: PE5, PE6, H5
Targeted Performance Bands: E3-E4

Criteria	Marks
• finds the stationary point	1
• identifies intercept	1
• sketches the correct function	1

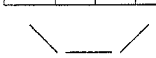
Sample Answer:

$$f'(x) = \frac{(x-1)e^x - e^x}{(x-1)^2}$$
$$= \frac{e^x(x-2)}{(x-1)^2}$$

$f'(x) = 0$ when $x = 2$ and $y = e^2$

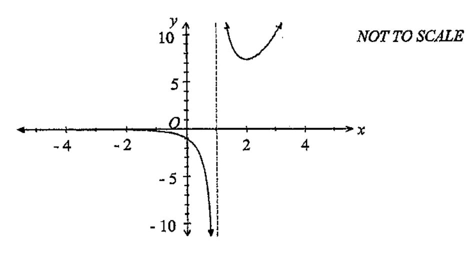
testing nature:

x	2^-	2	2^+
$f'(x)$	–	0	+



$\therefore (2, e^2)$ is a minimum point

when $x = 0,\ y = \frac{e^0}{0-1} = -1 \quad \therefore\ y$ intercept is $(0, -1)$



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(a) (iii) (1 mark)
Outcomes assessed: HE5
Targeted Performance Bands: E2-E3

Criteria	Marks
• gives correct domain	1

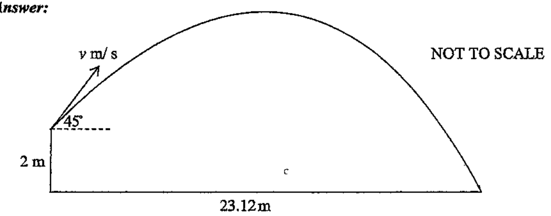
Sample Answer:

For an inverse function domain is $x \geq 2$

(b) (i) (2 marks)
Outcomes assessed: HE3
Targeted Performance Bands: E2-E3

Criteria	Marks
• finds the correct differential equations for the horizontal motion	1
• finds solves the correct differential equations for the vertical motion	1

Sample Answer:



Horizontal

$\ddot{x} = 0$
$\dot{x} = c_1$
at $t = 0$, $\dot{x} = v\cos 45° \Rightarrow c_1 = v\cos 45°$
$\therefore \dot{x} = \dfrac{v}{\sqrt{2}}$
$x = \dfrac{vt}{\sqrt{2}} + c_2$
at $t = 0$, $x = 0 \Rightarrow c_2 = 0$
$x = \dfrac{vt}{\sqrt{2}}$

Vertical

$\ddot{y} = -10$
$\dot{y} = -10t + c_3$
at $t = 0$, $\dot{y} = v\sin 45° \Rightarrow c_3 = v\sin 45°$
$\therefore \dot{y} = -10t + \dfrac{v}{\sqrt{2}}$
$y = -5t^2 + \dfrac{vt}{\sqrt{2}} + c_4$
at $t = 0$, $y = 2 \Rightarrow c_4 = 2$
$y = -5t^2 + \dfrac{vt}{\sqrt{2}} + 2$

(b) (ii) (2 marks)
Outcomes assessed: HE3
Targeted Performance Bands: E3-E4

Criteria	Marks
• determines the Cartesian equation of motion or other significant progress	1
• substitutes and solves for v	1

Sample Answer:

$t = \dfrac{\sqrt{2}x}{v}$

$\therefore y = -5 \times \dfrac{2x^2}{v^2} + x + 2$

at world record range $x = 23.12$ and $y = 0$

$\therefore 0 = -\dfrac{10 \times 23.12^2}{v^2} + 25.12$

$\dfrac{5345.344}{v^2} = 25.12$

$v^2 = 212.7923567$
$v = 14.59$ m/s (2 decimal places)

(b) (iii) (2 marks)
Outcomes assessed: HE3
Targeted Performance Bands: E3-E4

Criteria	Marks
• finds the value of t in terms of v	1
• substitutes into y and solves	1

Sample Answer:

maximum height when $\dot{y} = 0$

i.e. $t = \dfrac{v}{10\sqrt{2}}$

$y = -5 \times \dfrac{v^2}{10^2 \times 2} + \dfrac{v^2}{10 \times 2} + 2$

$= \dfrac{v^2}{20} - \dfrac{v^2}{40} + 2$

$= \dfrac{v^2}{40} + 2$

$= \dfrac{212.7923567}{40} + 2$

$= 7.32$ m (2 decimal places)

Question 7 (12 marks)
(a) (i) (2 marks)
Outcomes assessed: H5, PE2
Targeted Performance Bands: E3-E4

Criteria	Marks
• Showing $\dfrac{\sec^2\theta - \tan^2\theta}{\sec\theta + \tan\theta}$ or other significant progress	1
• completing the proof	1

Sample Answer:

RTP $\sec\theta - \tan\theta = \dfrac{1}{\sec\theta + \tan\theta}$

LHS

$= \dfrac{(\sec\theta - \tan\theta)(\sec\theta + \tan\theta)}{(\sec\theta + \tan\theta)}$

$= \dfrac{\sec^2\theta - \tan^2\theta}{\sec\theta + \tan\theta}$

$= \dfrac{1}{\sec\theta + \tan\theta}$ = RHS

(a) (ii) (2 marks)
Outcomes assessed: PE2, HE7
Targeted Performance Bands: E3-E4

Criteria	Marks
• establishing $\sec\theta + \tan\theta \geq 1$ or other significant progress	1
• justifying the inequality	1

Sample Answer:

From the graphs $\sec\theta \geq 1$ and $\tan\theta \geq 0$ for $0 \leq \theta < \dfrac{\pi}{2}$

$\therefore \sec\theta + \tan\theta \geq 1$

$\therefore 0 < \dfrac{1}{\sec\theta + \tan\theta} \leq 1$

i.e. $0 < \sec\theta - \tan\theta \leq 1$ using (i)

(a) (iii) (3 marks)
Outcomes assessed: HE7
Targeted Performance Bands: E3-E4

Criteria	Marks
• progress towards obtaining a quadratic equation e.g. establishing $2\cos\theta = 1 + \sin\theta$	1
• obtaining the correct quadratic in terms of $\sin\theta$ or equivalent progress	1
• correct solution	1

Sample Answer:

$\sec\theta - \tan\theta = \dfrac{1}{2}$

i.e. $\dfrac{1}{\sec\theta + \tan\theta} = \dfrac{1}{2}$

$\therefore \sec\theta + \tan\theta = 2$

$\dfrac{1}{\cos\theta} + \dfrac{\sin\theta}{\cos\theta} = 2$

i.e. $2\cos\theta = 1 + \sin\theta$

Square and solve the quadratic:

$4\cos^2\theta = 1 + 2\sin\theta + \sin^2\theta$
$4(1 - \sin^2\theta) = 1 + 2\sin\theta + \sin^2\theta$
$5\sin^2\theta + 2\sin\theta - 3 = 0$
$(5\sin\theta - 3)(\sin\theta + 1) = 0$

since $\sin\theta$ is positive in the interval, $0 \leq \theta < \dfrac{\pi}{2}$

$\sin\theta = \dfrac{3}{5}$

i.e. $\theta = 0.644$ radians

OR

• progress towards use of auxiliary angle method e.g. establishing $2\sin\theta+\cos\theta=2$	1
• obtaining the correct value for R or α using the auxiliary angle method	1
• correct solution	1

$\sec\theta-\tan\theta=\frac{1}{2}$

$\frac{1}{\cos\theta}-\frac{\sin\theta}{\cos\theta}=\frac{1}{2}$

$2-2\sin\theta=\cos\theta$

$2\sin\theta+\cos\theta=2$

let $2\sin\theta+\cos\theta\equiv R\sin(\theta+\alpha)$ where R is positive and α is acute

i.e. $2\sin\theta+\cos\theta\equiv R\sin\theta\cos\alpha+R\cos\theta\sin\alpha$

$\Rightarrow R\cos\alpha=2$ and $R\sin\alpha=1$

i.e. $\tan\alpha=\frac{1}{2}$ and $R=\sqrt{5}$

$\therefore \sqrt{5}\sin\left(\theta+\tan^{-1}\frac{1}{2}\right)=2$

$\sin\left(\theta+\tan^{-1}\frac{1}{2}\right)=\frac{2}{\sqrt{5}}$

$\left(\theta+\tan^{-1}\frac{1}{2}\right)=\sin^{-1}\frac{2}{\sqrt{5}}$

$\theta=\sin^{-1}\frac{2}{\sqrt{5}}-\tan^{-1}\frac{1}{2}$

$\theta=0.644$ radians

OR

From simplifying to $2\cos\theta=1+\sin\theta$ i.e. $2\cos\theta-\sin\theta=1$

let $2\cos\theta-\sin\theta\equiv R\cos(\theta+\alpha)$ where R is positive and α is acute

i.e. $2\cos\theta-\sin\theta\equiv R\cos\theta\cos\alpha-R\sin\theta\sin\alpha$

$\Rightarrow R\cos\alpha=2$ and $R\sin\alpha=1$

i.e. $\tan\alpha=\frac{1}{2}$ and $R=\sqrt{5}$

$\therefore \sqrt{5}\cos\left(\theta+\tan^{-1}\frac{1}{2}\right)=1$

$\cos\left(\theta+\tan^{-1}\frac{1}{2}\right)=\frac{1}{\sqrt{5}}$

$\left(\theta+\tan^{-1}\frac{1}{2}\right)=\cos^{-1}\frac{1}{\sqrt{5}}$

$\theta=\cos^{-1}\frac{1}{\sqrt{5}}-\tan^{-1}\frac{1}{2}$

$\theta=0.644$ radians

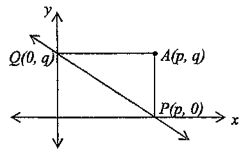
(b) (i) (1 mark)

Outcomes assessed: HE7

Targeted Performance Bands: E2-E3

Criteria	Mark
• finds the correct equation of PQ in any form	1

Sample Answer:



$y=-\frac{q}{p}x+q$

$py=-qx+pq$

$\frac{x}{p}+\frac{y}{q}=1$

(b) (ii) (3 marks)

Outcomes assessed: HE7

Targeted Performance Bands: E3-E4

Criteria	Marks
• determines the quadratic or other significant progress towards the solution	1
• uses $\Delta=0$ when PQ is a tangent to the curve	1
• establishes the relationship	1

Sample Answer:

If PQ is a tangent to the parabola then there is one point of intersection.

solve $\frac{x}{p}+\frac{y}{q}=1$ and $y=\frac{x^2}{4a}$ simultaneously

$\frac{x}{p}+\frac{x^2}{4aq}=1$

$4aqx+px^2=4apq$

$px^2+4aqx-4apq=0$

for PQ to be a tangent then $\Delta=0$ in this quadratic (i.e. only one root/solution)

i.e. $16a^2q^2+16ap^2q=0$

$16aq(aq+p^2)=0$

i.e. $aq+p^2=0$

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(b) (iii) (1 mark)

Outcomes assessed: HE7

Targeted Performance Bands: E3-E4

Criteria	Mark
• finds the locus	1

Sample Answer:

Coordinates of A $\Rightarrow$ $x=p$ and $y=q$

$\therefore$ since $aq+p^2=0$ from (ii) then $ay+x^2=0$

i.e. $x^2=-ay$ is the locus of A

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